Finding the refraction law using the ant colony algorithm and the Fermat's principle

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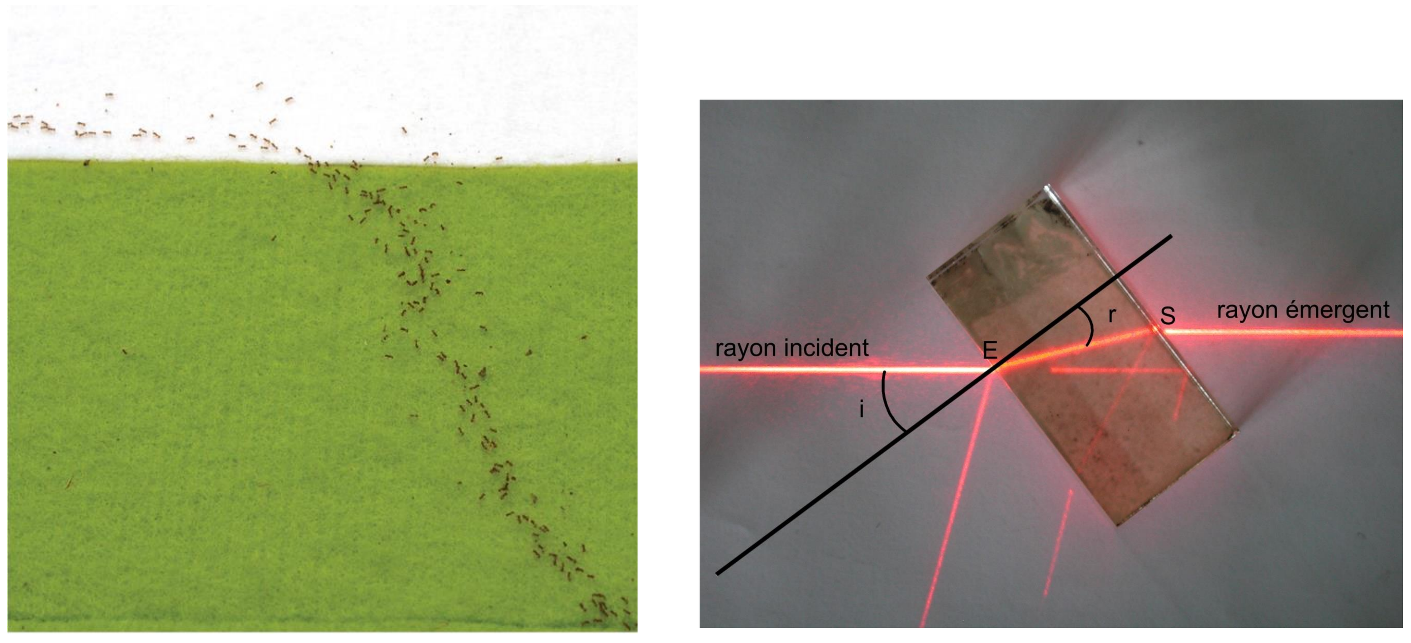
The Ant Colony Algorithm (ACA) is applied to classical optics to find the Snell law of refraction back. For sufficiently small simulation space, the algorithm rapidly converges to the optimal solution and the refraction law is recovered.

1. Context

The ACA is an optimization algorithm, proposed by Dorigo in 1996 [1], which is based on the collective behavior of ants which are able to find the shortest path between their colony and food [2] (figure 1A). Each ant leaves some pheromones on its way which influences the decisions of the other ants. After a while, the shortest path is covered by the largest amount of pheromones and most of the ants follow this optimal way. This algorithm has been applied with success to a large number of optimization problems such as the travelling salesman problem.



(A) **(B)**

Figure 1 – (A) Ants follow the fastest path and are submitted to a « refraction » when they cross a medium which influences their speed [2] (B) Refraction of a laser beam

In classical optics, the Fermat principle states that light always chooses the path (between one departure and one arrival points) with the least duration time. This principle leads to the refraction law (figure 1B) : when a ray of light crosses two media with different refractive indexes, a change in its direction occurs which can be characterized by the Snell law

$$n_1 \sin i = n_2 \sin r$$

where n_1, n_2 are the refractive indexes of medium 1 and 2, i and r are the angles of incidence and refraction. Refraction indexes can be linked to the speed of light v in the related medium through the relation $n = c/v$ where c is the speed of light in the vacuum.

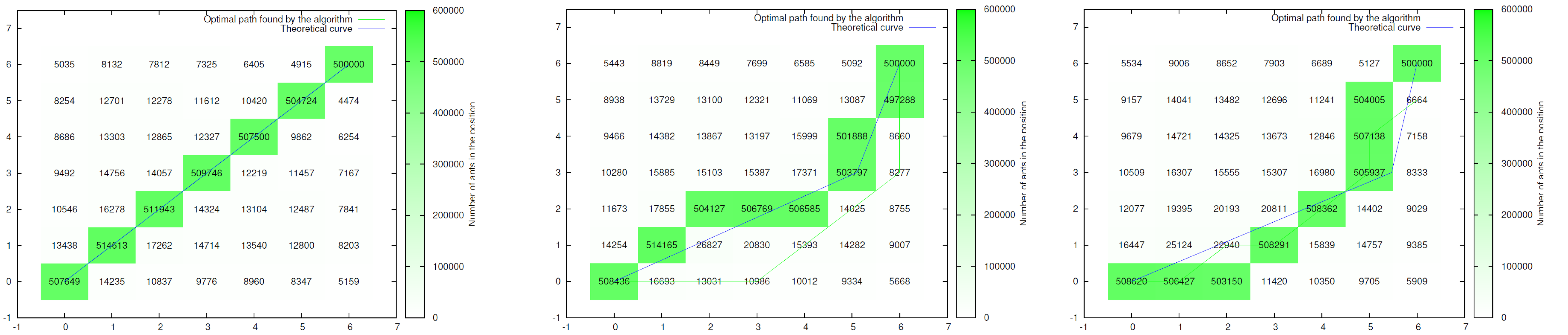
The goal of this work is to apply the ACA on the Fermat's principle and compare the solution proposed by the algorithm with the prediction of the Snell law.

2. Algorithm

1. Initial conditions : the simulation space is discretized into a square grid and is divided into two different media. Arrival and departure points are defined by the user. All the transition probabilities are initially equal.
2. For each iteration, a number n_a of ants independently travels in the grid.
3. At each time step, each ant, initially at position i, chooses a new neighbored position j with a probability p_{ij} given by $p_{ij} \propto \tau_{ij}^{\alpha}$ where τ_{ij} is the quantity of pheromones associated to this transition. The path of each ant is stopped when it arrives at the final position.
4. At the end of the path, the old pheromones are evaporated with a rate ρ and are then updated depending on the ant path : $\tau_{ij} \rightarrow \tau_{ij} + \left(\frac{Q}{T}\right)^{\chi}$ where T is the duration of the ant path.
5. At the end of the program, the least duration path and a grid providing the frequency at which the ants get to each position are given.

3. Results

Simulation grids were composed of 7x7 points and simulation parameters were chosen as $\alpha = 1$, $\chi = 2$, $Q = 1/7n_a$, $n_a = 50$, $\rho = 25\%$ and 10000 iterations. The algorithm was first tested for a homogeneous medium (n_1=n_2) : as expected, the optimal path is given by a straight line between the arrival and the departure points. Different refraction indexes (n_1/n_2 ratios of 1/3 and 1/5) were then tested showing that in average the ants seem to follow the optimal path close to the theoretical prediction. However the shortest path found by the simulation is not always close to the theoretical one. Our simulations and conclusions are limited by the small size of our grid. Larger grids would need more time computation to reach a good precision.



(A) **(B)** **(C)**

Figure 2 – Simulation results for n_1/n_2 ratios of (A) 1/1, (B) 1/3 and (C) 1/5

4. Conclusion

Our preliminary results for the application of ACA on light refraction are promising and show that it could provide a good method for more complex optic problems or more general optimization problems involving a path optimization. However, current computers are not fast enough to provide a sufficiently good precision.

[1] Dorigo, M., Maniezzo, V., Colorni, A., "Ant system: optimization by a colony of cooperating agents", *IEEE Transactions on Systems, Man, and Cybernetics, Part B (Cybernetics)*, *26*(1), pp 29-41, 1996.
[2] Oettler, J., Schmid, V. S., Zankl, N., Rey, O., Dress, A., Heinze, J., "Fermat's principle of least time predicts refraction of ant trails at substrate borders", *PloS one*, *8*(3), e59739, 2013.