Suppression of low-frequency fluctuations and stabilization of a semiconductor laser subjected to optical feedback from a double cavity: theoretical results

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We demonstrate numerically that low-frequency fluctuations (LFF's) observed in a laser diode subjected to a first optical feedback with a short delay are suppressed by means of an adequate second optical feedback.   The general idea of this technique is based on the observation that second feedback can suppress the antimodes that are responsible for the crises in the LFF regime.   Furthermore, we observe that the second optical feedback can steer an unstable laser that is biased near threshold into a stable regime. © 1999 Optical Society of America

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A semiconductor laser subjected to external optical feedback can present a large variety of dynamic behaviors, such as periodic and quasi-periodic oscillations, chaos,[1] coherence collapse,[2] and low-frequency fluctuations[3] (LFF's) that degrade the laser characteristics. LFF's occurs mainly when a laser that is subjected to moderate feedback is biased near the solitary laser threshold. This regime is characterized by a succession of sudden dropouts and slow recoveries of the laser's mean intensity.

From an application point of view it is interesting to investigate practical methods of suppressing or controlling chaos and LFF's. Among several methods (Refs. 4–6 and references therein) that are available, a dynamic targeting technique was proposed by Wieland *et al.*,[4] who showed numerically that a single-mode semiconductor laser that suffers from coherence collapse when subjected to optical feedback can be steered into the maximum gain mode by adjustment of the feedback phase as the feedback strength is varied. Because the maximum gain mode never undergoes a Hopf bifurcation,[7] the laser operates in a stable regime. Recently Hohl and Gavrielides[5] applied a dynamic targeting technique to the experimental control of a chaotic semiconductor laser biased near threshold. Although Wieland *et al.* proposed adjusting the feedback phase by accurate positioning of the reflection source[4] (generally a mirror) within one half of an optical wavelength, they achieved adjustment of the feedback phase by slightly varying the pump current.[5]

One major difficulty of the targeting technique is that, because of spontaneous-emission noise and the smallness of the attraction basin, the laser can be kicked out of the maximum gain mode at the beginning of the procedure that was proposed in Ref. 4 and jump to the nearest mode, which is often but not always stable. Furthermore, in both studies it was assumed that either the positioning of the mirror[4] or the laser

pump current[5] could be adjusted, whereas in practical cases these parameters cannot be modified easily. It is therefore interesting to investigate alternative methods of chaos and LFF suppression that do not require modifications of the laser or feedback parameters. A method in which a second optical feedback[6] was used was thus proposed for stabilizing a chaotic laser diode that was pumped far above threshold (up to twice the threshold current of the solitary laser).

In this Letter we investigate specifically the stabilizing effects of a second optical feedback, as proposed in Ref. 6, but with particular attention to the bifurcation diagram of the steady-state solutions. We show numerically for what is believed to be the first time that a second optical feedback can suppress LFF that is observed in a laser diode biased near threshold and subjected to a first optical feedback with a short delay. In addition, we show that one can steer a laser biased near threshold to lock it into the stable maximum gain mode without any modifications to the laser or the first-feedback parameters. Because the positioning of the second external mirror does not need to be accurate, our technique is easier from an experimental point of view than dynamic targeting. Moreover, our technique works whatever the first-feedback strength is.

The key ideas of our work are based on the following observations. According to Sano,[8] the dropouts of the optical power in the LFF regime are caused by crises, i.e., collisions of the system trajectory in phase space with antimodes. Each crisis is preceded by chaotic itinerancy of the system trajectory among the attractor ruins of external-cavity modes, with a drift toward the maximum gain mode. For small feedback delays (as in the case under study), a few antimodes are responsible for the crises. These observations suggest that one might be able to suppress LFF by shifting these antimodes away from the other

external-cavity modes or, better yet, by inducing them to disappear. Without modifying any parameters of the first optical feedback or the laser diode, one can achieve this by means of a second optical feedback.

Moreover, using a single-optical-feedback configuration with a short delay, Hohl and Gavrielides[5,9] recently observed that the laser underwent a cascade of bifurcations as the feedback strength was increased, exhibiting alternately stable and unstable behaviors, such as chaos and LFF. In the stable regions of the bifurcation cascade the laser was locked into the stable maximum gain mode. The second idea of our work is that such stable regions might also be observed in a double-cavity configuration as the second-feedback strength is increased, whatever the strength of the first is. Consequently laser stabilization will always be achieved in several ranges of the second-feedback strength.

A single-mode semiconductor laser that is subjected to weak or moderate external optical feedback is described by the Lang–Kobayashi equations.[10] Extended to the problem of a laser subjected to optical feedback from a double cavity (see Fig. 1), those equations can be formulated in dimensionless form as[11]

$$dE/ds = (1 + i\alpha)NE + \kappa_1 E(s - \tau_1)\exp(-i\Omega\tau_1)$$
$$+ \kappa_2 E(s - \tau_2)\exp(-i\Omega\tau_2),$$
$$T(dN/ds) = P - N - (1 + 2N)\,|E|^2, \tag{1}$$

where s is time measured in units of the photon lifetime, $E(s) = A(s)\exp[i\phi(s)]$ is the normalized slowly varying complex electric field, $N(s)$ is the normalized excess carrier number, κ_1 and κ_2 are the normalized feedback strengths of the first and second external cavities, respectively, τ_1 and τ_2 are the ratios of the round-trip time to the photon lifetime for both external cavities, α is the linewidth-enhancement factor, and Ω is the dimensionless angular frequency of the solitary laser. P is the dimensionless pumping current above the solitary laser threshold, and T is the ratio of the carrier lifetime to the photon lifetime. Here we choose parameter values of a 780-nm laser diode, first-feedback phase $\Omega\tau_1$, and delay τ_1 that are identical to those described in Ref. 5: $\alpha = 4, P = 0.001, T = 1000,$ $\Omega\tau_1 = -1.45,$ and $\tau_1 = 1000.$ We also choose the first feedback strength $\kappa_1 = 4.6 \times 10^{-3}$ at which LFF is observed in a single-feedback configuration (i.e., $\kappa_2 = 0$).[5] The second cavity round-trip time and the feedback phase are $\tau_2 = 200$ and $\Omega\tau_2 = 0.8$, respectively.

The steady-state solutions of Eqs. (1) are of the form $E = A_s\exp[i(\Delta - \Omega)s]$ and $N = N_s$, where Δ is a stationary angular frequency and the corresponding A_s and N_s are constants:

$$\Delta = \Omega - \kappa_1[\alpha\cos(\Delta\tau_1) + \sin(\Delta\tau_1)]$$
$$- \kappa_2[\alpha\cos(\Delta\tau_2) + \sin(\Delta\tau_2)], \tag{2}$$

$$A_s{}^2 = \frac{P - N_s}{1 + 2N_s},$$

$$N_s = -\kappa_1\cos(\Delta\tau_1) - \kappa_2\cos(\Delta\tau_2). \tag{3}$$

Figure 2(a), obtained from Eq. (2), shows the typical evolution of the product of the stationary angular fre-

quencies and the first-feedback delay $\Delta\tau_1$ with respect to κ_2. New steady-state solutions are created in pairs by a saddle-node bifurcation with one external-cavity mode and one antimode; the latter is always unstable. Similarly to the single-feedback case,[12] antimodes also satisfy the condition $d\Omega/d\Delta < 0$ in the double-feedback case. But, contrary to the single-feedback case, pairs of steady-state solutions can disappear when the feedback strength κ_2 of the second cavity increases for a given feedback strength κ_1 of the first cavity. In Fig. 2(b) we present the corresponding bifurcation diagram of the phase-difference function $\phi(t) - \phi(t - \tau_1) + \Omega\tau_1$ for $\kappa_1 = 4.6 \times 10^{-3}$, with κ_2 as the bifurcation parameter. The choice of this phase-difference function for the bifurcation diagram is convenient, since it reduces to $\Delta\tau_1$ for stationary behaviors and can be directly compared with Fig. 2(a). For the value of κ_1 chosen here, LFF is observed in the single-feedback configuration (i.e., $\kappa_2 = 0$).[5] Figure 3(a) shows chaotic itinerancy of the phase trajectory among successive external-cavity modes, followed by collision of the trajectory with the antimode that corresponds to the stable maximum gain mode.
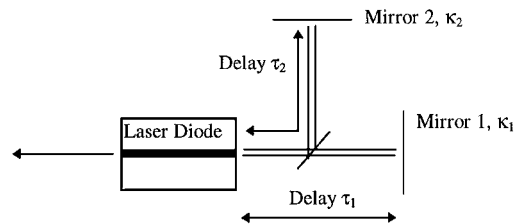

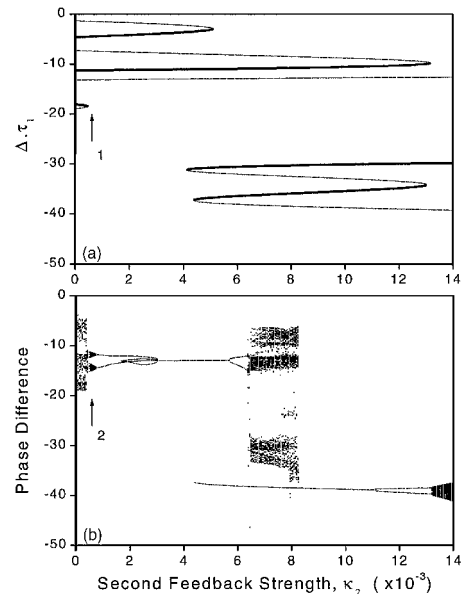Fig. 1.   Schematic configuration of a laser diode subjected to optical feedback from a double cavity.

Fig. 2. (a) Stationary angular frequencies Δ as a function of κ_2. Thick curves, antimodes; thin curves, external-cavity modes. (b) Bifurcation diagram of the phase-difference function $\phi(t) - \phi(t - \tau_1) + \Omega\tau_1$. The second-feedback strength κ_2 is the bifurcation variable. Arrow 1 indicates the disappearance of the antimode that is responsible for the crisis, and arrow 2 indicates the corresponding LFF suppression.
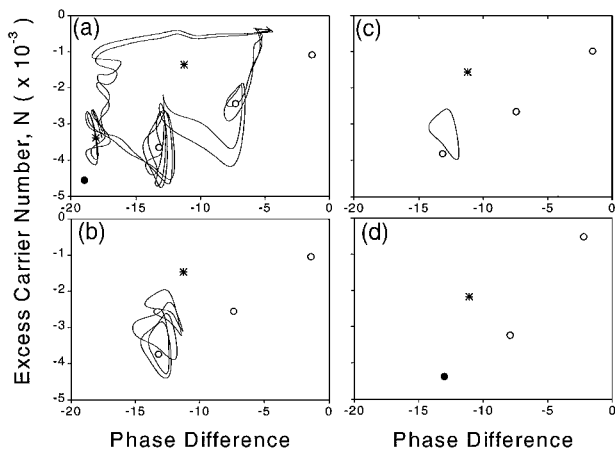
Fig. 3. Phase trajectories observed in the space $[\phi(t) - \phi(t - \tau_1) + \Omega\tau_1, N(t)]$. Asterisks, antimodes; open circles, unstable external-cavity modes; filled circles, stable external-cavity modes. (a) LFF for $\kappa_2 = 0$. (b) Quasiperiodic behavior with frequency locking for $\kappa_2 = 0.5 \times 10^{-3}$. (c) Limit cycle corresponding to periodic behavior for $\kappa_2 = 1 \times 10^{-3}$. (d) Stationary behavior for $\kappa_2 = 4 \times 10^{-3}$: The laser locks into a stable external-cavity mode. The first-feedback strength is $\kappa_1 = 4.6 \times 10^{-3}$ in all cases.

The trajectory then is repelled to higher values of the excess carrier number N, where the chaotic itinerancy toward lower values of the phase-difference function starts again.

The maximum gain mode and its corresponding antimode collide and disappear for $\kappa_2 = 0.45 \times 10^{-3}$ [arrow 1 in Fig. 2(a)]. As this pair of modes disappears, LFF, which is observed from $\kappa_2 = 0$ to $\kappa_2 = 0.45 \times 10^{-3}$, suddenly stops, and chaotic behavior is observed [arrow 2 in Fig. 2(b)] because the phase trajectory can no longer collide with the antimode and stays close to the nearest mode. This chaotic behavior is observed in a very small range of κ_2, and, as κ_2, increases further, quasi-periodic behaviors appear [from $\kappa_2 = 0.5 \times 10^{-3}$ to $\kappa_2 = 0.8 \times 10^{-3}$ in Fig. 2(b)] with frequency locking near $\kappa_2 = 0.5 \times 10^{-3}$ [Fig. 3(b)]. The quasi-periodic regime is followed by periodic behaviors up to $\kappa_2 = 3 \times 10^{-3}$. Figure 3(c) shows the limit cycle that corresponds to the periodic behavior observed for $\kappa_2 = 1 \times 10^{-3}$. Figures 2(a) and 2(b) show that, from $\kappa_2 = 3 \times 10^{-3}$ to $\kappa_2 = 5.7 \times 10^{-3}$, the laser locks into a stable external-cavity mode whatever the initial conditions are [see, for instance, Fig. 3(d)]. For further increases of κ_2 the laser undergoes a cascade of bifurcations composed of successive intervals within which it exhibits unstable behavior, such as periodic, quasi-periodic, and chaotic behavior and LFF (e.g., from $\kappa_2 = 5.7 \times 10^{-3}$ to $\kappa_2 = 8.3 \times 10^{-3}$ in the bifurcation diagram) and stable behavior when it locks into the new stable maximum gain mode (e.g., from $\kappa_2 = 8.3 \times 10^{-3}$ to $\kappa_1 = 10.9 \times 10^{-3}$).

We performed numerical calculations for larger first-feedback strength, as great as $\kappa_2 = 5 \times 10^{-2}$, and always observed LFF suppression. Physically, this result could be interpreted in terms of an effective reduction of the linewidth-enhancement factor. In a laser with a single optical feedback the large value of the linewidth-enhancement factor is propitious for LFF since it causes the external-cavity modes with the lowest stationary angular frequencies to lie close to antimodes. With a second optical feedback, however, LFF is avoided because the antimodes that are responsible for the crises are suppressed or shifted away from external-cavity modes, as can be verified on the basis of Eqs. (2) and (3). This second feedback effect is analogous to an effective reduction of the linewidth-enhancement factor. Indeed, the same equations show that the reduction of this parameter also moves antimodes away from external-cavity modes.

In all cases the laser diode undergoes a cascade of bifurcations as the second-feedback strength is increased with successive regions in which it exhibits chaos or LFF and regions in which it exhibits stable behavior. In these regions the system trajectories in phase space are captured by the new stable maximum gain mode and stay there. We emphasize the fact that LFF suppression and laser stabilization are achieved whatever the second-feedback phase is, and therefore accurate positioning of the second mirror is not required. By contrast, the second-feedback delay must be sufficiently short for one to observe a cascade of bifurcations with stable regions.

In summary, we have shown numerically that LFF in a laser diode that is subjected to a single external optical feedback with a short delay and biased near threshold can be suppressed by means of a second external optical feedback.

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