A large-N_c PNJL model with explicit $\mathbf{Z}_{N_c}$ symmetry

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Abstract

A PNJL model is built, in which the Polyakov-loop potential is explicitly Z_{N_c}-symmetric in order to mimic a Yang-Mills theory with gauge group SU(N_c). The physically expected large-N_c and large-T behaviours of the thermodynamic observables computed from the Polyakov-loop potential are used to constrain its free parameters. The effective potential is eventually U(1)-symmetric when N_c is infinite. Light quark flavours are added by using a Nambu-Jona-Lasinio (NJL) model coupled to the Polyakov loop (the PNJL model), and the different phases of the resulting PNJL model are discussed in 't Hooft's large-N_c limit. Three phases are found, in agreement with previous large-N_c studies. When the temperature T is larger than some deconfinement temperature T_d, the system is in a deconfined, chirally symmetric, phase for any quark chemical potential μ. When $T < T_d$ however, the system is in a confined phase in which chiral symmetry is either broken or not. The critical line $T_\chi(\mu)$, signalling the restoration of chiral symmetry, has the same qualitative features than what can be obtained within a standard $N_c = 3$ PNJL model.

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I. INTRODUCTION

The structure of the QCD phase diagram is intimately related to our understanding of fundamental features of QCD, like for example confinement dynamics and chiral symmetry breaking, and to their interplay with in-medium effects like a nonzero temperature or quark density. This is the reason why a lot of effort is devoted to study this field, either on the theoretical side, to which the present work belongs, or on the experimental side through heavy-ion-collision experiments. Among the various effective frameworks used to study the QCD phase diagram (see *e.g.* the review [1]), we will mostly focus on two of them: Polyakov-loop effective models for the pure gauge part of QCD, and the Nambu-Jona-Lasinio (NJL) model for the quark part.

The Polyakov loop is defined as

$$L(T,\vec{x}) = P\,\mathrm{e}^{i\,g\int_0^{1/T}d\tau A_0(\tau,\vec{x})}, \tag{1}$$

in which P is the path-ordering, g the strong coupling constant, $A_0 = A_0^a\,T_a$ the temporal component of the Yang-Mills field, T_a the generators of the gauge algebra, and T the temperature. The integral runs on the compactified timelike dimension. The Polyakov loop is a precious tool to study the phase structure of a given Yang-Mills theory since $\langle L(T,\vec{x})\rangle = 0$ ($\neq 0$) when the theory is in a (de)confined phase [2]. Moreover, global gauge transformations belonging to the center of the gauge algebra only cause $L(T,\vec{x})$ to be multiplied by an overall factor. That is why it has been conjectured that the confinement/deconfinement phase transition in a Yang-Mills theory with gauge algebra $\mathfrak{g}$ might be related to the spontaneous breaking of a global symmetry related to the center of $\mathfrak{g}$ [3]. In the particular case of $\mathrm{SU}(N_c)$, deconfinement might thus be driven by the breaking of a global Z_{N_c} symmetry. The order parameter of the deconfinement phase transition should then be the traced Polyakov loop

$$\phi = \frac{1}{N_c}\mathrm{Tr}_c L, \tag{2}$$

where the trace Tr_c is taken over the colour indices. The thermodynamic properties of pure gauge SU(3) QCD can then be studied by resorting to an effective scalar field theory where the potential energy density is Z_3-symmetric [4]

$$U = T^4\,\lambda\left[-\frac{b_2(T)}{2}|\phi|^2 + \frac{b_4}{4}\,|\phi|^4 + \frac{b_6}{6}(\phi^3 + \phi^{*3})\right]. \tag{3}$$

The real coefficients b_i can be fitted on lattice data. Various applications of this formalism can be found for example in [5]. Note that, in the following, ϕ and L will generally be indifferently called Polyakov loop.

The NJL model is based on the Lagrangian [6]

$$\mathcal{L}_{NJL} = \bar{q}(i\gamma^\mu \partial_\mu - m_q)q + \frac{G}{2}\left[(\bar{q}q)^2 + (\bar{q}i\gamma_5\vec{\tau}q)^2\right], \tag{4}$$

where q is the quark field, m_q the mass matrix, and $\vec{\tau}$ the Pauli matrices when an SU(2) flavour symmetry is considered. The interaction terms are such that the Lagrangian is chirally symmetric. The NJL model is designed to model chiral symmetry breaking and study many related phenomenological problems; the interested reader may consult the review [7] for more information. In the original NJL model, quarks are not coupled to the gauge field: As shown in [8], the coupling of this model to the Polyakov loop can be achieved by minimally coupling the quark field to a gauge field of the form $A_\mu = A_0\,\delta_{\mu 0}$, that formally appears as an imaginary quark chemical potential. The so-called PNJL model resulting in this coupling has motivated a lot of studies devoted to the QCD phase diagram [9, 10], including cases with a nonzero magnetic field [11].

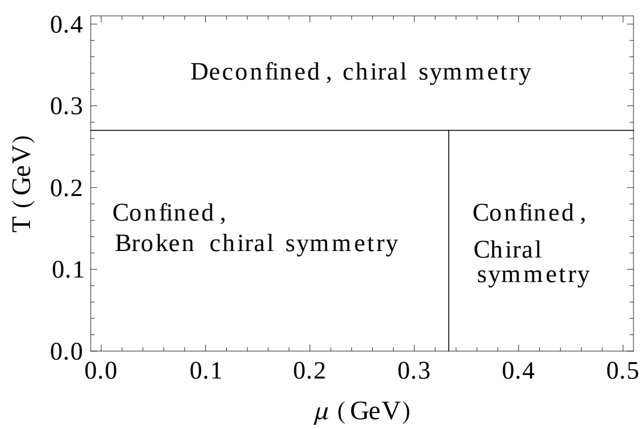

FIG. 1. Phase diagram obtained by taking the large-N_c limit of the PNJL model used in [12]. The solid lines signal first-order phase transitions.

The phase structure of the PNJL model at arbitrary N_c has been discussed in [12], as well as its large-N_c limit. One of the ingredients of this last work is to set $\lambda \propto (N_c^2 - 1)$ in (3) so that the gluon potential has the correct scaling in N_c. The phase diagram that has been found at large-N_c is given in Fig. 1. The chirally symmetric but confined phase that appears for quark chemical potentials larger than about a third of the nucleon mass,

$\mu \gtrsim M_N/3$, can presumably be identified with the quarkyonic phase, that has been first proposed in [13] and further studied in [14] in particular.

In the present work, we propose to re-build a PNJL model valid at large-N_c, but in which the Polyakov-loop potential is explicitly Z_{N_c} symmetric, that is typically with a term in $\phi^{N_c} + \phi^{*N_c}$ instead of the standard Z_3-symmetric term $\phi^3 + \phi^{*3}$. Such a potential is proposed in Sec. II, and the corresponding PNJL model is written in Sec. III. Then, the issue of deconfinement and chiral symmetry restoration when varying T and μ are discussed in Sec. IV in 't Hooft's large-N_c limit. The obtained phase diagram and concluding comments are given in Sec. V.

II. PURE GAUGE SECTOR

A. Explicit Z_{N_c}-symmetry

The simplest effective potential energy density depending on ϕ, defined in (2), and being explicitly Z_{N_c}-invariant has been proposed in [15] and reads

$$V_g(T, N_c, \phi, \phi^*) = A(T, N_c)\, |\phi|^2 + B(T, N_c)\, |\phi|^4 + C(T, N_c)\, (\phi^{N_c} + \phi^{*N_c}), \tag{5}$$

where, besides a mass term ($|\phi|^2$) and a simple interaction term ($|\phi|^4$), the term in $\phi^{N_c} + \phi^{*N_c}$ accounts for the Z_{N_c}-symmetry [16]. The real coefficients A, B, and C appearing in (5) are functions of T and N_c a priori, and their explicit form will be specified in the following. Note that ϕ, which depends on T, N_c, and $\vec{x}$ a priori, is here assumed to be independent of $\vec{x}$.

Various parametrizations of Z_3-symmetric potentials, fitted on pure gauge lattice data, have been proposed so far [8, 9, 17], the one of [9] being the most widely used. Here, we are rather interested in obtaining an effective potential valid at large N_c, *i.e.* $N_c > 4$ at least. All these values of N_c have in common that the asymptotic (large $|\phi|$) behaviour of V_g is driven by the Z_{N_c}-symmetric term, a feature that will be implicitly assumed in the analysis below. The present formalism will thus not be valid for $N_c = 3$ in particular, where the asymptotic behaviour is driven by the interaction term. Nevertheless, our results in the large-N_c limit will be compared to $N_c = 3$ lattice data as it is usually done in the literature. The following expected qualitative behaviours have to be imposed in order to constrain the shape of the functions A, B, and C:

- The pressure $p_g = -\min_\phi(V_g)$ is proportional to $N_c^2 T^4$ at large N_c and T in order to recover asymptotically the Stefan-Boltzmann limit of a free gluon gas.
- The norm, $|\phi_0|$, of the optimal value of the Polyakov loop, $\phi_0 = |\phi_0|\,e^{i\delta_0}$, is N_c-independent at the dominant order. $|\phi_0| = 0$ in the confined phase, and > 0 in the deconfined phase.
- There exists a critical temperature T_d signalling a first-order phase transition. At the critical temperature, $|\phi_0| = 0$ and $1/2$ are two degenerate minima of V_g. This last value is chosen so that it will ensure a good compatibility between our model and existing lattice data but it has only to be nonzero in order to lead to a deconfined phase. T_d has to be seen as a typical value for the deconfinement temperature in SU(N_c) Yang-Mills theory since the deconfinement temperature appears to be N_c-independent up to corrections in $1/N_c^2$ [18].

The above constraints are actually satisfied by the following Lagrangian

$$V_g = N_c^2 T^4\, a(T) \left[|\phi|^2 - 4|\phi|^4 + \frac{l(T)^{2-N_c}}{N_c} [8l(T)^2 - 1](\phi^{N_c} + \phi^{*N_c}) \right], \tag{6}$$

where

$$l(T) > \frac{1}{\sqrt{8}}, \quad l(T_d) = \frac{1}{2}, \quad l'(T) > 0. \tag{7}$$

Explicit forms of $a(T)$ and $l(T)$ will be given in the next section. The potential (6) has the following absolute minimum: $\phi_0(T < T_d) = 0$ and $\phi_0(T \geq T_d) = |\phi_0(T)|\, e^{2i\pi k/N_c}$, where $k = 0, \ldots, N_c - 1$ and where $|\phi_0(T)|$ is a solution of

$$1 - 8|\phi_0(T)|^2 + l(T)^{2-N_c} \left[8l(T)^2 - 1\right] |\phi_0(T)|^{N_c - 2} = 0. \tag{8}$$

It is straightforwardly checked that

$$|\phi_0(T)| = l(T) \tag{9}$$

actually solves (8).

A more compact expression for the optimal value of the Polyakov loop is thus

$$\phi_0 = l(T)\, e^{2i\pi k/N_c}\, \Theta(T - T_d), \tag{10}$$

where Θ is the Heaviside function. The constraints (7) ensure not only that the absolute minimum of V_g is always 0 below T_d, but also that the phase transition is of first order because of the discontinuity in $|\phi_0|$. As seen from (8), $|\phi_0|$ only depends on T as required.

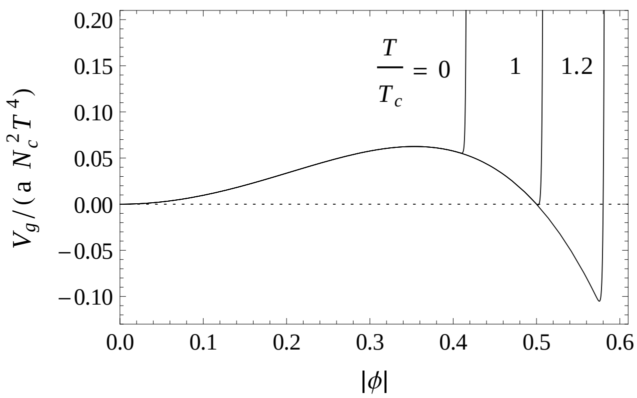

FIG. 2. Schematic evolution of the effective potential (11) versus the temperature (solid lines).

Restricting ourselves to the values $\phi = |\phi|\, \mathrm{e}^{2i\pi k/N_c}$, we get at the limit $N_c \to \infty$ a quite simple shape for the effective potential (6), namely

$$\frac{V_g}{N_c^2 T^4} \equiv \frac{\omega_g}{T^4} = a(T)\, |\phi|^2(1 - 4|\phi|^2) \qquad |\phi| \leq l(T), \\ \to +\infty \qquad |\phi| > l(T). \tag{11}$$

Hence, a U(1) invariance is recovered at infinite N_c as a limiting case of the Z_{N_c}-symmetry. The schematic evolution of the large-N_c limit of V_g with the temperature is plotted in Fig. 2; the behaviour (11) is readily observed, as well as the change of global minimum in $T = T_d$. Finally, the large-N_c limit of the pressure reads

$$p_g(T, N_c) = N_c^2 T^4\, a(T)\, l(T)^2 \left[4l(T)^2 - 1\right]. \tag{12}$$

Provided that $l(\infty) = 1$ according to the large-T behaviour of the Polyakov loop, p_g would tend toward the Stefan-Botlzmann limit for a free gluon gas if $a(\infty) = \pi^2/135$.

B. Numerical data

The function $l(T)$ is constrained by the relations (7) in order for the structure of the potential and its evolution with the temperature to have the required behaviour. Moreover, $l(T)$ is equal to the norm of the Polyakov loop as soon as $T > T_d$. The physical constraint $l(\infty) = 1$ could be imposed, but it is not sufficient to write down an explicit expression for $l(T)$. That is why a better way of proceeding is to fit available lattice computations of the Polyakov loop in pure Yang-Mills theory. To our knowledge, large-N_c values have not been

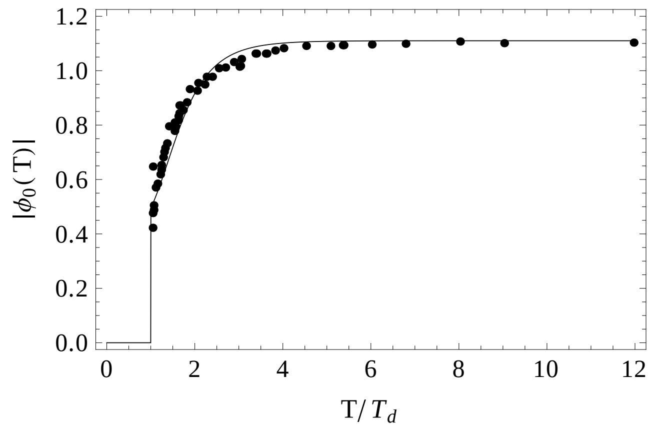

FIG. 3. Norm of the Polyakov loop minimizing the potential (6) versus the temperature in units of T_d (solid line). The norm of the Polyakov loop computed in pure gauge SU(3) lattice QCD has been added for comparison (points); data are taken from [19].

obtained so far, but accurate SU(3) ones have been computed in [19]. Since the Polyakov loop should not depend on N_c at the dominant order, it is relevant to fit $l(T)$ on SU(3) data; the ad hoc form

$$l(T) = 0.74 - 0.37\,\tanh\left(1.41\frac{T_d}{T} - 0.60\frac{T}{T_d}\right) \tag{13}$$

leads to a satisfactory parametrization of the results of [19] as it can be seen in Fig. 3. It is also worth noting that Fig. 2 has been obtained using the form (13) for $l(T)$, with $T = 1.2\,T_d$ for the curve labelled $T > T_d$.

The function $a(T)$ is only present as an overall factor in V_g, so it does not come into play in the qualitative features of the effective potential. However, it is relevant in view of reproducing the absolute value of the pressure in pure gauge QCD, for which lattice data are known at N_c = 3, 4, 5, 6, 8 and ∞ through an extrapolation of these data [20]. The simple empirical choice

$$a(T) = \frac{0.053}{l(T)^4} \tag{14}$$

leads to a good agreement between the lattice data of [20] and formula (12), as shown in Fig. 4.

It is worth saying that the forms (13) and (14) are not meant to be valid at asymptotically high temperatures. They have been chosen instead to agree with current lattice data up to 12 T_d (for $l(T)$) and 3 T_d (for $a(T)$). It is not problematic since the rest of this work will be devoted to study the phase structure of our PNJL model at large N_c: As it will be shown

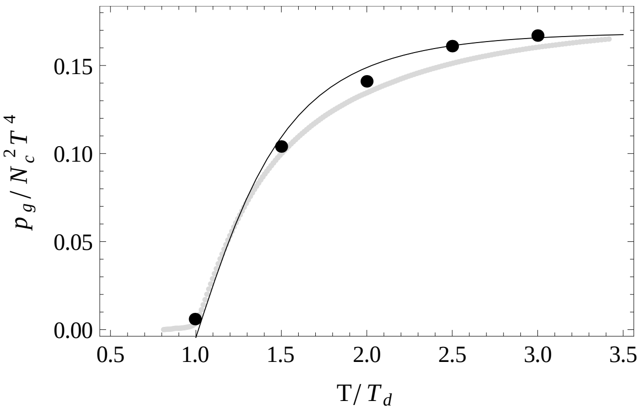

FIG. 4. Large-N_c pure gauge pressure computed from Eq. (12) and normalized to $N_c^2 T^4$ (solid line). The corresponding lattice data, taken from [20], are plotted for comparison in the case $N_c = 3$ (gray points) and $N_c \to \infty$ (black points).

in the next sections, no phase transition is expected to occur at energy scales well above T_d.

III. PNJL MODEL

As shown in [8], a minimal coupling of the NJL Lagrangian (4) to a gauge field of the form $A_\mu = A_0\,\delta_{\mu 0}$ makes eventually appear the Polyakov loop in the quark grand potential. In the mean field approximation, one is led indeed to the quark potential [8]

$$\frac{V_q(\mu, T, \sigma, L, L^\dagger)}{N_c\,N_f} = \frac{\sigma^2}{2g} - 2\int \frac{d^3p}{(2\pi)^3} \times \left\{ E_p + \frac{T}{N_c}\mathrm{Tr}_c\ln\left[1 + L\,\mathrm{e}^{-(E_p-\mu)/T}\right] + \frac{T}{N_c}\mathrm{Tr}_c\ln\left[1 + L^\dagger\,\mathrm{e}^{-(E_p+\mu)/T}\right]\right\}, \tag{15}$$

where the Polyakov loop L has been defined in (1). In the above equality,

$$E_p = \sqrt{p^2 + (m_q - \sigma)^2} \tag{16}$$

is the quark dispersion relation, with m_q the quark bare mass and σ related to the chiral condensate as follows

$$\sigma = G\,\langle \bar{q}\,q\rangle\,. \tag{17}$$

The coupling G has to scale as $(N_c\,N_f)^{-1}$ in order for the potential (15) to scale as $N_c\,N_f$, so it is convenient to define the coupling g as

$$g = G\,N_c\,N_f. \tag{18}$$

Since the pure gauge part of the potential only involves the traced Polyakov loop ϕ, it is interesting to express V_q in terms of ϕ rather than L. Terms of the form $\text{Tr}_c\ln\left[1+z\,L\right]$ can be expressed as functions of $\text{Tr}_c L \propto \phi$, $\text{Tr}_c L^2$, $\text{Tr}_c L^3$, ... through a Taylor expansion. A possible way of proceeding is to expand the quark potential at the first order in L. This eventually leads to formulas in which only ϕ appears in V_q [12]. This scheme has the advantage of being independent of the parametrization of L. Here we adopt an inequivalent procedure, consisting in choosing the following ansatz for L:

$$
\begin{aligned}
L &= \text{diag}(\underbrace{\mathrm{e}^{i\theta},\dots,\mathrm{e}^{i\theta}}_{(N_c-1)/2},1,\underbrace{\mathrm{e}^{-i\theta},\dots,\mathrm{e}^{-i\theta}}_{(N_c-1)/2}) \qquad \text{odd}-N_c \\
&= \text{diag}(\underbrace{\mathrm{e}^{i\theta},\dots,\mathrm{e}^{i\theta}}_{N_c/2},\underbrace{\mathrm{e}^{-i\theta},\dots,\mathrm{e}^{-i\theta}}_{N_c/2}) \qquad \text{even}-N_c.
\end{aligned}
\tag{19}
$$

This ansatz is such that $L^\dagger L = \mathbf{1}$ and $\det L = 1$ as demanded for an SU(N_c) element. Moreover, it reduces to the parametrization of [8] at $N_c = 3$. It has also the advantage of allowing to compute exactly the color traces appearing in (15), but the price to pay is that L is assumed to depend only on one real parameter θ.

It is readily computed that

$$
\begin{aligned}
\phi &= \tfrac{1}{N_c}\left[1+(N_c-1)\cos\theta\right] \qquad \text{odd}-N_c \\
&= \cos\theta \qquad \text{even}-N_c
\end{aligned}
\tag{20}
$$

by using of the ansatz (19) in (2). Moreover,

$$
\begin{aligned}
\text{Tr}_c\ln\left[1+L\,\mathrm{e}^{-(E_p-\mu)/T}\right] &= \ln\det{}_c\left[1+L\,\mathrm{e}^{-(E_p-\mu)/T}\right] \\
&= \frac{N_c-1}{2}\ln\left[1+2\frac{N_c\phi-1}{N_c-1}\mathrm{e}^{-(E_p-\mu)/T}+\mathrm{e}^{-2(E_p-\mu)/T}\right] \\
&\quad +\ln\left[1+\mathrm{e}^{-(E_p-\mu)/T}\right] \qquad \text{odd}-N_c, \\
&= \frac{N_c}{2}\ln\left[1+2\phi\mathrm{e}^{-(E_p-\mu)/T}+\mathrm{e}^{-2(E_p-\mu)/T}\right] \\
&\qquad \text{even}-N_c,
\end{aligned}
\tag{21}
$$

and, taking into account a cutoff for the momentum integration of the vacuum term, one finally arrives at the quark potential, whose large-N_c limit is given by

$$
\begin{aligned}
\omega_q(\mu,T,\sigma,\phi) &= \frac{V_q(\mu,T,\sigma,\phi)}{N_c\,N_f} \\
&= \frac{\sigma^2}{2g}-\frac{1}{\pi^2}\int_0^\Lambda dp\,p^2E_p-\frac{T}{2\pi^2}\int_0^\infty dp\,p^2\left\{\ln\left[1+2\phi\mathrm{e}^{-(E_p-\mu)/T}+\mathrm{e}^{-2(E_p-\mu)/T}\right]\right. \\
&\quad \left.+(\mu\to-\mu)\right\}.
\end{aligned}
\tag{22}
$$

This last potential reduces to the genuine NJL potential once $\phi = 1$, as it is the case in previous studies [8, 12].

The total potential of the large-N_c PNJL model under study is finally given by

$$\mathcal{V}(\mu, T, \sigma, \phi) = N_c^2 \omega_g(T, \phi) + N_c\, N_f\, \omega_q(\mu, T, \sigma, \phi). \tag{23}$$

IV. PHASE DIAGRAM AT LARGE N_c

In 't Hooft's large-N_c limit, the number of quark flavours stays finite and $\mathcal{V}$ is dominated by the gluonic contribution. Consequently, the optimal value ϕ_0 can be found by minimizing ω_g only. According to (11), the large-N_c solution reads

$$\phi_0(T) = l(T)\, \Theta(T - T_d). \tag{24}$$

The physical value of σ, denoted σ_0 and depending on T and μ, is then such that it minimizes $\omega_q(T, \mu, \sigma, \phi_0(T))$. ω_g does not depend on σ. Since $\sigma \propto \langle \bar{q}q \rangle$, chiral symmetry is present when $\sigma_0 = 0$ and broken when $\sigma_0 \neq 0$. As a consequence of (24), the deconfined phase appears as soon as $T > T_d$, independently of the value of μ: As pointed out in [13], quarks have no influence on the deconfinement phase transition at large-N_c because of the suppression of internal quark loops in this limit.

As a consequence of the large-N_c limit, the confined/deconfined phases are straightforwardly identified in our model. The situation is less simple as far as chiral symmetry is concerned; numerical computations are needed. As a first step, the parameters of the model have to be fixed. The values

$$m_q = 5.5 \text{ MeV},\ g = 60.48 \text{ GeV}^{-2},\ \Lambda = 651 \text{ MeV},\ T_d = 270 \text{ MeV}, \tag{25}$$

are used in most of the PNJL studies and will be taken in the following also. The first three parameters have been fitted so that the zero-temperature pion mass and decay constant are reproduced within the standard NJL model with $N_c = 3$ and $N_f = 2$ [7, 21]. T_d is a typical value for the deconfinement temperature in SU(N_c) Yang-Mills theory.

Using the parameters (25), the optimal value σ_0 can now be computed for any couple (μ, T), and can be linked to the quark condensate thanks to (17)

$$\langle \bar{q}q \rangle (\mu, T) = \frac{N_c\, N_f}{g} \sigma_0(\mu, T). \tag{26}$$

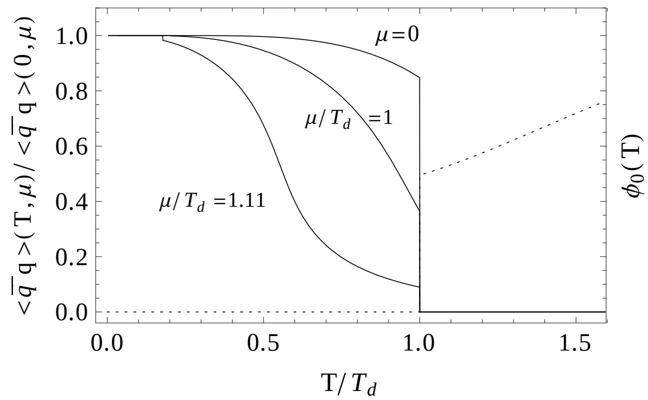

FIG. 5. Chiral condensate, normalized to its zero temperature value, versus T in units of T_d, and plotted for $\mu/T_d = 0$, 1, and 1.11 (solid lines). The optimal value of the Polyakov loop is also plotted (dotted line).

In the limit where T and μ both tend toward zero, we get

$$\lim_{\mu,T\to 0} \langle \bar{q}q \rangle (\mu, T) = -N_c\, N_f\, 5.29\, 10^6\, \text{MeV}^3, \tag{27}$$

corresponding to a quite common value of -(317 MeV)^3 for $N_c = 3$ and $N_f = 2$. Notice that, although formally valid in the large-N_c limit, our model leads to a satisfactory estimate for the chiral condensate at $N_c = 3$ also.

The chiral condensate versus the temperature is plotted in Fig. 5 for some values of the quark chemical potential. The most salient feature of this plot is the simultaneity of the first-order deconfinement phase transition and of the complete restoration of chiral symmetry through a first-order phase transition occuring at $T_\chi = T_d$. However, when $\mu/T_d \gtrsim 0.8$ ($\mu \gtrsim$ 200 MeV), the quick decrease of the chiral condensate suggests a progressive restoration of chiral symmetry through a crossover at temperatures smaller than T_d. As shown in [8], the crossover temperature can be computed thanks to the determination of the peak position in the dimensionless quark susceptibility reading, at large-N_c,

$$\chi_{qq}(T, \mu) = \frac{\Lambda\, T}{\partial^2_\sigma \omega_q\big|_{\sigma=\sigma_0}}. \tag{28}$$

A plot of $\chi_{qq}(T, \mu)$ for some values of μ/T_d is given in Fig. 6. Several observations can be made by observing this figure together with Fig. 5. First, the peak of the quark susceptibility is located in T_d when $\mu/T_d \leq 0.79$; this corresponds to a first-order-type chiral symmetry restoration in the deconfined phase. The point $(0.79, 1) \times T_d$ actually corresponds to a

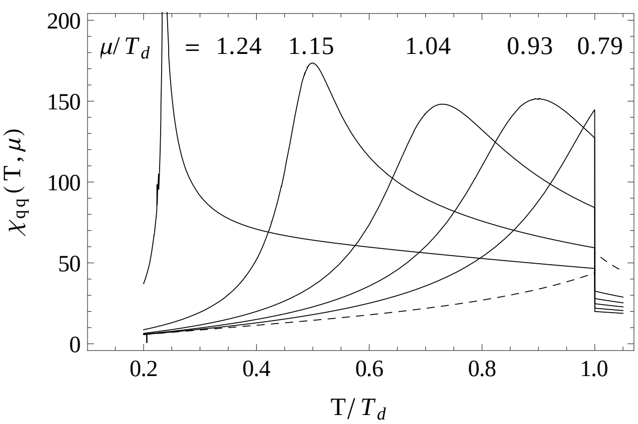

FIG. 6. Dimensionless quark susceptibility (28) versus the temperature in units of T_d (solid lines) with, from left to right, μ/T_d =1.24, 1.15, 1.04, 0.93, 0.79. $\chi_{qq}(T,0)$ is also ploted for completeness (dashed line).

triple point in the (μ, T)-plane: At large μ the peak of χ_{qq} is located below T_d – a larger μ corresponds to a lower peak position –, leading to the existence of a confined phase in which chiral symmetry is progressively restored through a crossover. A careful look at σ_0 actually shows that the chiral phase transition below T_d becomes of first order when $\mu/T_d \geq 1.24$: There exists thus a critical-end-point that we find to be $(1.23, 0.26) \times T_d$ in the (μ, T)-plane.

Gathering all these observations, the phase diagram of our model in the (μ, T)-plane can be established; it is shown in Fig. 7. The three phases we find correspond to those found in [12], see Fig. 1, but the structure of the chiral phase transition is a bit more involved under T_d: The chemical potential at which chiral symmetry is restored now depends on T, and there exists a critical-end-point at large enough μ. Although the deconfining phase transition corresponds to what is expected in the large-N_c limit of QCD from generic arguments [13], the critical line $T_\chi(\mu)$ we find under T_d quite resembles to what can be observed within previously known $N_c = 3$ PNJL studies [8, 9].

V. CONCLUSION

Effective "Polyakov-loop-based" approaches have proven to be a relevant tool in view of modelling the thermodynamic properties of pure gauge QCD. The traced Polyakov loop is then the order parameter associated to confinement, itself seen as correlated with a global center symmetry, Z_{N_c} when the gauge group is SU(N_c). Following a suggestion made in [15],

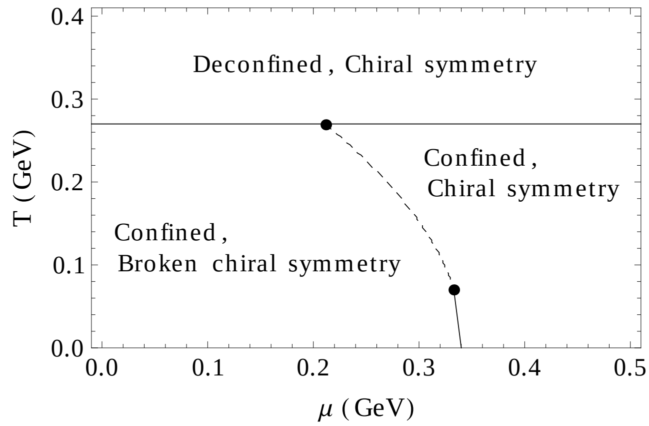

FIG. 7. Phase diagram of the large-N_c PNJL model (23) with explicit Z_{N_c} symmetry. The solid lines denote first-order phase transitions while the dashed line denotes a crossover. The triple point (0.212, 0.270) GeV and the critical end-point (0.335, 0.063) GeV have been also plotted. The end of the lower curve is reached at (0.343, 0) GeV.

an explicitly Z_{N_c}-symmetric potential involving the traced Polyakov loop has been built and leads to: A first-order phase transition at large-N_c, a gluonic pressure scaling as N_c^2, and an N_c-independent optimal value for the Polyakov loop. The coupling of the pure gauge sector to light quarks has then been performed within a PNJL approach. Thanks to a particular ansatz for the Polyakov loop, the quark potential is such that in only involves the traced Polyakov loop that appears in the pure gauge potential. It has to be said that the resulting PNJL model is designed to be relevant in the large-N_c limit, although it compares favourably with some $N_c = 3$ observables. It is moreover inequivalent to the model proposed in [12].

The phase diagram of our PNJL model has been explored in 't Hooft's large-N_c limit. At any μ, it shows a deconfined, chirally symmetric, phase above the deconfinement temperature (T_d =270 MeV). The deconfinement phase transition and the restoration of chiral symmetry are found to be simultaneous first-order phase transitions, in agreement with [12, 13]. At temperatures lower than T_d, thus in the confined phase, there is a critical line $T_\chi(\mu)$ separating a phase with broken chiral symmetry at small μ and a chirally symmetric phase at large μ. The phase transition is found to be a crossover from the triple point (0.212, 0.270) GeV to the critical-end-point (0.335, 0.063) GeV. It is then of first order until the boundary (0.343, 0) GeV is reached, corresponding to the estimate $\mu \approx M_N/3$ [13]. It is worth saying that a confined, chirally symmetric, phase has also been found by solving Schwinger-Dyson equations at nonzero μ in [22], and that evidences for the existence of such a phase has been found in Coulomb gauge QCD calculations [23].

Finally, we remark that the large-N_c limit of the proposed pure gauge effective potential has a U(1) symmetry, which emerges as the limit of a Z_{N_c} symmetry. This large-N_c effective potential could be used in an approach where the traced Polyakov loop is allowed to depend on the position, typically via a Lagrangian of the type $\mathcal{L} \approx \partial_\mu \phi \, \partial^\mu \phi^* - V_g$. Of particular interest would then be to search for localised, solitonic, solutions of $\mathcal{L}$: One could then take advantage of the fact that finding solutions of a complex scalar field theory with a U(1) invariance is a topic that has attracted a considerable attention, mostly since Coleman's work on Q-balls [24], and for which many results are already available. We hope to present such a study in future works.

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