Multicriteria Choice and Ranking Using Decision Rules Induced from Rough Approximation of Graded Preference Relations

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Abstract. The approach described in this paper can be applied to support multicriteria choice and ranking of actions when the input preferential information acquired from the decision maker is a graded pairwise comparison (or ranking) of reference actions. It is based on decision-rule preference model induced from a rough approximation of the graded comprehensive preference relation among the reference actions. The set of decision rules applied to a new set of actions provides a fuzzy preference graph, which can be exploited by an extended fuzzy net flow score, to build a final ranking.

Keywords: Multicriteria choice and ranking, Decision rules, Dominance-based rough sets, Graded preference relations, Fuzzy preference graph, Fuzzy net flow score, Leximax

1 Introduction

Construction of a logical model of behavior from observation of agent's acts is a paradigm of artificial intelligence and, in particular, of inductive learning. The set of rules representing a decision policy of an agent constitutes its *preference model*. It is a necessary component of decision support systems for multicriteria choice and ranking problems. Classically, it has been a utility function or a binary relation – its construction requires some *preference information* from the agent called *decision maker* (DM), like substitution ratios among criteria, importance weights, or thresholds of indifference, preference and veto. In comparison, the preference model in terms of decision rules induced from *decision examples* provided by the DM has two advantages over the classical models: (i) it is intelligible and speaks the language of the DM, (ii) the preference information comes from observation of DM's decisions.

Inconsistency often present in the set of decision examples cannot be considered as simple error or noise – they follow from hesitation of the DM, unstable character of his/her preferences and incomplete determination of the family of criteria. They can convey important information that should be taken into account in the construction of

S. Tsumoto et al. (Eds.): RSCTC 2004, LNAI 3066, pp. 510–522, 2004.

the DM's preference model. Rather than correct or ignore these inconsistencies, we propose to take them into account in the preference model construction using the *rough set* concept [14, 15]. For this purpose, the original version of rough set theory has been extended in two ways : (i) substituting the classical indiscernibility relation with respect to attributes by a dominance relation with respect to criteria, and (ii), substituting the data table of actions described by attributes, by a pairwise comparison table, where each row corresponds to a pair of actions described by binary relations on particular criteria, which permits approximation of a comprehensive preference relation in multicriteria choice and ranking problems. The extended rough set approach is called dominance-based rough set approach [3,5,6,8,9,11,16].

Given a finite set A={x,y,z,…} of actions evaluated by a family of criteria $G=\{g_1,\ldots,g_n\}$, we consider the preferential information in the form of a pairwise comparison table (PCT) including pairs of some *reference actions* from a subset $A'\subseteq A$. In addition to evaluation on particular criteria, each pair $(x,y)\in A'\times A'$ is characterized by a *comprehensive preference relation* which is graded (true or false to some grade). Using the rough set approach to the analysis of the PCT, we obtain a rough approximation of the graded preference relation by a dominance relation. More precisely, the rough approximation concerns unions of graded preference relations, called upward and downward cumulated preference relations. The rough approximation is defined for a given level of consistency, changing from 1 (perfect separation of certain and doubtful pairs) to 0 (no separation of certain and doubtful pairs). The rough approximations are used to induce "*if ..., then ...*" decision rules. The resulting decision rules constitute a preference model of the DM. Application of the decision rules on a new set $M\subseteq A\times A$ of pairs of actions defines a preference structure in M in terms of fuzzy four-valued preference relations. In order to obtain a recommendation, we propose to use a Fuzzy Net Flow Score (FNFS) exploitation procedure adapted to the four-valued preference relations.

The paper is organized as follows. In section 2, we define the pairwise comparison table from the decision examples given by the DM. In section 3, we briefly sketch the variable-consistency dominance-based rough set approach to the analysis of PCT, for both cardinal and ordinal scales of criteria. Section 4 is devoted to induction of decision rules and section 5 characterizes the recommended procedure for exploitation of decision rules on a new set of actions. An axiomatic characterization of the FNFS procedure is presented in section 6. Section 7 includes an illustrative example and the last section groups conclusions.

2 Pairwise Comparison Table (PCT) Built of Decision Examples

For a representative subset of reference actions $A'\subseteq A$, the DM is asked to express his/her comprehensive preferences by pairwise comparisons. In practice, he/she may accept to compare the pairs of a subset $B\subseteq A'\times A'$. For each pair $(x,y)\in B$, the comprehensive preference relation $\succ$ assumes different grades h of intensity, hence denoted by $\succ^h$. Let H be the finite set of all admitted values of h, and H^+ (resp. H^-) the subset of strictly positive (resp., strictly negative) values of h. It is assumed that $h\in H^+$ iff $-h\in H^-$ and $h\in(0,1]$. Finally $H = H^- \cup\{0\}\cup H^+$ and $H\subset[-1,1]$.

For each pair $(x,y)\in A'\times A'$, the DM is asked to select one of the four possibilities:

1. action x is comprehensively preferred to y in grade h, i.e. $x \succ^h y$, where $h\in H^+$,
2. action x is comprehensively *not* preferred to y in grade h, i.e. $x \succ^h y$, where $h\in H^-$,
3. action x is comprehensively indifferent to y, i.e. $x\succ^0 y$,
4. DM refuses to compare x to y.

Although the intensity grades are numerically valued, they may be interpreted in terms of linguistic qualifiers, for example: "*very weak preference*", "*weak preference*", "*strict preference*", "*strong preference*" for h=0.2, 0.3, 0.7, 1.0, respectively. A similar interpretation holds for negative values of h. Let us also note that $x\succ^h y$ does not necessarily imply $y\succ^{-h}x$ and $x\succ^0 y$ does not necessarily imply $y\succ^0 x$.

An $m\times(n+1)$ *Pairwise Comparison Table* S_{PCT} is then created on the base of this information. Its first n columns correspond to criteria from set G. The last, (n+1)-th column of S_{PCT}, represents the comprehensive binary relation $\succ^h$ with $h\in H$. The m rows are pairs from *B*. If the DM refused to compare two actions, such a pair does not appear in S_{PCT}.

In the following we will distinguish two kinds of criteria – *cardinal* and *ordinal* ones. In consequence of this distinction, for each pair of actions in an S_{PCT} we have either a difference of evaluations on cardinal criteria or pairs of original evaluations on ordinal criteria. The difference of evaluations on a cardinal criterion needs to be translated into a graded marginal intensity of preference. For any cardinal criterion $g_i\in G$, we consider a finite set $H_i \equiv (H_i^- \cup \{0\} \cup H_i^+)$ of marginal intensity grades such that for every pair of actions $(x,y)\in A\times A$ exactly one grade $h\in H_i$ is assigned.

1. $x\succ_i^h y$, $h\in H_i^+$, means that action x is preferred to action y in grade h on criterion g_i,
2. $x\succ_i^h y$, $h\in H_i^-$, means that action x is *not* preferred to action y in grade h on criterion g_i,
3. $x\succ_i^0 y$, means that action x is similar (asymmetrically indifferent) to action y on criterion g_i.

Within the preference context, the similarity relation $\succ_i^0$, even if not symmetric, resembles indifference relation. Thus, in this case, we call this similarity relation "asymmetric indifference". Of course, for each cardinal criterion $g_i\in G$ and for every pair of actions $(x,y)\in A\times A$, $[\exists h\in H_i^+ : x\succ_i^h y] \Rightarrow [\nexists k\in H_i^+ : y\succ_i^k x]$ as well as $[\exists h\in H_i^- : x\succ_i^h y] \Rightarrow [\nexists k\in H_i^- : y\succ_i^k x]$. Observe that the binary relation $\succ^0$ is reflexive, but neither necessarily symmetric nor transitive, and $\succ^h$ for $h\in H\backslash\{0\}$ are neither reflexive nor symmetric and not necessarily transitive. $\bigcup_{h\in H}\succ^h$ is not necessarily complete.

Consequently, PCT can be seen as decision table $S_{PCT} = \langle B, G\cup\{d\}\rangle$, where $B\subseteq A\times A$ is a non-empty set of pairwise comparisons of reference actions and d is a decision corresponding to the comprehensive pairwise comparison (comprehensive graded preference relation).

3 Rough Approximation of Comprehensive Graded Preference Relations Specified in PCT

Let G^N be the set of cardinal criteria, and G^O – the set of ordinal criteria, such that $G^N \cup G^O = G$ and $G^N \cap G^O = \emptyset$. Moreover, for each $P \subseteq G$, we denote by P^N, P^O the same partitioning of P, i.e. $P^O = P \cap G^O$ and $P^N = P \cap G^N$. In order to define the rough approximations of comprehensive graded preference relations we need the concept of dominance relation between two pairs of actions with respect to (w.r.t.) a subset of criteria. This concept is defined below, separately for subsets of cardinal criteria and for subsets of ordinal criteria. In the case of cardinal criteria, the dominance is built on graded preference relations, and in the case of ordinal criteria, the dominance is built directly on pairs of evaluations.

A. Cardinal Criteria Let $P = P^N \subseteq G$ ($P \neq \emptyset$). Given $(x,y),(w,z) \in A \times A$, the pair of actions (x,y) is said to dominate (w,z) w.r.t. subset of cardinal criteria P (denoted by $(x,y)D_P(w,z)$) if x is preferred to y at least as strongly as w is preferred to z w.r.t. each $g_i \in P$. Precisely, "*at least as strongly as*" means "*in at least the same grade*", i.e. for each $g_i \in P$ and $k \in H_i$ such that $w \succ_i^k z$, there exist $h \in H_i$ such that $h \geq k$ and $x \succ_i^h y$. Let $D_{\{i\}}$ be the dominance relation confined to the single criterion $g_i \in P$. The binary relation $D_{\{i\}}$ is a complete preorder on $A \times A$. Since the intersection of complete preorders is a partial preorder and $D_P = \bigcap_{g_i \in P} D_{\{i\}}$, then the dominance relation D_P is a partial preorder on $A \times A$. Let $R \subseteq P \subseteq G$ and $(x,y),(u,v) \in A \times A$; then the following implication holds: $(x,y)D_P(u,v) \Rightarrow (x,y)D_R(u,v)$.

Given $P \subseteq G$ and $(x,y) \in A \times A$, we define:

- a set of pairs of actions dominating (x,y), called P-*dominating set*, $D_P^+(x,y) = \{(w,z) \in A \times A: (w,z)D_P(x,y)\}$,
- a set of pairs of actions dominated by (x,y), called P-*dominated set*, $D_P^-(x,y) = \{(w,z) \in A \times A: (x,y)D_P(w,z)\}$.

To approximate the comprehensive graded preference relation, we need to introduce the concept of *upward cumulated preference* (denoted by $\succ^{\geq h}$) and *downward cumulated preference* (denoted by $\succ^{\leq h}$), having the following interpretation:

- $x \succ^{\geq h} y$ means "x is comprehensively preferred to y by *at least* grade h", i.e. $x \succ^{\geq h} y$ if $x \succ^k y$, where $h \leq k \in H$,
- $x \succ^{\leq h} y$ means "x is comprehensively preferred to y by *at most* grade h", i.e. $x \succ^{\leq h} y$ if $x \succ^k y$, where $h \geq k \in H$.

The P-dominating sets and the P-dominated sets defined on *B* for all pairs of reference actions from *B* are "granules of knowledge" that can be used to express P-lower and P-upper approximations of cumulated preference relations $\succ^{\geq h}$ and $\succ^{\leq h}$, respectively:

- for $h \in H$, $\underline{P}(\succ^{\geq h}) = \{(x,y) \in B: D_P^+(x,y) \subseteq \succ^{\geq h}\}$, $\quad \overline{P}(\succ^{\geq h}) = \bigcup_{(x,y) \in \succ^{\geq h}} D_P^+(x,y)$.
- for $h \in H$, $\underline{P}(\succ^{\leq h}) = \{(x,y) \in B: D_P^-(x,y) \subseteq \succ^{\leq h}\}$, $\quad \overline{P}(\succ^{\leq h}) = \bigcup_{(x,y) \in \succ^{\leq h}} D_P^-(x,y)$.

It has been proved in [3] that for $h\in H$, $\underline{P}(\succ^{\geq h}) \subseteq \succ^{\geq h} \subseteq \overline{P}(\succ^{\geq h})$ and $\underline{P}(\succ^{\leq h}) \subseteq \succ^{\leq h} \subseteq \overline{P}(\succ^{\leq h})$. Furthermore, one has also that, for $h\in H$, $\underline{P}(\succ^{\geq h}) = B - \overline{P}(\succ^{\leq h})$ and $\underline{P}(\succ^{\leq h}) = B - \overline{P}(\succ^{\geq h})$. From the definition of the P-boundaries (P-doubtful regions) of $\succ^{\geq h}$ and of $\succ^{\leq h}$ for any $h\in H$, $Bn_P(\succ^{\geq h}) = \overline{P}(\succ^{\geq h}) - \underline{P}(\succ^{\geq h})$ and $Bn_P(\succ^{\leq h}) = \overline{P}(\succ^{\leq h}) - \underline{P}(\succ^{\leq h})$, it follows that $Bn_P(\succ^{\geq h})=Bn_P(\succ^{\leq h})$.

The concepts of the quality of approximation, reducts and core can be extended also to the approximation of cumulated preference relations. In particular, the *quality of approximation* of $\succ^{\geq h}$ and $\succ^{\leq h}$ for all $h\in H$, by $P\subseteq G$ is characterized by the coefficient $\gamma_P = \left| B - \left(\bigcup_{h\in H} Bn_P(\succ^{\geq h}) \right) \right| / |B| = \left| B - \left(\bigcup_{h\in H} Bn_P(\succ^{\leq h}) \right) \right| / |B|$, where $|\ |$ denotes cardinality of a set. It expresses the ratio of all pairs of actions $(x,y)\in B$ correctly assigned to $\succ^{\geq h}$ and to $\succ^{\leq h}$ by the set P of criteria to all the pairs of actions contained in B. Each minimal subset $P\subseteq G$, such that $\gamma_P = \gamma_G$, is a *reduct* of G (denoted by $RED_{S_{PCT}}$). Let us remark that S_{PCT} can have more than one reduct. The intersection of all B-reducts is the *core* (denoted by $CORE_{S_{PCT}}$).

In fact, for induction of decision rules, we consider the Variable Consistency Model on S_{PCT} [12,16] relaxing the definition of P-lower approximation of the cumulated preference relations $\succ^{\geq h}$ and $\succ^{\leq h}$, for any $h\in H$, such that $(1-l)\times 100$ percent of the pairs in P-dominating or P-dominated sets may not belong to the approximated cumulated preference relation: $\underline{P}^l(\succ^{\geq h}) = \left\{(x,y)\in B : \left|D_P^+(x,y)\cap \succ^{\geq h}\right| / \left|D_P^+(x,y)\right| \geq l\right\}$ and $\underline{P}^l(\succ^{\leq h}) = \left\{(x,y)\in B : \left|D_P^-(x,y)\cap \succ^{\leq h}\right| / \left|D_P^-(x,y)\right| \geq l\right\}$ where $l\in(0,1]$ is the required level of consistency.

B. Ordinal Criteria. In the case of ordinal criteria, the dominance relation is defined directly on pairs of evaluations $g_i(x)$ and $g_i(y)$, for all pairs of actions $(x,y)\in A\times A$. Let $P=P^O$ and $P^N=\emptyset$, then, given $(x,y),(w,z)\in A\times A$, the pair (x,y) is said to dominate the pair (w,z) w.r.t. subset of ordinal criteria P (denoted by $(x,y)D_P(w,z)$) if, for each $g_i\in P$, $g_i(x)\geq g_i(w)$ and $g_i(z)\geq g_i(y)$. Let $D_{\{i\}}$ be the dominance relation confined to the single criterion $g_i\in P^O$. The binary relation $D_{\{i\}}$ is reflexive, transitive, but non-necessarily complete (it is possible that *not* $(x,y)D_{\{i\}}(w,z)$ and *not* $(w,z)D_{\{i\}}(x,y)$ for some $(x,y),(w,z)\in A\times A$). Thus, $D_{\{i\}}$ is a partial preorder. Since the intersection of partial preorders is a partial preorder and $D_P=\bigcap_{g_i\in P} D_{\{i\}}$, $P=P^O$, then the dominance relation D_P is a partial preorder.

C. Cardinal and Ordinal Criteria. If subset of criteria $P\subseteq G$ is composed of both cardinal and ordinal criteria, i.e. if $P^N\neq\emptyset$ and $P^O\neq\emptyset$, then, given $(x,y),(w,z)\in A\times A$, the pair (x,y) is said to dominate the pair (w,z) w.r.t. subset of criteria P, (denoted by $(x,y)D_P(w,z)$) if (x,y) dominates (w,z) w.r.t. both P^N and P^O. Since the dominance relation w.r.t. P^N is a partial preorder on $A\times A$ and the dominance w.r.t. P^O is also a partial preorder on $A\times A$, then also the dominance D_P, being the intersection of these two dominance relations, is a partial preorder. In consequence, all the concepts related to rough approximations introduced in 3.1 can be restored using this specific definition of dominance relation.

4 Induction of Decision Rules from Rough Approximations

Using the rough approximations of relations $\succ^{\geq h}$ and $\succ^{\leq h}$, defined in Section 3, it is then possible to induce a generalized description of the preferential information contained in a given S_{PCT} in terms of decision rules. The syntax of these rules is based on the concept of *upward cumulated preferences w.r.t. criterion* g_i (denoted by $\succ_i^{\geq h}$) and *downward cumulated preferences w.r.t. criterion* g_i (denoted by $\succ_i^{\leq h}$), having similar interpretation and definition as for the comprehensive preference. Let also $G_i=\{g_i(x),\ x\in A\}$, $g_i\in G^O$, be a set of different evaluations on ordinal criterion g_i. The decision rules induced from S_{PCT} have then the following syntax:

1) **$D_{\geq}$*-decision rules***, which are induced with the hypothesis that all pairs from $\underline{P}^l(\succ^{\geq h})$ are positive and all the others are negative learning examples:

   if $x \succ_{i1}^{\geq h(i1)} y$ *and* ... $x \succ_{ie}^{\geq h(ie)} y$ *and* $g_{ie+1}(x)\geq r_{ie+1}$ *and* $g_{ie+1}(y)\leq s_{ie+1}$ *and* ... $g_{ip}(x)\geq r_{ip}$ *and* $g_{ip}(y)\leq s_{ip}$, th*en* $x\succ^{\geq h}y$,

2) **$D_{\leq}$*-decision rules***, which are induced with the hypothesis that all pairs from $\underline{P}^l(\succ^{\leq h})$ are positive and all the others are negative learning examples:

   if $x \succ_{i1}^{\leq h(i1)} y$ *and* ... $x \succ_{ie}^{\leq h(ie)} y$ *and* $g_{ie+1}(x)\leq r_{ie+1}$ *and* $g_{ie+1}(y)\geq s_{ie+1}$ *and* ... $g_{ip}(x)\leq r_{ip}$ *and* $g_{ip}(y)\geq s_{ip}$, *then* $x\succ^{\leq h}y$,

   where $P = \{g_{i1},...,g_{ip}\} \subseteq G$, $P^N = \{g_{i1},...,g_{ie}\}$, $P^O = \{g_{ie+1},...,g_{ip}\}$, $(h(i1),..., h(ie)) \in H_{i1}\times ... \times H_{ie}$ and $(r_{ie+1},...,r_{ip})$, $(s_{ie+1},...,s_{ip})\in G_{ie+1}\times...\times G_{ip}$;

Since we are working with variable consistency approximations, it is enough to consider the lower approximations of the upward and downward cumulated preference relations, namely $\underline{P}^l(\succ^{\geq h})$ and $\underline{P}^l(\succ^{\leq h})$. To characterize the quality of the rules, we say that a pair of actions *supports* a decision rule ρ if it matches both the condition and decision parts of ρ. On the other hand, a pair is *covered* by a decision rule ρ as soon as it matches the condition part of ρ. Let *Cover(*ρ*)* denote the set of all pairs of actions covered by the rule ρ. Finally, we define the credibility $\alpha_\rho\left(\succ^{\geq h}\right)$ of $D_{\geq}$-decision rule ρ as $\alpha_\rho\left(\succ^{\geq h}\right)=\dfrac{\left|Cover(\rho)\cap\succ^{\geq h}\right|}{\left|Cover(\rho)\right|}$. For $D_{\leq}$-decision rules, the credibility is defined analogously.

Let us remark that the decision rules are induced from P-lower approximations whose composition is controlled by user-specified consistency level *l*. It seems reasonable to require that the smallest accepted credibility of the rule should not be lower than the currently used consistency level *l*. Indeed, in the worst case, some pairs of actions from the P-lower approximation may create a rule using all criteria from P thus giving a credibility $\alpha_\rho\left(\succ^{\geq h}\right) \geq l$. The user may have a possibility of increasing this lower bound for credibility of the rule but then decision rules may not cover all pairs of actions from the P-lower approximations. Moreover, we require that each decision rule is minimal. Since a decision rule is an implication, by a *minimal* decision rule we understand such an implication that there is no other implication with an antecedent of at least the same weakness and a consequent of at least the same strength with a not worse credibility $\alpha_\rho\left(\succ^{\geq h}\right) \geq l$. The induction of variable-consistency decision rules can be done using the rule induction algorithm for VC-DRSA, which can be found in [13].

5 Use of Decision Rules for Decision Support

Application of the set of decision rules on a new subset $\mathcal{M}$=M×M⊆A×A of pairs of actions induces a specific preference structure in set M. In fact, each pair of actions $(u,v)\in\mathcal{M}$ can match several decision rules. The matching rules can state different grades of preference and have various credibilities. A synthesis of the matching rules for a given pair of actions results in a graded (fuzzy) four-valued preference relation of level 2 [2]. This means that not only the relation is a graded one but also that its α-cuts are fuzzy four-valued preference relations, because of information about preference and non-preference. The three steps of the exploitation procedure lead to final ranking in the set of actions M.

Step 1. By application of the decision rules on $\mathcal{M}$, we get for each pair $(u,v)\in\mathcal{M}$ a set of different covering rules (possibly empty) stating different conclusions in the form of cumulated preference relations $\succ^{\geq h}$ and $\succ^{\leq h}$. For all pairs $(u,v)\in\mathcal{M}$, the cumulated preference relations are stratified into preference relations $\succ^{h}$ of grade $h\in H$ and for each pair $u\succ^{h}v$ a confidence degree $\beta(u\succ^{h}v)$ is calculated. This means that, for each $h\in H$, $\succ^{h}$ is a fuzzy relation in $\mathcal{M}$, which may be represented by a fuzzy preference graph.

In general, several decision rules assigning pair (u,v) to different cumulated preference relations are taken into account. For each $h\in H$, a confidence $\beta\left(u \succ^{h} v\right)$ is committed to the pair $u\succ^{h}v$ computed as the difference between the positive and negative arguments $\beta\left(u \succ^{h} v\right)=\beta^{+}\left(u \succ^{h} v\right)-\beta^{-}\left(u \succ^{h} v\right)$ where $\beta^{+}\left(u \succ^{h} v\right)$ takes into account rules ρ_i matching the pair (u,v) (i=1,…,k) that assign (u,v) to the cumulated preference relation $\succ^{\geq s}$ (or $\succ^{\leq q}$) such that $h\geq s$ (or $h\leq q$):

$$\beta^+\left(u \succ^h v\right) = \frac{\left|(Cover(\rho_1) \cap \succ^h) \cup \ldots \cup (Cover(\rho_k) \cap \succ^h)\right|^2}{\left|Cover(\rho_1) \cup \ldots \cup Cover(\rho_k)\right| \left|\succ^h\right|}$$

$\beta^+\left(u \succ^h v\right)$ can be interpreted as a product of the credibility of the rule and its relative strength w.r.t. the graded preference relation. The confidence $\beta^-\left(u \succ^h v\right)$ is defined for matching decision rules ρ_i (i=k+1,…,m) that assign (u,v) to different graded preference relations than $\succ^h$ ($r_{k+1},\ldots,r_p > h$ and $r_{p+1},\ldots,r_m < h$).

Step 2. Pairs of relations ($\succ^h$, $\succ^{-h}$) are considered as providing information about both preference and non-preference in grade h. These contradictory pieces of information induce a four-valued relation for each $h \in H^+$. An advisable procedure to exploit any of these four-valued relations is an extension of the Fuzzy Net Flow Score. For each action $x \in M$: the net flow score is computed as $S^h_{nf}(x) = S^h_{++}(x) - S^h_{+-}(x) + S^h_{-+}(x) - S^h_{--}(x)$, where $S^h_{++}(x) = \sum_{y \in M} \beta(x \succ^h y)$, $S^h_{+-}(x) = \sum_{y \in M} \beta(y \succ^h x)$, $S^h_{-+}(x) = \sum_{y \in M} \beta(y \succ^{-h} x)$ and $S^h_{--}(x) = \sum_{y \in M} \beta(x \succ^{-h} y)$. This builds up a complete preorder $\trianglerighteq^h$ for each $h \in H^+$, such that $u \trianglerighteq^h v \;\Leftrightarrow\; S^h_{nf}(u) \geq S^h_{nf}(v)$

Step 3. The preorders $\trianglerighteq^h$, $h \in H^+$, are aggregated by the leximax procedure, i.e. resolving indifference in a preorder of grade h by a preorder of grade $k \in H^+$, where k is the highest grade among the grades smaller than h.

$$u \triangleright v \quad \Leftrightarrow \quad \exists h \in H^+ : \begin{cases} \forall k \in H^+ \text{ such that } k > h,\; u \trianglerighteq^k v \text{ and } v \trianglerighteq^k u \\ u \trianglerighteq^h v \text{ and not } v \trianglerighteq^h u \end{cases}$$

$$u \equiv v \quad \Leftrightarrow \quad \forall h \in H^+ : u \trianglerighteq^h v \text{ and } v \trianglerighteq^h u$$

where $\triangleright$ is the asymmetric part of $\trianglerighteq$ and $\equiv$ is the symmetric part of $\trianglerighteq$.

This lexicographic approach considers the set of preorders $\trianglerighteq^h$ for $h \in H^+$ as providing consistent hierarchical information on the comprehensive graded preference relation. Therefore, it gives priority to preorders $\trianglerighteq^h$ with high values of grade h. Indeed, the preorders with lower values of h are only called to break ties from high h-value preorders. For this reason, this lexicographic approach is called *leximax* procedure.

The final recommendation in ranking problems consists of the total preorder $\trianglerighteq$; in choice problems, it consists of the maximal action(s) of $\trianglerighteq$.

6 Axiomatic Characterization of the Fuzzy Net Flow Score procedure

In the context of four-valued relation, a ranking method resulting in the complete preorder $\trianglerighteq^h$ on A can be viewed as a function $\trianglerighteq(\succ^h, \succ^{-h})$ aggregating the pair of valued relations $\succ^h$, $\succ^{-h}$ on A×A into a single ranking. In the previous section, we proposed to rank alternatives by means of an extended Fuzzy Net Flow Score (FNFS) procedure, i.e. $u \trianglerighteq(\succ^h,\succ^{-h})\, v \;\Leftrightarrow\; S^h_{nf}(u) \geq S^h_{nf}(v)$. It can be shown that the axioms proposed in [1] (neutrality, strong monotonicity, circuit-independency) can be naturally extended to characterize the FNFS dealing with pairs of relations.

7 Illustrative Example

Let us consider the case of a Belgian citizen wishing to buy a house in Poland for spending his holidays there. The selling agent approached by the customer wants to rank all the available houses to present them in a relevant order to the customer. Thereby, the latter is proposed first to have a look at a short list of 7 houses (the reference actions), characterized by three criteria that seem important to the customer: Distance to the nearest airport, Price and Comfort (Table 1). While the two first criteria are cardinal (expressed in km and in , respectively), the last one is represented on a three-level ordinal scale (*Basic*, *Medium*, *Good*). The customer is then asked to give – even partially – his preferences on the set of 7 proposed houses, in terms of a comprehensive graded preference relation.

Table 1. Short list of the houses (reference actions).

Location of the house	Distance to the nearest airport (A1: [km])	Price (A2: [€])	Comfort (A3: [ ])
0: Poznan	3	60	*Good*
1: Kapalica	35	30	*Good*
2: Krakow	7	85	*Medium*
3: Warszawa	10	90	*Basic*
4: Wroclaw	5	60	*Medium*
5: Malbork	50	50	*Medium*
6: Gdansk	5	70	*Medium*

The customer gives his preferences by means of the graph presented in Fig. 1, where a thin arc represents a weak preference, and a bold arc, a strong preference. Thereby, this is a comprehensive graded preference relation, with 2 positive grades of preference, weak and strong ones. One may observe that the customer preference is allowed to be both not complete (there may exist pairs of houses without an arc; e.g., 5 and 4) and not completely transitive (e.g., 6 is preferred to 4 and 4 is preferred to 3, without evident preference between 6 and 3).

In order to build the PCT, differences of evaluations on cardinal criteria have been encoded in marginal graded preference relations ($\succ_i^h$), with H_i={–1, –0.5, 0, 0.5, 1}, i=1,2. While comparing two alternatives, x and y, a difference in Distance criterion

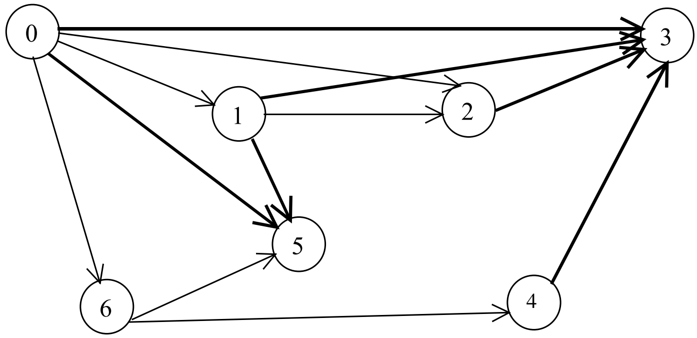

Fig. 1. Graph representation of the comprehensive graded preference relation in the set of reference actions.

smaller (in absolute value) than 3km is considered as non significant ($x \succ_1^0 y$ and $y \succ_1^0 x$). If the difference is between 4 and 10km in favor of x, then one weakly prefers x to y ($x \succ_1^{0.5} y$); finally, the preference is strong as soon as the difference is strictly greater than 10km ($x \succ_1^1 y$). As far as the Price criterion is concerned, an absolute difference smaller than 10 leads to indifference ($x \succ_2^0 y$ and $y \succ_2^0 x$), and the weak (resp. strong) preference appears as soon as the difference is strictly greater than 10 (resp. 30). For the sake of simplicity, we have assumed in this example that the marginal graded preference relations are symmetric, e.g. $x \succ_i^{0.5} y \Leftrightarrow y \succ_i^{-0.5} x$. As the Comfort criterion is ordinal, we have to take into account the pair of evaluations on this criterion instead of their difference. The piecewise comparison table (PCT) resulting from the above preference information is sketched in Table 2.

Table 2. A partial PCT corresponding to customer's preferences on the set of reference actions.

Pairs of reference actions (x,y)	h on A1: $x \succ_1^h y$	h on A2: $x \succ_2^h y$	Evaluations of (x;y) on A3	h on comprehensive preference relation: $\succ^h$
(0,0)	0	0	(*Good*; *Good*)	0
(0,1)	1	-0.5	(*Good*; *Good*)	0.5
(0,2)	0.5	0.5	(*Good*; *Medium*)	0.5
(0,3)	0.5	0.5	(*Good*; *Basic*)	1
(0,5)	1	0	(*Good*; *Medium*)	1
(0,6)	0	0	(*Good*; *Medium*)	0.5
…				

25 rules have been induced using the variable-consistency rule inducer [13], with a minimal consistency level l=0.85. Two examples of such rules are

if $x \succ_1^{\geq 1} y$ *and* $(x;y) \geq_3 (Good;Medium)$, *then* $x \succ^{\geq 1} y$;

if $x \succ_1^{\leq -1} y$ *and* $x \succ_2^{\leq 0.5} y$, *then* $x \succ^{\leq -0.5} y$

Suppose that the selling agent has found four other houses, presented in Table 3, and would like to see how these houses will be ranked by the customer. He may use to this end the preference model of the customer in form of the above decision rules on the set of new houses. According to Step 1 of our exploitation procedure presented in section 5, application of the rules on all possible pairs of the new houses results in fuzzy relation $\succ^h$, corresponding to fuzzy preference graphs (h=1 and 0.5). Then, according to Step 2, complete preorder $\trianglerighteq^h$ in the set of new houses is obtained by the Fuzzy Net Flow Score procedure. The fuzzy net flow score for h=1 and the corresponding complete preorder $\trianglerighteq^1$ are shown in the two last columns of Table 3. In fact, according to Step 3, since no pair of actions (x,y) have the same fuzzy net flow score at grade h=1, this grade is sufficient to define the final ranking of the new houses ($\trianglerighteq = \trianglerighteq^1$).

The dominance-based rough set approach gives a clear recommendation:

- for the **choice problem**, it suggests to **select house 2'** having the highest score,
- for the **ranking problem**, it suggests the **ranking** presented in the last column of Table 3:
  (2') → (3') → (0') → (1')

Table 3. The set of new houses and their ranks in the final ranking.

Location of the house	Distance to the nearest airport (A1: [km])	Price (A2: [€])	Comfort (A3: [ ])	Fuzzy Net Flow Score (h=1)	Final rank
0': Kornik	50	40	*Medium*	0.23	3
1': Rogalin	15	50	*Basic*	-5.17	4
2': Lublin	8	60	*Good*	3.42	1
3': Torun	100	50	*Medium*	1.52	2

8 Summary and Conclusions

We presented a complete methodology of multicriteria choice and ranking based on decision rule preference model. By complete we mean that it starts from acquisition of preference information, then it goes through analysis of this information using the Dominance-based Rough Set Approach (DRSA), followed by induction of decision rules from rough approximations of preference relations, and ends with a recommendation of the best action in a set or of a ranking of given actions.

The preference information is given by the Decision Maker (DM) in form of pairwise comparisons (or ranking) of some reference actions – comparison means specification of a grade of comprehensive preference of one reference action on another. DRSA aims at separating consistent from inconsistent preference information, so as to express certainly (P-lower approximation) or possibly only (P-upper approximation) the comprehensive graded preference relations for a pair of actions in terms of evaluations of these actions on particular criteria from set P. The inconsistency concerns the basic principle of multicriteria comparisons that says: if for two pairs of actions, (x,y) and (w,z), action x is preferred to action y at least as much as action w is preferred to z on all criteria from P, then the comprehensive preference of x over y should not be weaker than that of w over z. The rough approximations of comprehensive graded preference relations prepare the ground for induction of decision rules with a warranted credibility. Upon acceptance of the DM, the set of decision rules constitutes the preference model of the DM, compatible with the pairwise comparisons of reference actions. It may then be used on a new set of actions, giving as many fuzzy preference relations $\succ^h$ in this set (fuzzy preference graphs) as there are grades of the comprehensive graded preference relation. Exploitation of these relations with the Fuzzy Net Flow Score procedure leads to complete preorders $\trianglerighteq^h$ for particular grades. Aggregation of these preorders using the leximax procedure gives the final recommendation, that is, the best action or the final ranking $\trianglerighteq$.

Acknowledgements

The research of the second author has been supported by the Italian Ministry of University and Scientific Research (MURST). The third author wishes to acknowledge financial support from the Ministry of Science and from the Foundation for Polish Science.

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