3D LuGre Model for Multibody Systems Involving Contacts with Stick-slip

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EXTENDED ABSTRACT

1 Introduction

Friction is an extremely complex mechanism, which involves micro-interactions between the surfaces in contact. However, in multibody systems the general behaviour of friction at the macroscopic level is generally sufficient. Among the commonly used friction models [1], the LuGre one is very popular in multibody system and motion control as it is computationally efficient and able to reproduce most of phenomena observed in friction.

This paper presents an adaptation of the LuGre model, inspired namely from [2] for the management of the varying normal force and from [3] for the management of the direction of the slip velocity.

2 Generalized 3D LuGre model

Let us consider a contact between a point A attached to body j with local position vector $\vec{r}_A$ and a plane attached to body i (Figure 1) defined by the local position vector $\vec{r}_P$ of a point P of the plane and its normal unit vector $\vec{n}$. A contact between a plane and a sphere could be managed in a similar manner. The inter-bodies penetration δ and the penetration rate $\dot{\delta}$ can be computed as:

$$\delta = (\vec{e}_P - \vec{e}_A)\cdot\vec{n} \qquad \dot{\delta} = -\vec{V}_{rel}\cdot\vec{n} \tag{1}$$

$\vec{e}_P$ and $\vec{e}_A$ being the respective position vectors with respect to the global reference coordinate system and $\vec{V}_{rel}$ the relative velocity of body j with respect to body i (the plane) computed e.g. at the middle of the penetration zone (point M in Figure 1).

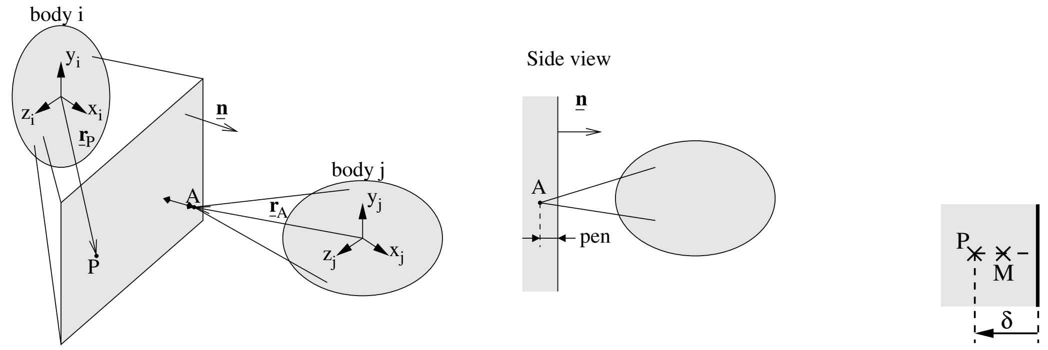

Figure 1: Contact between a plane and a point

The relative velocity is decomposed along the tangential ($\vec{V}_t$) and normal ($\vec{V}_n$) components

$$\vec{V}_n = (\vec{V}_{rel}\cdot\vec{n})\vec{n} \qquad \vec{V}_t = \vec{n}\times(\vec{V}_{rel}\times\vec{n}) \tag{2}$$

The normal force (N) is usually computed in terms of δ e.g. according to the Hunt and Crossley formula

$$N = K_{contact}\,\delta^{p_K} + D_{contact}\,\delta^{p_D}\,\dot{\delta} \tag{3}$$

with $K_{contact}$ the contact stiffness, $D_{contact}$ the contact damping, and p_K and p_D fitting coefficients.

The friction force on the plane is computed according to a vector version of the LuGre equations. The bristle deformation is represented as a vector $\vec{Z}$ possibly taking place in all directions, and driven by

$$\dot{\vec{Z}} = \vec{V}_t - \vec{Z}\frac{\sigma_0^M V_t}{G^M(V_t)} \quad \text{with} \quad G^M(V_t) = \mu_d + (\mu_s - \mu_d)e^{(-|\frac{V_t}{V_{St}}|^{\alpha})} \tag{4}$$

with μ_d and μ_s the dynamic and static (or breakaway) friction coefficients respectively, $G_M(V_t)$ the function describing the friction coefficient in terms of the magnitude V_t of the sliding velocity, V_{st} the Stribeck velocity and α ranging from 0.5 to 2 [4]. Finally, the friction force on the plane is calculated from

$$\vec{F} = \left(\sigma_0^M\vec{Z} + \sigma_1^M\dot{\vec{Z}} + \sigma_2^M\vec{V}_t\right)N \tag{5}$$

with σ_0 [m^{-1}] the micro-stiffness, σ_1 [s/m] the micro damping, and σ_2 [s/m] the viscous effect, all of them per unit of normal force.

The previous vector expressions will lead to 3 scalar equations, corresponding to the projections along the X, Y, Z coordinates of a coordinate system, a priori arbitrary but usually attached to one of the contact surfaces.

With respect to the original LuGre model, the bristle deflection is 3D but, as it is driven by the tangential velocity vector, the friction force is naturally aligned with the latter. The same technique was used in [3] but in 2D only as the contact plane (the road) is known. Moreover, the normal force N is transferred from the expression of G_M to equation 5, as in [2]. This allows to manage more properly varying normal forces.

It can be demonstrated that under a constant sliding velocity $\vec{V}_t$, the steady-state friction force reads

$$\vec{F}_{ss} = \left(\left(\mu_d + (\mu_s - \mu_d) e^{(-|\frac{V_t}{V_{St}}|^{\alpha})} \right) \frac{\vec{V}_t}{V_t} + \sigma_2^M \vec{V}_t \right) N \tag{6}$$

which corresponds to the so-called GKF model [4].

3 Applications

The proposed model was first tested on a cube lying on the X-Y plane, subjected to gravity and a varying lateral force, the contact being implemented through 6 contact points regularly spaced on a circle. The example demonstrated the ability of the projected equations to be used in all directions, with multiple contact points and with varying normal force. The results perfectly agree with the ones presented in [2].

The second example is more complex and corresponds to the experimental setup investigated in [5] (Fig. 2). The setup is composed of 3 identical beams linked by a tie-boss, expected to represent turbine blades connected by friction elements. The purpose of the arrangement is to induce relative slip at the contacts to dissipate energy by friction when the blades are excited by frequencies close to resonance during speed up or slow down of the turbine. Out of resonance, the parts stick to each other and no dissipation, and consequently no wear, takes place. The right part of Figure 2 shows the time history of the sliding velocity exhibited by the LuGre model at some interface when the system is excited by a force of 0.5 N that follows a logarithmic swept sine from 30 Hz to 80 Hz. It can be observed that sliding takes place around resonance. Out of resonance, microslips take place, related to the bristle deformation, physically corresponding to deformation of the asperities in the contact. Comparison with the regularized GKF model will also be developed.

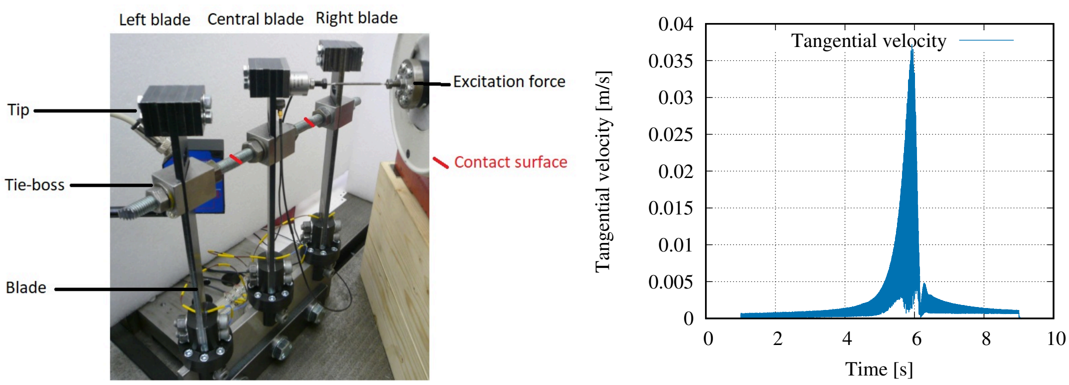

Figure 2: Experimental setup presented in [5] and example of simulation result

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