Robust state estimation for a linear reaction-convection-diffusion equation under unknown disturbances

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***Abstract*— We propose a robust state estimation approach for a linear reaction-convection-diffusion equation under bounded unknown disturbances. Inspired by sliding mode theory, an adequate discontinuous input function is designed to compensate for the effect of the unknown disturbances. Based on Filippov's solutions theorem, we report the existence of generalized solutions to the estimation error system subject to the discontinuous input. Based on a Lyapunov stability analysis, we show the asymptotic convergence of the estimation error. The observer is then designed under more relaxed and realistic assumptions by replacing the discontinuous input by a continuous approximation and by using adaptive techniques to compensate for the upper bound on the bounded disturbances which are rather assumed to be unknown. Numerical simulations are performed to illustrate the effectiveness of the proposed robust estimation approach.**

Keywords **Infinite dimensional systems, linear reaction-convection-diffusion equation, partial differential equations, robust observer design.**

I. Introduction

Distributed parameter systems are modeled by partially differential equations (PDEs) which depend on time and space variables. Many engineering systems may modeled by PDEs especially in bio(chemical) engineering systems consisting on axial dispersion (bio)chemical reactor models with convection and first order kinetics: see for instance [9], [20]. In this paper, we consider the problem of robust observation for a linear reaction-convection diffusion equation subject to norm-bounded unknown disturbances. The problem of robust observer design has been widely investigated in the literature of "control theory" in the case of lumped finite-dimensional systems subject to unknown inputs: design methods are based on Lyapunov analysis, solving linear matrix inequalities and sliding modes theory (see for instance [5], [6], [7], [18]). However, this issue was not sufficiently investigated in the case of distributed parameter systems except some papers such [1] where the authors proposed a robust adaptive neural observer for a class of parabolic PDE systems and [2] in which an uncertainty and disturbance estimator (UDE) is designed for an unstable parabolic PDE with disturbances. As a consequence this issue still needs to be addressed thoroughly: the problem consists in designing a dynamical system (a robust observer) which estimates the state of the considered infinite-dimensional system and reject the effect of the unknown inputs function. The dependence on the spacial variable make this problem much more complex since the available measurements usually occur only on the boundary of the spatial domain.

Inspired by sliding modes theory ([16], [13],[14]), we propose in this paper a robust observer for a linear reaction-convection diffusion equation under bounded unknown inputs based on the only available measurement on the boundary. A discontinuous input function is appropriately synthesized to ensure persistent unknown inputs rejection. The trajectories of the state observation error described by PDEs are shown to be well-posed in the sense of Filippov (see mathematical setting on Filippov's solutions concept in both Banach and Hilbert spaces, respectively, in [12] and [10]). Based on a constructive Lyapunov functional, we also establish the asymptotic convergence of the error trajectories to the origin, uniformly in the space variable. The result is then generalized to the more realistic case where the discontinuous input function is approximated by its continuous counter part and where an appropriate adaptive law is designed to compensate for the norm-upper bound of the unknown disturbance which is rather assumed unknown.

The paper is organized as follows: in the following section, we recall the concept of Filippov's solutions in Banach space; in Section III, we state the problem formulation; in Section IV, the observer is designed based on a Lyapunov analysis; in Section V, we prove the existence and the asymptotic convergence of the estimation error trajectories and in Section VI, a robust adaptive estimation approach is developed under more realistic and relaxed assumptions. To illustrate the theoretical results, some numerical simulations results are presented in Section VII and finally concluding remarks are given in Section VIII.

Notations. $\mathcal{C}^1([0,\tau])$ denotes the class of continuous functions on $[0,\tau]$ whose derivative is again continuous on $[0,\tau]$. $\mathcal{H}^l(0,1)$, with $l = 1,2,\ldots$, denotes the Sobolev space of absolutely continuous scalar functions $z(x)$ on $(0,1)$ with square integrable derivatives $z^{(i)}(x)$ for $i = 1,\ldots,l$. This space is equipped with the standard $\mathcal{H}^l-$norm. The standard notation $\mathcal{H}^0(0,1) = L^2(0,1)$ will also be used. Finally, $|.|$ denotes a norm on any vector space, since its interpretation will be clear from the context.

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II. Preliminaries

In this section, we recall the main features of the concept of Filippov's solutions in Banach spaces. Let us consider the following class of differential equations in Banach spaces:

$$\dot{x}(t) + Ax(t) = f(x(t)), \quad (1)$$
$$x(0) = x_0, \quad (2)$$

under the following assumptions:
Assumption 1: $(X, |.|)$ is a reflexive Banach space.
Assumption 2: The operator $-A : \mathcal{D}(A) \subset X \to X$ generates a C_0-semigroup $(K(t))_{t\geq 0}$ of bounded linear operators on X. Therefore $-A$ is closed and densely defined.
Assumption 3: (Growth condition) The function $f : \mathcal{D}(f) \subset X \to X$ is densely defined and satisfies the following linear growth condition for some nonnegative constants M and N:

$$|f(x)| \leq M\,|x| + N, \quad \text{for all} \quad x \in \mathcal{D}(f). \quad (3)$$

Le us now introduce the concept of generalized solutions for semilinear differential equations with discontinuous right-hand side on a Banach space.
Definition 1: The generalized solution of the equation

$$\dot{x}(t) + Ax(t) = f(x(t)), \quad (4)$$
$$x(0) = x_0 \quad (5)$$

is a mild solution of the differential inclusion

$$\dot{x}(t) + Ax(t) \in F(x(t)), \quad (6)$$
$$x(0) = x_0, \quad (7)$$

where $F(x) = \bigcap_{\epsilon>0} \overline{co}\ f(\overline{\mathcal{B}}(x,\varepsilon) \cap \mathcal{D}(f))$, $\overline{co}(E)$ denotes the closed convex hull of a set E and $\overline{\mathcal{B}}(x,\varepsilon)$ denotes the closed ball of center x and radius ε.

A mild solution $x(.)$ of the differential inclusion (6)-(7) is a continuous function $x : [0,T] \to X$ such that for some $T > 0$:

$$x(t) = K(t)x_0 + \int_0^t K(t-s)g(s)ds, \qquad \text{for } t \in [0,T], \quad (8)$$

where $g : [0,T] \to X$ is in $L^1(0,T;X)$ and satisfies $g(s) \in F(x(s))$ for almost every $s \in [0,T]$.

If $S \subset X$ and $x_0 \in S$, a viable solution $x(.)$ of (6)-(7) is a mild solution such that, for all t, $x(t) \in S$. A generalized viable solution of (4)-(5) is a viable solution $x(.)$ of (6)-(7). A particular class of semilinear differential equations is described by controlled differential equations of the form

$$\dot{x}(t) + Ax(t) = Bu(x(t)), \quad x(0) = x_0, \quad (9)$$

where x is the state variable and u is a control variable taking its values in U and given by

$$u(x) = N(x)\,\frac{Cx}{|Cx|}, \quad (10)$$

where $Y \equiv U$ and N is a continuous real-valued function, under the following assumptions:
Assumption 4: $(U, |.|)$ is a Banach space.
Assumption 5: $B : U \to X$ is a bounded linear operator.
Assumption 6: $u : \mathcal{D}(u) \subset X \to U$ is a densely defined function satisfying the growth condition (Assumption 3).
Assumption 7: $(Y, |.|)$ is a Banach space, $C : X \to Y$ is a bounded linear operator such that $C \neq 0$ and $\mathcal{D}(u) = X \backslash S$, where $S = Ker\{C\}$.
Now, we are ready to state an existence theorem of generalized solutions to (9)–(10). This result is a straightforward consequence of the proof of [10, Corollary 3.1].

Theorem 1 (Filippov's solutions): Under Assumptions 1–7, for any $x_0 \in S \cap \mathcal{D}(A)$, there exist generalized viable solutions on S of (9)–(10) if for all $x \in S \cap \mathcal{D}(A)$,

$$Ax \in F(x) - S, \quad (11)$$

where $F(x) = \bigcap_{\epsilon>0} \overline{co}\ Bu(\overline{\mathcal{B}}(x,\varepsilon) \cap \mathcal{D}(u))$.

Moreover, any viable solution $x(\cdot)$ satisfies

$$x(t) = K(t)x_0 + \int_0^t K(t-s)g(s)ds, \quad \forall t \geq 0, \quad (12)$$

where $g(s) \in Ax(s) + S$ for almost every s.

III. Problem statement

We consider the following perturbed linear reaction-convection-diffusion equation:

$$z_t(x,t) = Dz_{xx}(x,t) - z_x(x,t) - k_0 z(x,t) + \psi(t) \quad (13)$$

with initial and boundary conditions given by:

$$z(x,0) = z^0(x), \quad (14)$$
$$Dz_x(0,t) = z(0,t), \quad (15)$$
$$z_x(1,t) = 0. \quad (16)$$

where $z(x,t)$ is a space- and time-varying scalar field (e.g. a time-dependent concentration profile) over the spatial variable $x \in [0,1]$ and time variable $t \geq 0$, and D is a real-valued parameter. For example, D can be given by $D = \dfrac{D_a}{\nu}$ where ν and D_a are real-valued parameters, which can be interpreted as a fluid superficial velocity and as a dispersion/diffusion coefficient, respectively. The dynamical model equation (13) is a simplified model for real problems. Indeed, the reaction kinetics is usually nonlinear and the linear term $k_0 z(x,t)$ is its linear approximation. See e.g. [9], [20] and references cited therein. In this particular case, D^{-1} is the well-known Peclet number, for a spatial interval of length 1. $y(t) = z(0,t)$ is the boundary measurement. $\psi \in \mathcal{C}^1([0,\tau];\mathcal{H}^2(0,1))$ corresponds to an unknown disturbance satisfying the following boundedness assumption:

Assumption 8: There exists a known constant K such that

$$|\psi(t)| \leq K, \text{ for all } t \geq 0. \tag{17}$$

According to Curtain and Zwart [4, Theorem 3.1.3], the system (13)–(16) possesses a unique continuously differentiable classical solution.

The problem that is considered in this paper consists in designing a robust observer for system (13)–(16), which is robust in the sense that it estimates the state variable $z(x,t)$ despite the unknown perturbation $\psi(t)$ such that the only available measurement for the system (13)–(16) is its state at $x = 0$ or equivalently that the output is given by $y(t) = z(0,t)$.

IV. ROBUST STATE ESTIMATION APPROACH

We consider the dynamical system described by the following equations:

$$\begin{aligned}
\hat{z}_t(x,t) &= D\hat{z}_{xx}(x,t) - \hat{z}_x(x,t) - k_0\hat{z}(x,t) \\
&\quad + u(t), && (18)\\
\hat{z}(x,0) &= \hat{z}^0(x), && (19)\\
D\hat{z}_x(0,t) &= \hat{z}(0,t), && (20)\\
\hat{z}_x(1,t) &= 0 && (21)\\
\hat{y}(t) &= \hat{z}(0,t), && (22)
\end{aligned}$$

where $\hat{z}(x,t)$ denotes the estimated state, $\hat{z}^0(x)$ is the initial estimated state, $\hat{y}(t)$ is the estimated output and $u(t)$ is an input function whose main objective is to compensate the effect of the unknown disturbance and which is to be designed later. Let $e(x,t) = z(x,t) - \hat{z}(x,t)$ denotes the state observation error. Defining $e^0 = z^0 - \hat{z}^0$ to be the initial state observation error, the dynamics of the state observation error $e(x,t)$ can be written as

$$\begin{aligned}
e_t(x,t) &= De_{xx}(x,t) - e_x(x,t) - k_0 e(x,t) \\
&\quad + \psi(t) - u(t) && (23)\\
e(x,0) &= e^0(x) && (24)\\
De_x(0,t) &= e(0,t) && (25)\\
e_x(1,t) &= 0. && (26)
\end{aligned}$$

Based on a Lyapunov analysis, our aim is to synthesize the input function $u(t)$ in such away that the effect of the unknown disturbance is compensated and that the error system is exponentially stable. To that end, let us consider the following positive definite Lyapunov functional:

$$V(t) = \frac{1}{2}\theta e_x(0,t)^2 + \frac{1}{2}\int_0^1 e_x(s,t)^2 ds. \tag{27}$$

It can be shown that its time derivative along the state error trajectories of (23)–(26) is given by

$$\begin{aligned}
\dot{V}(t) &= (\theta - D)e_x(0,t)e_{tx}(0,t) - D\int_0^1 e_{xx}(s,t)^2 ds \\
&\quad - \frac{1}{2}(e_x(0,t))^2 - k_0 e_x(0,t)e(0,t) \\
&\quad - k_0\int_0^1 (e_x(s,t))^2 ds + \psi(t)e_x(0,t) \\
&\quad - u(t)e_x(0,t). && (28)
\end{aligned}$$

Let us now design the input function as follows:

$$u(t) = \theta e_{tx}(0,t) + \lambda\frac{e_x(0,t)}{|e_x(0,t)|} + \bar{\theta}k_0 e(0,t) - e_t(0,t), \tag{29}$$

where λ , θ, $\bar{\theta}$ are constant design parameters such that

$$\begin{aligned}
\bar{\theta} = \frac{\theta}{D} - 1 \quad &\text{and} \quad 0 < \theta \leq 2D && (30)\\
K \leq \lambda \quad &. && (31)
\end{aligned}$$

Replacing the discontinuous input $u(t)$ by its expression (29) in equation (28) and using Assumption (8),

$$\begin{aligned}
\dot{V}(t) &\leq -\lambda|e_x(0,t)| + |\psi(t)|\,|e_x(0,t)| \\
&\quad - k_0\int_0^1 (e_x(s,t))^2 ds - \frac{1}{2}(e_x(0,t))^2 \\
&\leq -\frac{1}{2}(e_x(0,t))^2 - k_0\int_0^1 (e_x(s,t))^2 ds, && (32)
\end{aligned}$$

where we have also used condition (31) and equation (25). Hence one gets

$$\dot{V}(t) \leq -\tau_1 V(t), \tag{33}$$

where $\tau_1 = \min\left\{\frac{1}{\theta}, 2k_0\right\}$.
At this stage, we deduce that the error system is exponentially stable such that $|e_x(x,t)|_0$ tends to 0 as t tends to infinity. Moreover, we deduce that $\lim_{t\to\infty}|e_x(0,t)| = 0$. As a consequence, it follows from the equation (25) that

$$\lim_{t\to\infty}|e(0,t)| = 0. \tag{34}$$

Later, we will use the property (34) to establish that the error trajectories of the error system $e(x,t)$ converge to zero as t tends to the infinity, uniformly in x.

V. EXISTENCE AND CONVERGENCE OF THE SOLUTIONS TO THE ERROR SYSTEM

This section is divided in two main parts. First, we report the existence of solutions to the error system (23)–(26) by applying Theorem 1 on Filippov's solutions. Second, we report the convergence of the error system trajectories.

A. Existence of solutions to the error system

We specify the operator C corresponding to the boundary measurement introduced in Section 2: the operator C is the bounded linear functional on $\mathcal{H}^1(0,1)$ which is given for all $\omega \in \mathcal{H}^1(0,1)$ by $C\omega = \omega(0)$. The main result of this subsection is based on Theorem 1:

Theorem 2: For any initial state error $e^0 \in Ker\{C\}$, there exist generalized viable solutions on $Ker\{C\}$ of the state error system (23)–(26) without disturbance, i.e. $\psi(t) \equiv 0$, and with input $u(t)$ given by (29). Any generalized viable solution $e(\cdot,t)$ of this system satisfies

$$e(.,t) = K(t)e^0 + \int_0^t K(t-s)g(s)ds, \quad t \geq 0, \tag{35}$$

where $g(s) \in Ae(.,s) + S$ for almost every s and where $(K(t))_{t\geq 0}$ is the C_0-semigroup generated by the operator $-A$ which is defined on its domain $\mathcal{D}(-A) = \{h \in \mathcal{H}^l(0,1) : h$ and $\frac{dh}{dx}$ are absolutely continuous, $\frac{d^2h}{dx^2} \in \mathcal{H}^l(0,1)$, $\frac{dh}{dx}(1) = 0$ and $\theta\frac{dh}{dx}(0) = h(0)\}$ by

$$-A = D\frac{\partial^2}{\partial x^2} - \frac{\partial}{\partial x} - k_0 I\ . \tag{36}$$

In addition, the corresponding generalized viable solution $e(\cdot,t)$ of the state error system (23)–(26) with disturbance $\psi(t)$ is given by

$$\begin{aligned} e(.,t) &= K(t)e^0 + \int_0^t K(t-s)\psi(s)ds \\ &+ \int_0^t K(t-s)g(s)ds, \quad t \geq 0. \end{aligned} \tag{37}$$

B. Convergence of the error system trajectories

The convergence analysis of the solution of the error system as well as the robust state estimation for the linear reaction-convection-diffusion equation are based on the following auxiliary result:

Lemma 1: [14, Lemma 1] Let $\xi(x) \in \mathcal{H}^1(0,1)$. Then, the following inequality holds

$$|\xi(\cdot,t)|_0^2 \leq 2(\xi^2(0) + |\xi_x(\cdot,t)|_0^2), \tag{38}$$

where $|.|_0$ denotes the usual norm on $L^2(0,1)$.

Applying Lemma 1 with $\xi(\cdot,t) = e(\cdot,t)$, using equation (25), and inequality (33), one can show that the state error system (23)–(26) is globally exponentially stable in the Sobolev space $\mathcal{H}^1(0,1)$, i.e. $|e(\cdot,t)|_1$ converges towards zero exponentially fast as time goes to infinity, for every initial condition e^0. Then, using successively Agmon's and Young's inequalities, one can conclude that $e(x,t) \to 0$ as $t \to \infty$, uniformly in $x \in [0,1]$, since $e(0,t) \longrightarrow 0$ as $t \to \infty$ (See (34)).

The previous Lyapunov analysis of the robust state estimation for the linear reaction-convection-diffusion equation is summarized in the following theorem.

Theorem 3: Consider the linear reaction-convection- diffusion equation (13)–(16) under Assumption 8 and the robust observer defined by (18)-(22) with the discontinuous input (29). Then the solution $e(\cdot,t)$ of the resulting state error system (23)–(26) is globally exponentially stable in the Sobolev space $\mathcal{H}^1(0,1)$. Moreover $e(x,t) \to 0$ as $t \to \infty$, uniformly in $x \in [0,1]$.

VI. Adaptive robust state estimation approach under relaxed assumptions

In practice, imperfections always occur on the discontinuous input such that imprecision in sensors, delays and disturbances which gives rise to the well-known chattering phenomenon. In order to improve our methodology in real life applications, we choose to apply the continuous approximation method to reduce chattering. This approach consists in approximating the discontinuous input by a continuous one. Furthermore, in order to extend the range of applications of our estimation approach, we assume that the upper-bound on the disturbance $\psi(t)$ is rather unknown in this section. Adaptive techniques are employed to compensate for the unknown upper-bound on the disturbance $\psi(t)$. The continuously-implemented counter-part of the discontinuous input u in (29) is given by

$$u(t) = \theta e_{tx}(0,t) + \hat{\lambda}\frac{e_x(0,t)}{|e_x(0,t)| + \epsilon} + \bar{\theta}k_0 e(0,t) - e_t(0,t). \tag{39}$$

where $\hat{\lambda}$ is an adaptive parameter such that

$$\dot{\hat{\lambda}} = \gamma\,|e_x(0,t)| - \sigma\hat{\lambda}. \tag{40}$$

Note that, in order to prevent $\hat{\lambda}$ to increase unboundedly, we have used the so-called σ-modification technique consisting in adding the term $-\sigma\hat{\lambda}$ in equation (40).

Corollary 1: Consider the linear reaction-convection-diffusion equation (13)–(16) under Assumption 8 and the infinite dimensional sliding mode observer defined by (18)-(22) with the continuously-implemented counter-part (39) of the discontinuous input u in (29), then the solutions $e(\cdot,t)$ of the resulting error boundary-value problem converge to a neighborhood of the origin by choosing appropriately the design parameters ϵ, γ and σ.

Hints for the proof:
The proof goes along the lines of the previous section by considering the following augmented Lyapunov functional defined by

$$\begin{aligned} W(t) &= V(t) + \frac{1}{2\gamma}\tilde{\lambda}^2 &(41)\\ &= \frac{1}{2}\theta e_x^2(0,t) + \frac{1}{2}\int_0^1 e_x^2(s,t)ds + \frac{1}{2\gamma}\tilde{\lambda}^2. &(42) \end{aligned}$$

where $\tilde{\lambda} = K - \hat{\lambda}$. One can show that

$$\dot{W}(t) \quad \leq \quad -2K_\tau W(t) + K_\Delta, \tag{43}$$

where K_τ is a positive constant and

$$K_\Delta(\sigma, \gamma, \epsilon) := \frac{\sigma}{2\gamma}K^2 + K\epsilon + \epsilon^2\frac{\gamma}{\sigma} \tag{44}$$

to deduce that $|e(x,t)|_0$ as well as $|e(0,t)|$ are bounded. Using again Agmon's and Young's inequalities, respectively, we can establish that the estimation error $e(x,t)$ converges to a small neighborhood of the origin which may be reduced at will by selecting appropriately the design parameters σ, γ ans ϵ by reducing K_Δ (See (44)), uniformly in $x \in [0,1]$. Moreover the adaptation error $\tilde{\lambda}(t)$ is bounded.

VII. Numerical simulations

The reaction-convection-diffusion equation given by the equations (13)–(16) can be employed to model many applications. Here we consider a Peclet number $Pe = D^{-1} = 1000$ and a decay coefficient $k_0 = 1$ (for instance the decay of a chemical compound in a stream of water). The unknown disturbance is given by $\psi(t) = -H(t-2) + 3H(t-4)$ where $H(t)$ represents the unitary step signal.

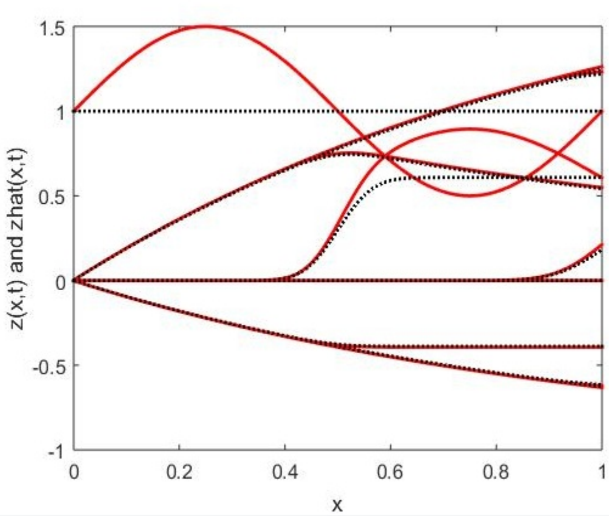

Fig. 1. Evolution of the original system spatial profiles (solid red line) and of the observer (black dotted line) at $t = 0, 0.5, ..., 8$

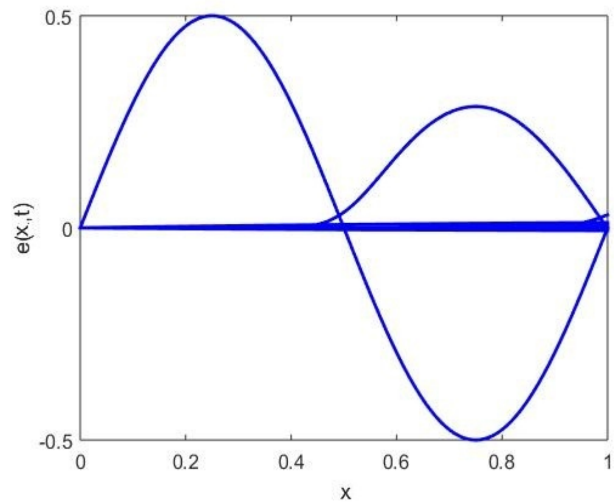

Fig. 2. Time evolution of the spatial profiles of the observation error $e(x,t)$ at $t = 0, 0.5, ..., 8$

To overcome the problem of on-line computation of the time derivative of the boundary measurement output, we assume that the PDE can be approximated by

$$z_t(x,t) \approx -z_x(x,t) \tag{45}$$

near this boundary, i.e., that the system is dominated by convection at the inlet boundary.
Using the the latter PDE approximation (45) and the equation (25), the correction term (39) is modified and replaced by the following new expression:

$$u(t) \quad = \quad \bar{\theta}e_t(0,t) + \bar{\theta}k_0 e(0,t) + \hat{\lambda}\frac{e(0,t)}{|e(0,t)| + \epsilon} \tag{46}$$

$$\approx \quad -\frac{\bar{\theta}}{D}e(0,t) + \bar{\theta}k_0 e(0,t) + \hat{\lambda}\frac{e(0,t)}{|e(0,t)| + \epsilon}. \tag{47}$$

Note that using the latter solution (the PDE approximation), the sensitivity with respect to the measurement noise is considerably reduced which clearly improves the performances of the proposed sliding mode observer in the case where the output signal is corrupted by disturbances.
The observer equations are solved using the *MatMol* library (www.matmol.org): see for instance the reference [19]. Spatial operators are approximated using finite difference schemes and the resulting set of ODEs are solved using the Matlab integrator *ode23*. The observer is initiated with $\hat{z}(x,0) = 1$ whereas the actual initial condition is $z(x,0) = 1 + 0.5sin(2\pi x)$. The design parameters of the observer are set to $\theta = 1.8D$, $\gamma = 1000$, $\sigma = 10$ and $\epsilon \approx 10^{-16}$ the floating-point relative accuracy. The convergence is illustrated in Figures 1 and 2. It is to be noticed that thanks to the continuous approximation technique applied to the input term $u(t)$ (See Equation (39) which represents the continuously-implemented counter-part of the input $u(t)$), the well-known chattering phenomena is avoided as it is clearly illustrated in Figures 1 and 2.

VIII. Concluding remarks.

We presented a robust estimation approach for a linear reaction-convection-diffusion equation under unknown disturbances based on the only available measurements occurring on the boundary of the spacial domain. We proved the existence of generalized Filippov's solutions and based on a Lyapunov analysis, we established both state estimation and unknown inputs rejection. Continuous approximation method and adaptive techniques were employed to extend the applicability of our approach. Theoretical results were illustrated via some simulations.

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