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Anticipating synchronization of two chaotic laser diodes by incoherent optical coupling and its application to secure communications

F. Rogister [a,*], D. Pieroux [b], M. Sciamanna [a], P. Mégret [a], M. Blondel [a]

[a] *Advanced Research in Optics, Service d'Electromagnétisme et de Télécommunications, Faculté Polytechnique de Mons, 31 Boulevard Dolez, B-7000 Mons, Belgium*

[b] *Optique Nonlinéaire Théorique, Université Libre de Bruxelles, Campus Plaine C.P. 231, B-1050 Bruxelles, Belgium*

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Abstract

We demonstrate anticipating synchronization between two chaotic laser diodes respectively subjected to incoherent optical feedback and incoherent optical injection. We investigate the robustness of the synchronization with respect to small parameter mismatches and spontaneous emission noise. We show that the gain saturation strongly enhances the synchronization quality. We demonstrate that anticipating synchronization can be applied to cryptographic purposes. We check how difficult it is to intercept a message encoded by chaos shift keying without an adequate replica of the transmitter laser. © 2002 Elsevier Science B.V. All rights reserved.

1. Introduction

Synchronization of two chaotic oscillators coupled in a master–slave configuration and its application to secure communications have attracted considerable interest this last decade [1–12]. In this type of communication, the chaotic output of the transmitter oscillator is used to carry a message in an encoded way. The message can then be decoded at the receiver oscillator provided that the two oscillators synchronize. The encoding can be achieved in several ways. In chaotic masking [4,6,7], the message is added to the chaotic output of the transmitter. In chaos modulation [9], the transmitter output is modulated by the message. In both cases, the decoding is based on the synchronization of the receiver on the chaotic part of the signal rather than on the full signal. The message can then be recovered by comparing the output of the receiver and the received signal. In chaos shift keying [5–8,12], the output of the chaotic transmitter itself directly carries the message. The transmitter switches between different chaotic orbits that correspond respectively to the bits "0" and "1" as one control parameter is modulated by the bit stream. In this case, message decoding is achieved by measuring the synchronization error

* Corresponding author. Tel.: +32-65-374198; fax: +32-65-374199.

E-mail address: rogister@telecom.fpms.ac.be (F. Rogister).

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between the outputs of the two oscillators. Most proposed cryptosystems have been implemented by electronic circuits [3–12]. Synchronization and chaotic cryptography have also been demonstrated with electrooptical circuits [13] and different types of lasers among which, for instance, solid-state lasers [14], fiber ring lasers [15–17] and laser diodes [18–33]. In recent years, much attention has been devoted to the synchronization of chaotic semiconductor lasers since these lasers are the most popular optical sources in high-speed optical communication systems. The cryptosystems involving semiconductor lasers exploit different means to drive them to chaos, the two most used being by coherent optical feedback [18,20, 23,25,28,29] and optical injection [19,31]. Other schemes involve semiconductor lasers combined with a nonlinear optical component and an optoelectronic feedback loop [21,22]. Schemes implementing laser diodes with optical feedback are of particular interest. On the one hand, the all-optical feedback is not bandwidth reduced. On the other hand, the time-delayed feedback generates a high-dimensional chaos [24]. This in turn leads to a potentially high security level since the known chaos-encrypted decoding technics [35–38] are much less efficient in the case of high-dimensional chaos. Several experiments have demonstrated the feasibility of laser diode synchronization and of message encoding/decoding by coherent optical feedback induced chaos [26–30,32]. However, numerical simulations have revealed that the detuning between the free-running frequencies of the transmitter and the receiver lasers degrades the synchronization performance [33]. From a practical point of view, it is therefore important to investigate alternative cryptographic schemes that do not require fine tuning of the optical frequencies.

In the previous work [39], we have numerically demonstrated anticipating synchronization [24,40] between a first diode subjected to incoherent optical feedback and a second diode driven by the first one through incoherent optical injection. In this scheme, the feedback and injected fields act on the carrier population in the diode active layers but do not interact with the intracavity lasing fields. As a consequence, the phases of the feedback and injection fields do not intervene on the lasers dynamics. For that reason, this synchronization scheme requires no fine tuning of the diode optical frequencies. This is a clear advantage in regard to other schemes based on coherent optical feedback and injection. It is therefore attractive for experimental realization. We have also shown that this synchronization scheme can be applied to chaos-embedded communication. The present paper extends our previous work. In particular, we investigate the robustness of anticipating synchronization with respect to spontaneous emission noise and parameter mismatches. Moreover, we show that the synchronization robustness is strongly affected by the gain saturation. With a view to secure communications, we check how difficult it is to intercept a message encoded by chaos shift keying without an adequate replica of the transmitter laser.

This paper is organized as follows. In Section 2, we describe the synchronization scheme. In Section 3, we present the corresponding model. In the next two sections, we give the necessary condition for anticipating synchronization and we show that the synchronization is robust to noise and to small mismatches of parameters. In Section 6, we demonstrate that our scheme allows the encoding/decoding of a 250 Mbit/s message and that the communication is secure at least with respect to naïve interceptions.

2. Synchronization scheme

In the scheme we propose (Fig. 1), the linearly polarized output field of the transmitter laser first undergoes a 90° polarization rotation through an external cavity formed by a Faraday rotator (FR) and a mirror. It is then split by a non-polarizing beam splitter (BS). One part is fed back into the transmitter laser and the other one is injected into the receiver laser. Polarization directions of feedback and injection fields are orthogonal to those of transmitter and receiver output fields respectively. In other words, the transmitter laser is subjected to incoherent optical feedback while the receiver laser is subjected to incoherent optical injection. An optical isolator (ISO) shields the transmitter from parasitic reflections from the receiver. Tunable attenuators (not shown in Fig. 1) adjust the

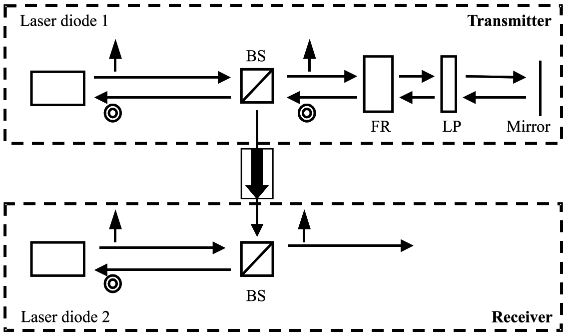

Fig. 1. Schematic representation of the synchronization scheme. See text for definitions.

respective strengths of the feedback and the injection. If necessary, a linear polarizer (LP) may be placed between the FR and the mirror to prevent coherent feedback induced by a second round-trip in the external cavity.

3. Model

The model we use to describe the dynamics of our scheme is an extension of models that have been proposed by Otsuka and Chern [41] for semiconductor lasers subject to incoherent optical feedback and semiconductor lasers with mutual incoherent coupling [42]:

$$\frac{\mathrm{d}P_1(t)}{\mathrm{d}t} = \left(G_1 - \frac{1}{\tau_{p1}}\right)P_1(t) + \beta_1 N_1(t) + F_1(t), \qquad (1)$$

$$\frac{\mathrm{d}N_1(t)}{\mathrm{d}t} = \frac{I_1}{e} - \frac{N_1(t)}{\tau_{s1}} - G[P_1(t) + \gamma P_1(t-\tau)], \qquad (2)$$

and

$$\frac{\mathrm{d}P_2(t)}{\mathrm{d}t} = \left(G_2 - \frac{1}{\tau_{p2}}\right)P_2(t) + \beta_2 N_2(t) + F_2(t), \qquad (3)$$

$$\frac{\mathrm{d}N_2(t)}{\mathrm{d}t} = \frac{I_2}{e} - \frac{N_2(t)}{\tau_{s2}} - G_2[P_2(t) + \sigma P_1(t-\tau_c)]. \qquad (4)$$

The gains are given by $G_j = G_{Nj}[1 - \varepsilon_j P_j][N_j - N_{0j}]$. In these equations, P_j and N_j are the photon number and the electron–hole pair number in the active region of laser j, with $j = 1$ for the transmitter and $j = 2$ for the receiver. N_{0j} is the value of N_j at transparency. τ_{pj}, τ_{sj}, I_j, G_{Nj} and ε_j are respectively the photon lifetime, the carrier lifetime, the injection current, the gain coefficient and the gain saturation coefficient of laser j. e is the electronic charge. F_j is a Langevin noise force that accounts for stochastic fluctuations arising from spontaneous emission process. The Langevin forces satisfy the relations $\langle F_j(t) \; F_{j'}(t')\rangle = 2N_jP_j\beta_j\delta(t-t')\delta_{jj'}$, where β_j is the spontaneous emission rate. The operating parameters γ, τ and σ are respectively the strength and the delay of the feedback at the transmitter, and the coupling strength at the receiver. The duration taken by the light emitted by the transmitter to reach the receiver is τ_c. We use typical values for the internal parameters of the transmitter laser: $\tau_{p1} = 2$ ps, $\tau_{p1} = 2$ ns, $G_{N1} = 1 \times 10^4$ s^{-1}, $N_{01} = 1.1 \times 10^8$, $\beta_1 = 5 \times 10^2$ s^{-1} and $\varepsilon_1 = 7.5 \times 10^{-8}$. In a first step, the parameters at the receiver are chosen identical to those of the transmitter. Afterwards, we will consider slight differences between the corresponding parameters.

4. Anticipating synchronization

For vanishing stochastic terms F_j and identical internal and operating parameters, the exact synchronous solution,

$$P_2(t) = P_1(t - \Delta t), \qquad (5)$$

$$N_2(t) = N_1(t - \Delta t) \qquad (6)$$

with $\Delta t = \tau_c - \tau$ the synchronization lag, exists if the coupling strength at the receiver equals the feedback strength at the transmitter, i.e. if $\sigma = \gamma$. It should be noted that this condition is necessary but not sufficient to observe synchronization between the two lasers since the solution (5), (6) can be stable or unstable. In the former case, this implies that the state of the receiver at time t can synchronize to the state of the transmitter at time $t - \tau_c + \tau$. In other words, the receiver anticipates the signal that will be injected at time $t + \tau$. The anticipation time is τ, the feedback delay at the transmitter. Anticipating synchronization has been demonstrated recently to result from the interaction between delayed feedback and dissipation and to be a rather universal phenomenon in nonlinear dynamical systems with unidirectional coupling [40]. It has also been predicted in coupled laser diodes subject to delayed coherent optical feedback [24,33,34,43] and experimentally observed in laser diodes subject to delayed optoelectronic feedback [44].

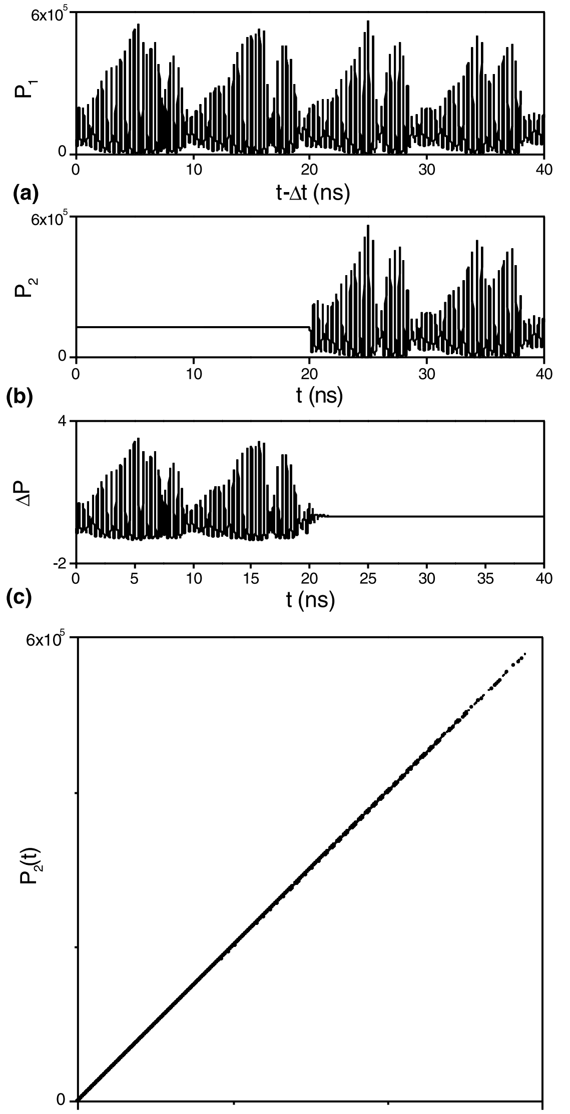

Fig. 2. Numerical simulation illustrating synchronization. The parameters of the transmitter and receiver diodes are all identical. Stochastic terms $F_j(t)$ are neglected. (a) Output of the transmitter. (b) Output of the receiver. (c) Normalized synchronization error. The transmitter output is shifted by Δt. (d) Synchronization diagram of the receiver output $P_2(t)$ versus the transmitter output $P_1(t-\Delta t)$. This diagram corresponds to a simulation over 1 μs long. The transient has been discarded.

We first consider the synchronization of two identical lasers when the stochastic terms $F_{1,2}(t)$ are neglected. The transmitter parameters are $\gamma = 0.41$, $\tau = 9$ ns, $I_1 = 1.8 \times I_{\mathrm{th1}}$ where I_{th1} is the threshold value. To begin with, the receiver is shielded from the transmitter by setting $\sigma = 0$. Due to the incoherent optical feedback, the output

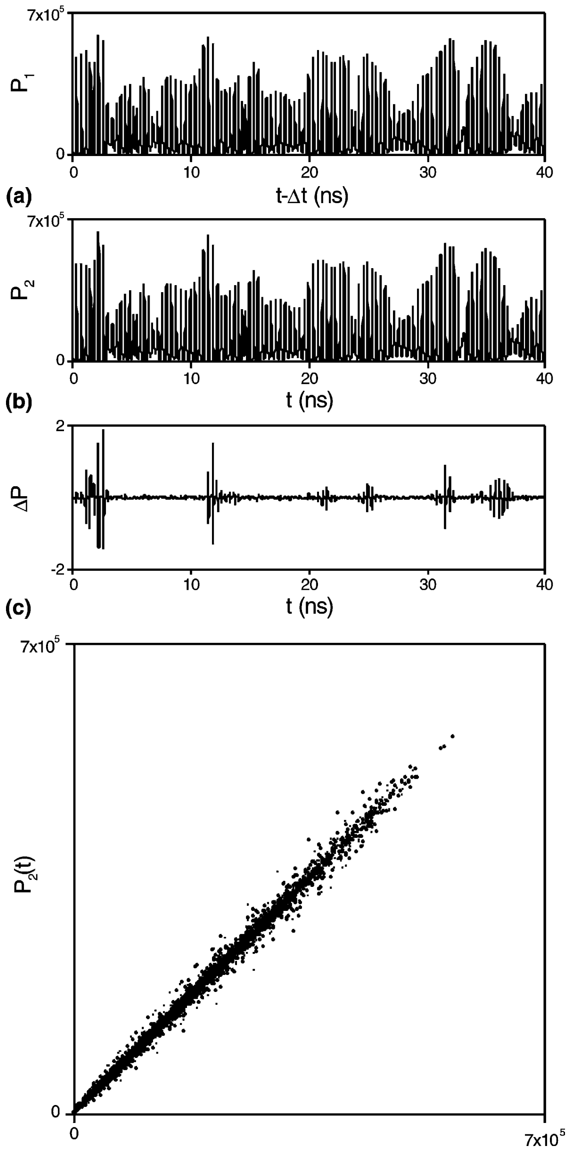

Fig. 3. Same as Fig. 2 but with the stochastic terms $F_j(t)$ included.

of the transmitter laser is chaotic for these parameter values (Fig. 2(a)), whereas that of the uncoupled receiver is steady (Fig. 2(b)). Then, at time $t = 20$ ns, we set $\sigma = \gamma$ and the beam of the transmitter enters the receiver with a coupling strength that equals the transmitter feedback strength. The receiver is then driven into chaos and synchronizes perfectly to the transmitter after a short transient. As seen in Fig. 2(c), the normalized synchronization error, which is defined as

$$\Delta P(t) = \frac{P_1(t - \Delta t) - P_2(t)}{P_0} \quad (7)$$

with P_0 the stand-alone receiver mean output, vanishes. This perfect synchronization is also shown in the synchronization diagram that displays the receiver output at time t versus the transmitter output at time $t - \Delta t$ (Fig. 2(d)). After the transient, all the points lie on the diagonal line.

We consider now a more realistic case by accounting for the noise. Results are shown in Fig. 3. Although the output of the receiver laser (Fig. 3(b)) is still similar to that of the transmitter (Fig. 3(a)), the synchronization error is no longer null (Fig. 3(c)). As a result, the points lie no more exactly on the diagonal line in the synchronization diagram (Fig. 3(d)). Since they would lie on it in case of perfect synchronization, the quality of the synchronization can be characterized by the linear correlation coefficient r: the better the synchronization, the closer r to unity. In the present case, $r = 0.993$ indicating a good level of synchronization. This lets us conclude that anticipating synchronization between the two lasers is robust with respect to stochastic fluctuations induced by spontaneous emission noise.

5. Robustness of the synchronization to parameter mismatches

For cryptographic purposes, synchronization between the transmitter and receiver must be restricted to very similar internal laser parameters and close working conditions. Sensitivity of the synchronization to mismatches of the parameters should lead to a high level of security due to the difficulty to replicate the receiver. However, under real world conditions, internal parameters of the two laser diodes never match exactly even if they are produced on the same wafer. Moreover, the operating parameters cannot be perfectly

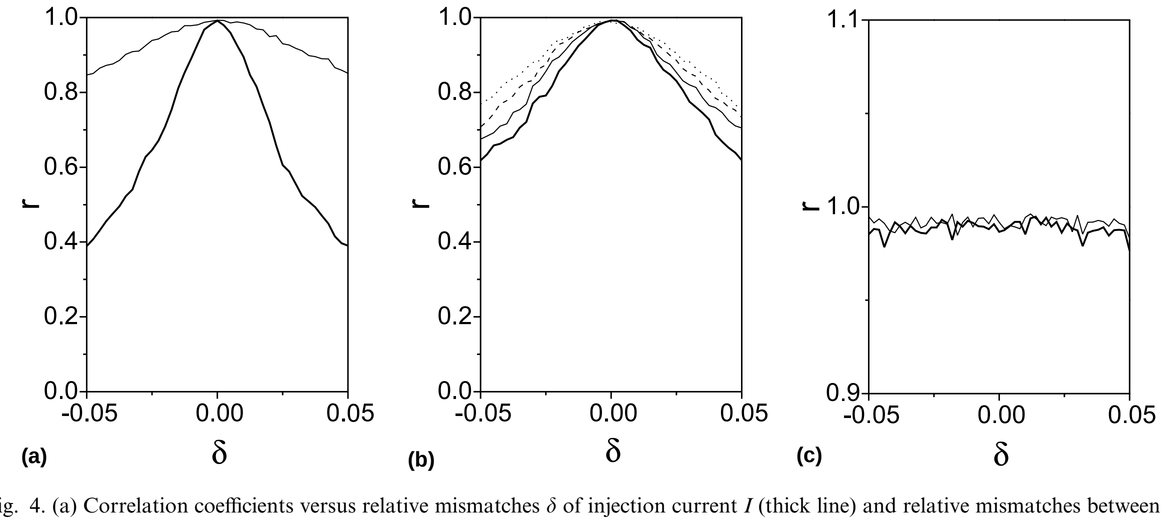

Fig. 4. (a) Correlation coefficients versus relative mismatches δ of injection current I (thick line) and relative mismatches between the feedback strength γ and the coupling strength σ (thin line), respectively. (b) Correlation coefficient versus the relative mismatches δ of carrier losses $1/\tau_s$ (thick line), photon losses $1/\tau_p$ (thin line), carrier number at transparency N_0 (dashed line), gain coefficient (dotted line). (c) Correlation coefficient versus the relative mismatches δ of gain saturation coefficient (thick line) and spontaneous emission rate (thin line). The correlation coefficients have been calculated from five series of 1 μs long.

controlled. In practical cases, synchronization must therefore occur also for small parameter mismatches.

Fig. 4(a) shows the dependence of the correlation coefficient r between the outputs of the transmitter and receiver lasers on the relative mismatch δ between the injection currents, on the one hand, and between the coupling strength at the receiver and the feedback strength at the transmitter, on the other hand. Fig. 4(b) shows the dependence of r on the mismatch between carrier and photon losses, carrier numbers at transparency and gain coefficients. The dependence of the correlation coefficient r on the relative mismatch between the gain saturation coefficients and spontaneous emission rates at the transmitter and receiver is presented in Fig. 4(c). The effect of the Langevin noise forces has been taken into account in these three figures. In each case, the correlation coefficient decreases as the mismatch increases but it remains above 0.9 if the discrepancies between the transmitter and the receiver parameters are smaller than 1%. The synchronization is therefore robust to small mismatches between corresponding parameters in both systems. The injection current is the most critical parameter, followed by the carrier and the photon lifetimes. By contrast, mismatches of several percents on the gain saturation coefficient and the spontaneous emission rate do not affect the synchronization. Worth noting is that a 1% mismatch between the gain coefficients corresponds to a frequency detuning of several hundreds GHz if the frequency dependence of the gain is taken into account [45].

In order to illustrate how large parameter mismatches can degrade the synchronization, we show the time evolution of both lasers, the synchronization error and the synchronization diagram in Fig. 5 for $I_2 = 1.1 \times I_1$ (i.e. a 10% mismatch). It is seen that the two lasers depart from each other and that the points in the synchronization diagram are strongly dispersed: the correlation coefficient is $r = 0.23$.

The gain saturation coefficient ε strongly affects the dependence of the synchronization quality on both parameter mismatches and noise level. The dependence of the correlation coefficient relative to injection current mismatches (Fig. 6(a)) and carrier losses (Fig. 6(b)) is displayed for three different values of ε, namely 8×10^{-8}, 4×10^{-8} and 2×10^{-8}. For a 1% mismatch on the carrier losses, the values of the correlation coefficient are respectively 0.95, 0.73 and 0.48. For a 1% mismatch on the injection currents, the values of the correlation coefficient are respectively 0.87, 0.62 and 0.43. Worth noting is that the noise impact

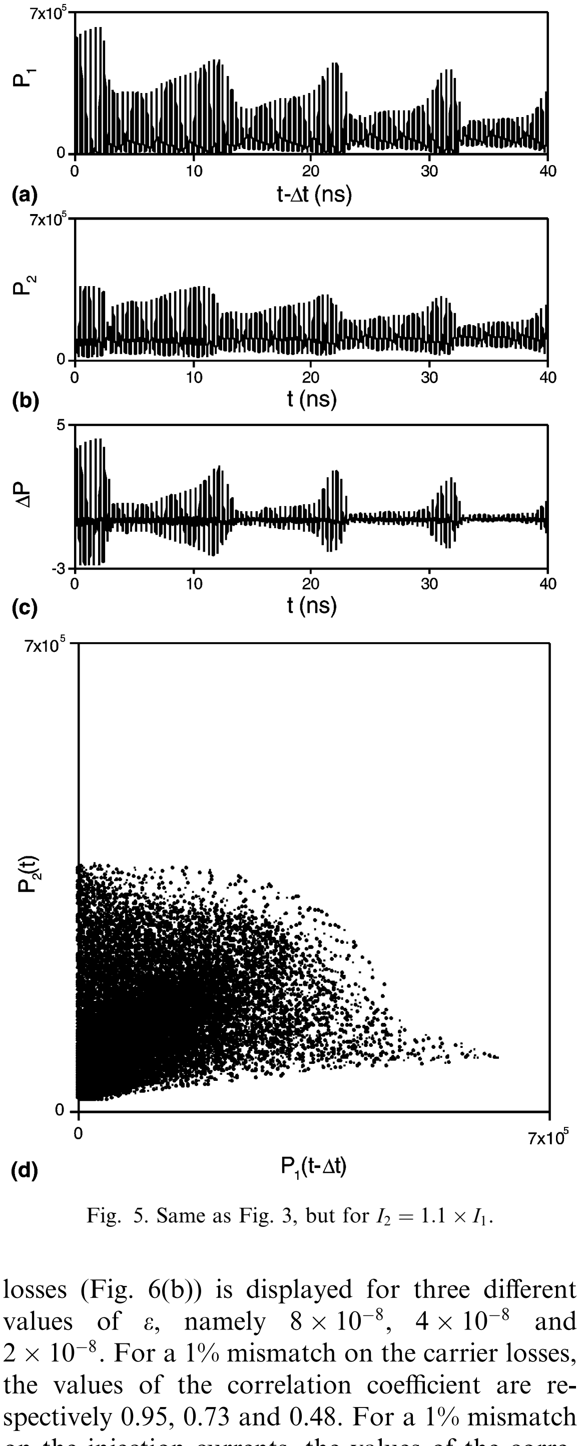

Fig. 5. Same as Fig. 3, but for $I_2 = 1.1 \times I_1$.

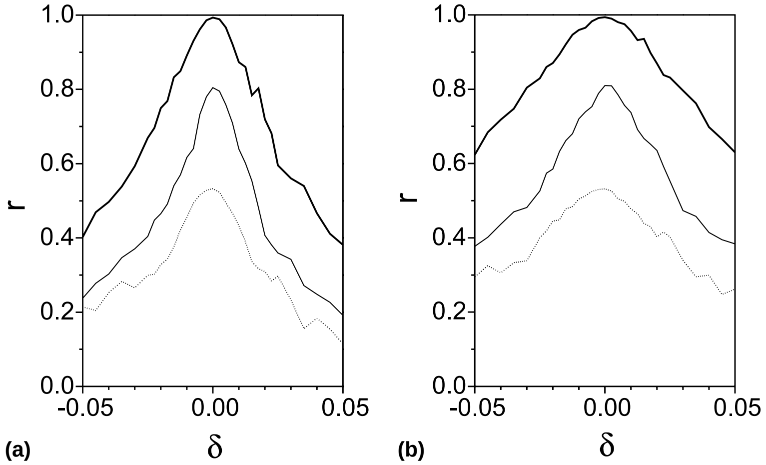

Fig. 6. Correlation coefficients versus the relative mismatches δ of injection current (a) and carrier losses (b) for three different values of the gain saturation coefficient ε, namely 8×10^{-8} (thick line), 4×10^{-8} (thin line) and 2×10^{-8} (dotted line). The correlation coefficients have been calculated from five series of 1 μs long.

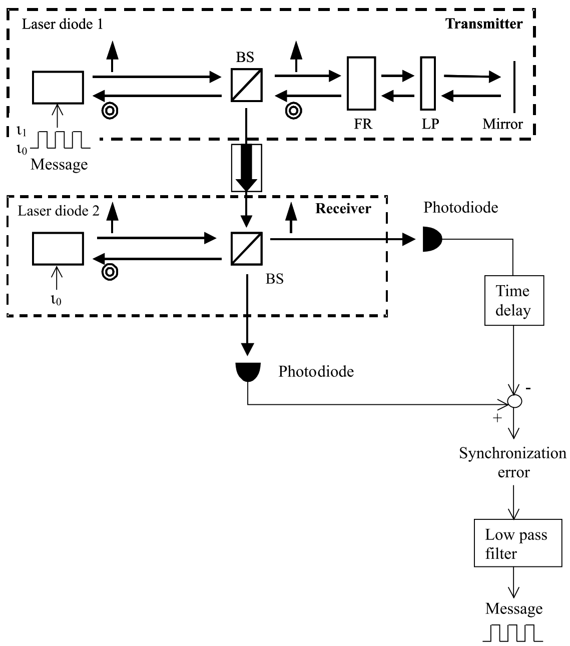

Fig. 7. Schematic representation of the secure communication scheme we propose. The message encoding is achieved by chaos shift keying, i.e., the bit stream modulates the transmitter diode injection current.

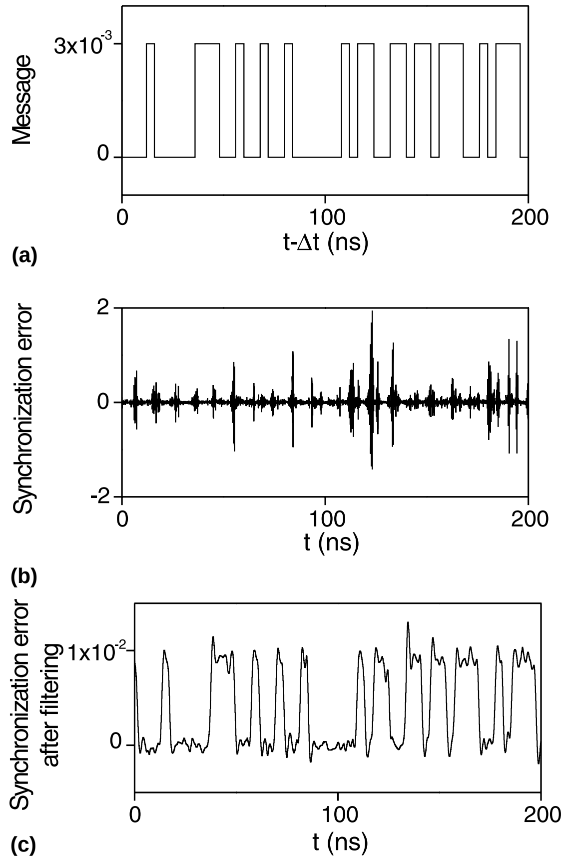

Fig. 8. Numerical simulations, (a) 250 Mbit/s input message, (b) synchronization error before filtering, (c) synchronization error after filtering.

increases as the gain saturation coefficient decreases: in absence of parameter mismatches (i.e. $\delta = 0$), the value of the correlation coefficient are respectively 0.99, 0.80 and 0.53 for $\varepsilon = 8 \times 10^{-8}$, 4×10^{-8} and 2×10^{-8} respectively. The role of ε on synchronization robustness can be qualitatively understood as follows. The linear damping rate increases drastically with the gain saturation coefficient ε [41,45]. This increase leads in turn to a more robust synchronization with respect to parameter mismatches. Indeed, as Voss [40] demonstrated, anticipating synchronization is the result of the interplay between delayed feedback and dissipation; moreover, the larger the damping resulting from dissipation, the more robust the synchronization to perturbations and parameter mismatches [40].

6. Cryptography implementing chaos shift keying

In this section, we consider message encoding by chaos shift keying (Fig. 7). The bit stream modulates the injection current at the transmitter, i.e. bits "0" and "1" correspond to two different values $\imath_0$ and $\imath_1$ of the injection current I_1. Here we choose $\imath_0 = 1.8 \times I_{\mathrm{th1}}$ and $\imath_1 = 1.003 \times \imath_0$ respectively. At the receiver, a replica of the transmitter laser is used. The injection current I_2 at the receiver is set to $\imath_0$. In absence of parameter mismatch and for noiseless conditions, message decoding is achieved by measuring the synchronization error. Whenever spontaneous emission noise and internal parameter mismatches are considered, the synchronization error fluctuates strongly and needs to be low-pass filtered in order to recover the message. We have tried different filters and found that a fourth-order Butterworth filter with a cut-off frequency of 1.3 times the bit rate is a convenient choice. Bits "0" are detected when the filtered synchronization error is close to zero, while bits "1" are detected when the synchronization error caused by the injection current mismatch is large. Fig. 8 illustrates a 250 Mbit/s message transmission for identical lasers in presence of noise.

The robustness of the synchronization to small mismatches between homologous internal and

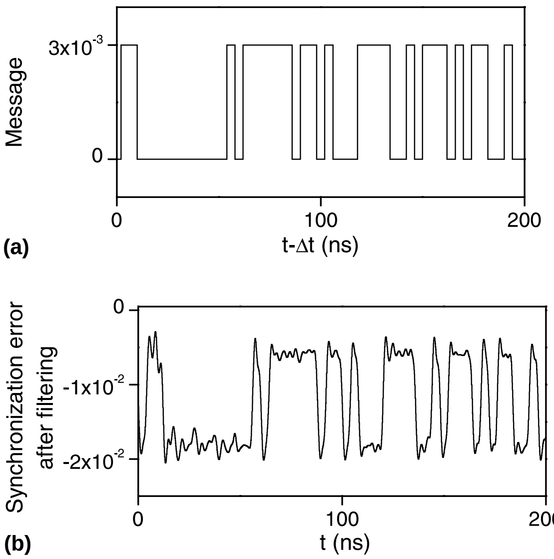

Fig. 9. Numerical simulations, (a) 250 Mbit/s input message, (b) synchronization error after filtering in the case of 0.5% mismatches on the carrier losses $1/\tau_s$ and the photon losses $1/\tau_p$.

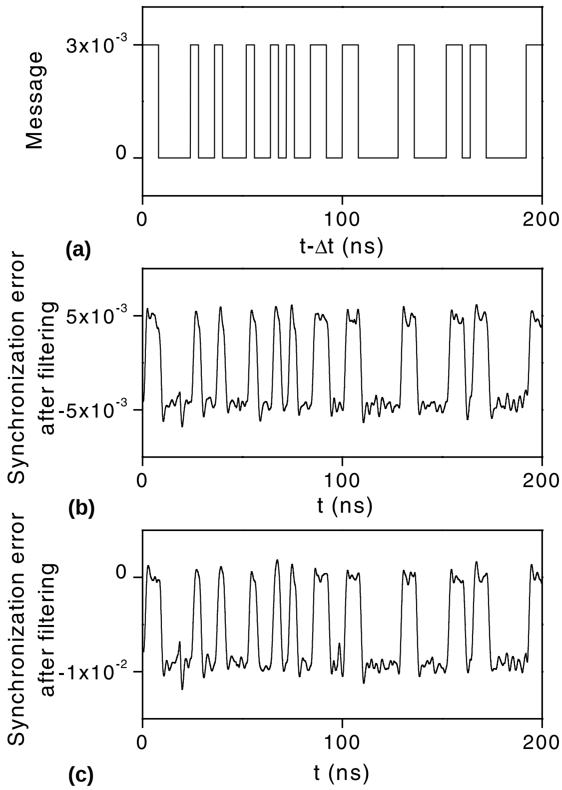

Fig. 10. Numerical simulations, (a) 250 Mbit/s input message, (b) synchronization error after filtering for $I_2 = (\imath_0 + \imath_1)/2$, (c) synchronization error after filtering for $I_2 = \imath_1$.

operating parameters makes the secure communication implementation practically feasible. In Fig. 9, we assume a 0.5% positive mismatch on the carrier and the photon losses: the message can still be recovered after filtering the synchronization error. The filtered synchronization error is shifted to lower values with respect to the case where the parameters match exactly. This result can be understood as follows: an increase of both photon and carrier losses leads to an increase of the threshold and therefore to a decrease of the average power.

One may think that even a very small error on I_2 could lead to the confusion of bits "0" and "1" since the corresponding values of the transmitter current, $\imath_0$ and $\imath_1$, are very close. For instance, what would happen if we take $I_2 = (\imath_0 + \imath_1)/2$? Would it result in the confusion of bits "0" and "1"? Figs. 10(b) and (c) are computed for the particular cases $I_2 = (\imath_0 + \imath_1)/2$ and $I_2 = \imath_1$ respectively. Both show that the message can be decoded and the bits are clearly resolved. The shift of the traces toward negative values results from the fact that when I_2 increases, the average power emitted by the receiver increases. This in turn leads to a decrease in average of the synchronization error (Eq. (7)) but does not degrade the message recovery.

The relative intensity noise (RIN) spectrum [45] of the transmitter output under injection current modulation by the message is displayed in Figs. 11(a) and (b) along with a reference RIN spectrum in absence of modulation (Figs. 11(c) and (d)). Although a slight difference can be observed at low frequencies, both spectra look very similar, making hard for the message to be extracted out of them. The encoded bits are also undetectable by direct observation in the time domain of the output of the transmitter (Fig. 12(b)). In order to check that the message can no longer be found by low-pass filtering the transmitted signal, Butterworth and Chebyshev filters have been used without success. As an example, Fig. 12(c) depicts the transmitter output after smoothing with a fourth-order low-pass Butterworth filter with 1.3 B cut-off frequency. The message cannot be recognized.

Assuming that the transmitter parameters are unknown, replication of the system by an

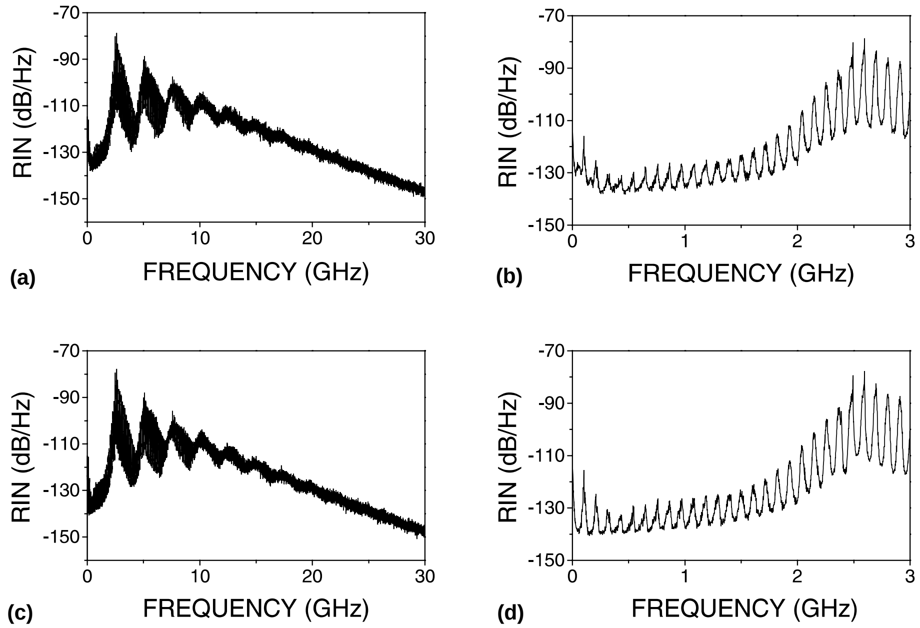

Fig. 11. RIN spectrum of the transmitter output: (a) the injection current is modulated by the message; (c) the injection current is not modulated by the message; (b) and (d) are low-frequency snapshots of (a) and (c) respectively.

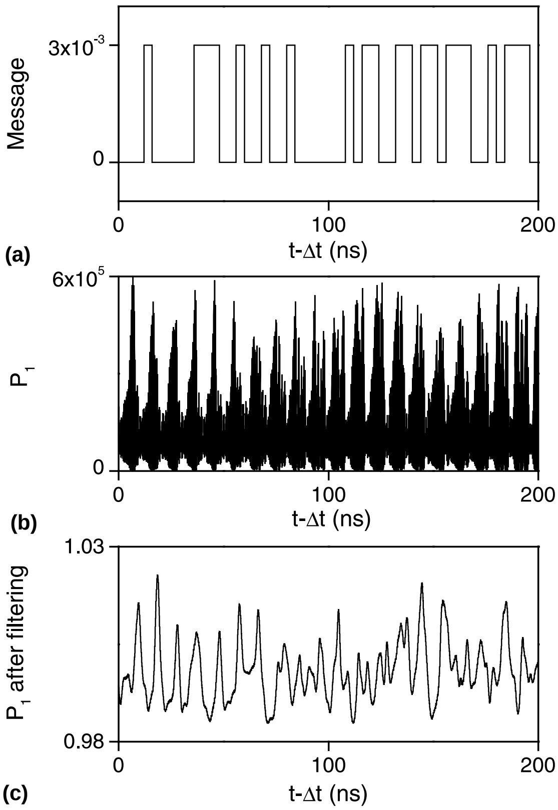

Fig. 12. Numerical simulations, (a) 250 Mbit/s input message, (b) output of the transmitter without filtering, (c) output of the transmitter after filtering.

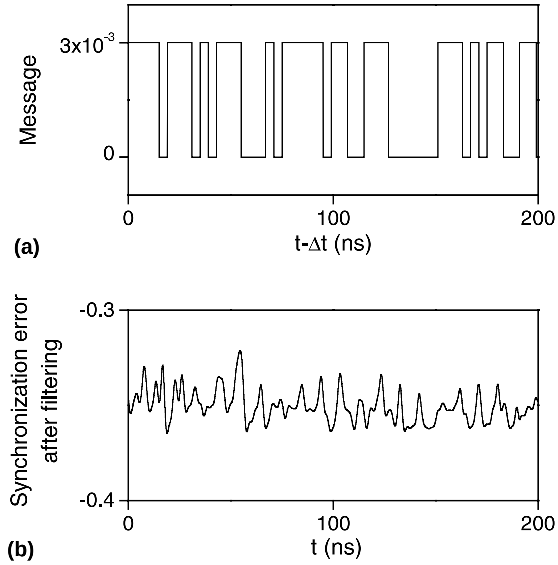

Fig. 13. Numerical simulations, (a) 250 Mbit/s input message, (b) synchronization error after filtering in the case of 5% mismatches on the carrier losses $1/\tau_s$ and the photon losses $1/\tau_p$.

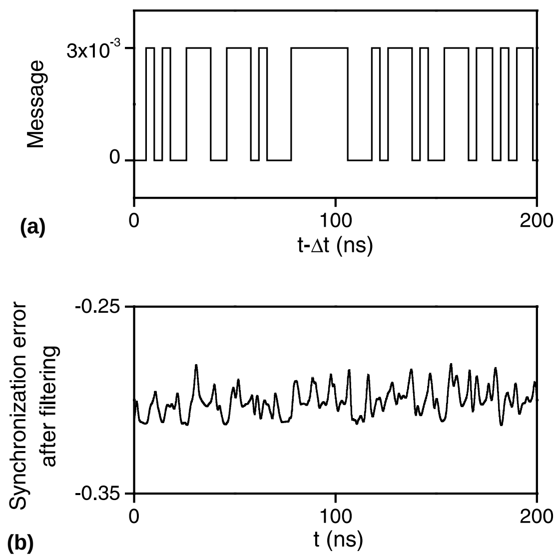

Fig. 14. Numerical simulations, (a) 250 Mbit/s input message, (b) synchronization error after filtering in the case of 5% mismatches on the injection current.

eavesdropper should be extremely difficult owing to a combination of two reasons. Firstly, parameter mismatches of only a few percents (typically 5%) lead to such severe degradations of the synchronization quality that recovery of the message is not possible. Secondly, from a laser chip to another, critical parameters such as carrier and photon lifetimes vary considerably (from 1 to 3 ns and 1 to 2 ps respectively [45]). In order to illustrate the difficulty of intercepting the encoded message without an adequate replica of the transmitter laser, we assume a 5% positive mismatch on the carrier losses and the photon losses (Fig. 13). We smooth the synchronization error with the same filter as above. Fig. 13(b) shows that the message is not recovered. In Fig. 14 we assume a 5% mismatch on the injection current; in this case neither, the message is not recovered. Finally, we have investigated the impact of both the bit rate and the amplitude of the current modulation on the quality of the message encoding–decoding process. In agreement with [19,20,23], we found that the period of modulation cannot be smaller than the transient time that the receiver laser takes to synchronize on the transmitter. The quality of the decoded message degrades as the bit rate increases until the message cannot longer be restored. Simulations show that a message with a bit rate larger than 1 Gbit/s cannot be decoded at the receiver. Furthermore, the modulation amplitude must be small enough to avoid a bit detection by direct observation of the chaotic signal. The limitation of the bit rate is a consequence of short bursts of desynchronization appearing in the synchronization error. These are observed whenever spontaneous emission noise or small parameter mismatches are assumed. Similar bursts of desynchronization have previously been reported between semiconductor lasers subject to coherent optical feedback and coupled in a coherent way [43]. The filtering of the synchronization error with a sufficiently low cut-off frequency filter partially washes out these bursts but leads to an unavoidable limitation of the bit rate.

7. Conclusion

We have proposed a synchronization scheme involving laser diodes subject to incoherent optical feedback and injection. This scheme is remarkable in that it requires no fine tuning of the laser optical frequencies, contrary to other schemes based on laser diodes subject to coherent optical feedback. This results from the absence of interaction between the intracavity fields and the injected and fed back fields. Our synchronization scheme is therefore attractive for experimental and operational investigations. We report on anticipating synchronization between the two lasers provided that the laser parameters and the operating parameters are adequately chosen. We have shown that anticipating synchronization is robust with respect to spontaneous emission noise and small parameter mismatches. We have also observed that the gain saturation drastically affects the robustness of the synchronization. Furthermore, by implementing the former synchronization scheme, we have numerically demonstrated message encoding/decoding by chaos shift keying. The difficulty to intercept the message without an adequate replica of the transmitter laser has also been checked.

A few issues of practical importance, including the effect of the transmission channel, the bit-error rate of secure communications and the dimension of the chaotic attractor, have not been addressed in this paper and will be the subjects of future investigations. The latter is worth knowing since a high-dimensional chaos is expected due to the delayed nature of the feedback. This should strongly complicate eavesdropping via reconstruction of the embedding phase space [21,35–38].

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