Quaternion Spline Interpolation for Suspension Kinematics and Dynamics

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EXTENDED ABSTRACT

1 Introduction

Problems of kinematics are, in many cases, characterized by a set of nonliner algebraic equations that have to be constructed and solved at each time step. The procedure can be computationally time consuming so that it is interesting to develop suitable methods to improve the simulation efficiency. Moreover, parametrization of finite rotations is an essential issue in multibody kinematics and dynamics. Among the available options, the concept of quaternions shows some interesting properties to describe body rotations, especially when dealing with interpolation.

The main idea is to solve in a preliminary step the kinematics of specific subsystems, i.e. to pre-compute the position and the orientation of important bodies of the subsystem, in terms of a set of independent parameters whose number corresponds to the number of degrees of freedom of the subsystem. The pre-computation leads to a look-up table from which the situation of each body, i.e. its X, Y, Z coordinates and the quaternions describing its orientation, can be computed by interpolation. The size of the table and the distribution of the pre-computed points are also addressed in this study: with an optimal distribution, the required accuracy can be obtained with a minimal size of the table, and consequently with lower memory requirements. The main motivation of the work is the efficient representation of suspension kinematics (see Figure 1) for the purpose of vehicle dynamics problems. It is particularly relevant for suspensions, whose up and down motion can be interpolated from only one parameter. For example, authors of [1] use two dimensional analytical functions to express suspension kinematics and then solve problems of vehicle dynamics.

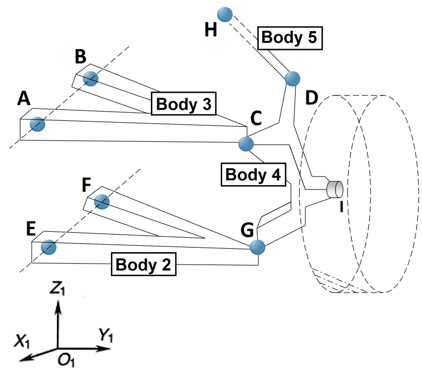

Figure 1: Scheme of a double wishbone suspension

2 Spline, B-spline and quaternion spline interpolation

Generally, *spline interpolation* is a form of interpolation where the interpolant is a special type of piecewise low degree polynomial called a spline. Spline interpolation provides lower interpolation error [3] and also avoids the problem of Runge's phenomenon, in which oscillation can occur when interpolating using high degree polynomials. B-spline as another spline function is a piecewise polynomial function. However, B-spline function is defined as a linear combination of control points p_i and **b**asis functions, which is also the origin of its name – **B**-spline. The function has several useful properties such as local support property; changing p_i affect the curve in the parameter range $x_i < x < x_{i+n}$ [4].

The base functions $B_i^k(t)$'s are defined by the following recurrence relation [5]:

$$B_i^k(t) = \frac{t-t_i}{t_{i+k-1}-t_i}B_i^{k-1}(t) + \frac{t_{i+k}-t}{t_{i+k}-t_{i+1}}B_{i+1}^{k-1}(t), \qquad \text{where} \qquad B_i^1(t) = \begin{matrix} 1 & t_i \le t \le t_{i+1}, \\ 0 & \text{otherwise.} \end{matrix} \tag{1}$$

The B-spline quaternion curve with a cumulative basis form is formulated as [5]

$$\hat{Q}(t) = \hat{q}_0^{\tilde{B}_0^k(t)} \prod_{i=0}^{n+1} (\hat{q}_{i-1}^{-1}\hat{q}_i)^{\tilde{B}_i^k(t)}, \qquad \text{where} \qquad \tilde{B}_i^k(t) = \sum_{j=i}^{n+1} B_i^k(t). \tag{2}$$

where the control points $\hat{q}_i$ are precomputed so as to reproduce a given sequence of data quaternions $\hat{Q}_i$ ($i = 0, 1, \ldots n$). We assume k = 4, so C^2 continuity is achieved [5].

3 Obtained results and their discussion

Every rotation in a three-dimensional Euclidean space can be parametrized by two quantities: a unit vector **e** indicating the direction of an axis of rotation, and the angle θ describing the magnitude of the rotation about the axis. Assuming 3 body configurations i, $i+1$ and $i+2$, we get 2 relative angles ($\theta_{i,i+1}$ and $\theta_{i+1,i+2}$), 2 rotation axes ($\mathbf{e}_{i-1,i}$ and $\mathbf{e}_{i,i+1}$), and 1 so-called axis difference ϕ_i defined as the angle between $\mathbf{e}_{i-1,i}$ and $\mathbf{e}_{i,i+1}$. Both, θ and ϕ measure the variation between successive body configurations and play an important role in the precision of the interpolation.

The interpolation methodology was studied on a double wishbone car suspension (Figure 1) whose kinematic solution was obtained from a solver based on the Cartesian coordinates approach. The precision of the interpolation was measured by angle error $\theta_{ex,in}$, which is defined as an angle between exact and interpolated orientation.

Different input data for the interpolation were generated and it turned out that the precision is not determined only by the values of θ and/or ϕ, but also by their smoothness. To demonstrate this phenomena the look-up table data generation was performed and a threshold was imposed in the same time for θ and ϕ. The kinematics had to be recomputed to fulfil desired $\theta_{0.2}$ and $\theta_{0.01}$, which are angle distances obtained by converting θ to a smoothing spline with a tuning (or smoothing) parameter 0.2 and 0.01 [6]. Figure 2 shows θ, $\theta_{0.2}$ and $\theta_{0.01}$, and Figure 3 shows the achieved angle errors of the interpolation while using look-up tables corresponding to θ, $\theta_{0.2}$ and $\theta_{0.01}$, respectively.

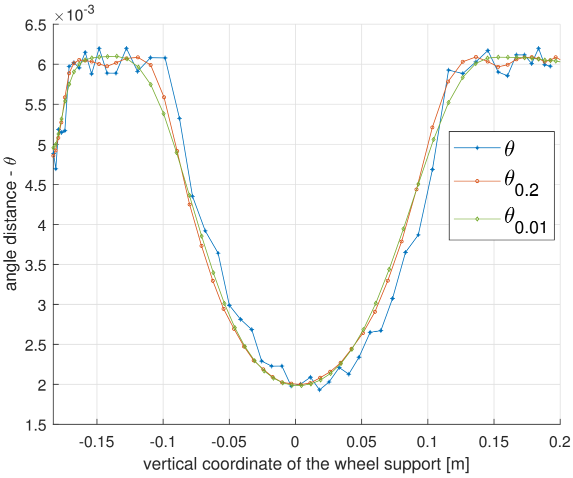

Figure 2: Angle distance θ

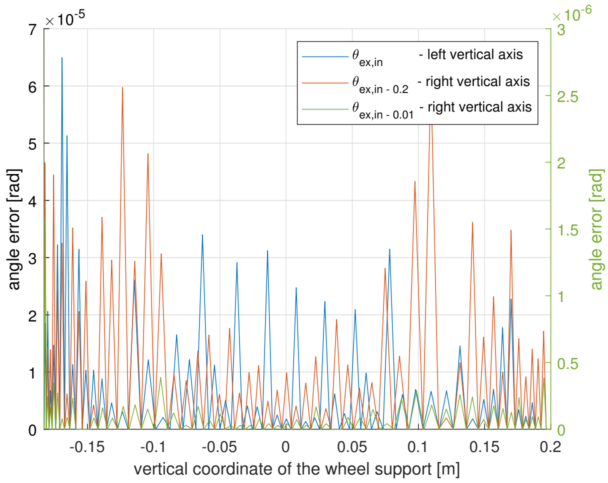

Figure 3: Angle error

4 Conclusion

This paper deals with the usage of quaternion interpolation in the multibody kinematics, which allows to reduce the computational costs for more complex dynamic analyses. The parametric study showed the importance of the continuity of input data. The smoother the input data are the lower the angle error is achieved. However, look-up table size remains the same. This proves the importance of the right choice of input data. Presented methodology is further implemented in EasyDyn for a testing simple vehicle (Figure 4). It is further planned to use the introduced approach for fast dynamical simulations during optimization processes needed in the Formula SAE development.

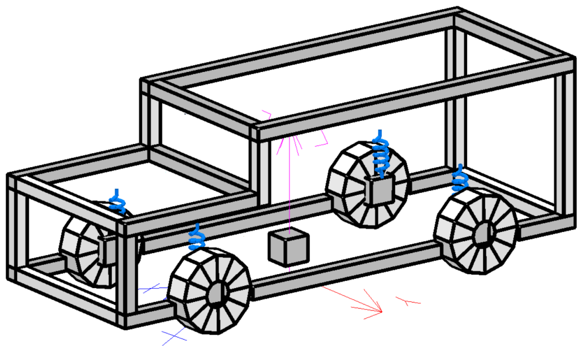

Figure 4: Ilustration of vehicle dynamic model in EasyDyn

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