Boson and neutron stars with increased density

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Abstract

We discuss boson stars and neutron stars, respectively, in a scalar-tensor gravity model with an explicitly time-dependent real scalar field. While the boson stars in our model – in contrast to the neutron stars – do not possess a hard core, we find that the qualitative effects of the scalarization are similar in both cases : the presence of the gravity scalar allows both type of stars to exist for larger central density as well as larger mass at given radius than their General Relativity counterparts. In particular, we find new types of scalarized neutron stars which have radii very close to the corresponding Schwarzschild radius and hence are comparable in density to black holes.

1 Introduction

With increased interest in astrophysical objects and, in particular, their gravitational properties, compact objects have come to the focus of theoretical research again. These objects are normally defined to have strong gravitational fields and as such are a good testing ground for the momentarily accepted best model of the gravitational interaction – General Relativity (GR) – as well as extensions thereof and even alternative gravity models. Compact objects come in two varieties: either they are star-like with a globally regular space-time or they possess a physical singularity shielded from observation by an event horizons. The former are neutron stars and boson stars, respectively, the latter black holes. While neutron stars and black holes are known to exist and can now be studied with unprecedented precision, boson stars [1] are hypothetical objects made principally of scalar bosonic particles. Evaluating and testing gravity theories is also vital in order to understand two of the great puzzles of current day physics : the nature of dark matter and dark energy. While dark matter is understood to be some kind of matter that interacts only gravitationally and probably has its origin in physics beyond the Standard Model of Particle physics, the nature of dark energy remains elusive. Consequently, suggestions for a modification of GR have been made on the ground of so-called "scalar-tensor" gravity models [2, 3, 4], an idea that relates back to Horndeski [5]. Classes of scalar-tensor gravity models have then been studied thoroughly and a classification, named "Fab Four", was achieved [6, 7]. In this paper, we are interested in a particular model dubbed "John" in this exact classification. As has been shown in [8], static, spherically symmetric black holes can carry scalar hair in this model if the scalar field is explicitly (and linearly) time-dependent. In particular, the Noether current associated to the shift symmetry of the Galileon-type gravity scalar does not diverge on the horizon in this model. In [9], neutron stars have been studied for a specific polytropic equation of state and it has been claimed that the astrophysical objects resulting from the model are viable and not in conflict with constraints from observations. Here, we revisit these results and compare them with those related to another EOS used in [10]. We find that the solutions obtained with the EOS of [10] (a) are in perfect agreement with results obtained in [10] and (b) only this EOS leads to neutron stars possessing the proper mass-radius relation. While neutron stars are matched to the Schwarzschild solution at the exterior radius, we also discuss boson stars in this paper that reach the Schwarzschild solution only asymptotically and hence do not possess a "hard core".

Our paper is organized as follows: in Section 2 we discuss the scalar-tensor gravity model coupled to an appropriate energy-momentum content. In Section 3, we present our results for boson stars, while Section 4 contains our findings for neutron stars. We summarize and conclude in Section 5.

2 The model

In this paper, we present our results for a scalar-tensor gravity model of Horndeski type coupled minimally to an appropriate matter content with Lagrangian density $\mathcal{L}_{\rm matter}$. The action reads :

$$\mathcal{S} = \int \left(\kappa\mathcal{R} + \frac{\eta}{2}G^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi + \mathcal{L}_{\rm matter}\right)\sqrt{-g}{\rm d}^4x \ , \tag{1}$$

where $\kappa = (8\pi G)^{-1}$. This action contains the standard Einstein-Hilbert term as well as a non-miminal coupling term – first discussed in [6, 7] – that couples a gravity scalar ϕ to the Einstein tensor $G_{\mu\nu}$ via a coupling constant η. For $\eta = 0$, we recover standarad General Relativity (GR).

In the following, we will assume the matter content of the model to be that of (a) a complex valued scalar field and (b) a perfect fluid with a given equation of state, respectively. In the latter case, the model has solutions in the form of neutron stars, while the complex scalar field in curved space-time describes boson stars. The gravity equations then read

$$\kappa G_{\mu\nu} + \eta\left(\partial_\alpha\phi\partial^\alpha\phi G_{\mu\nu} - \frac{1}{2}\epsilon_{\mu\alpha\sigma\rho}R^{\sigma\rho\gamma\delta}\epsilon_{\nu\beta\gamma\delta}\nabla^\alpha\phi\nabla^\beta\phi + g_{\mu\alpha}\delta^{\alpha\rho\sigma}_{\nu\gamma\delta}\nabla^\gamma\nabla_\rho\phi\nabla^\delta\nabla_\sigma\phi\right) = T_{\mu\nu} \ , \tag{2}$$

where $T_{\mu\nu}$ denotes the energy-monentum tensor of the matter content. The model has a shift symmetry $\phi \to \phi + c$, where c is a constant, which leads to the existence of a locally conserved Noether current

$$J^\mu = -\eta G^{\mu\nu}\nabla_\nu\phi \ \ , \ \ \nabla_\mu J^\mu = 0 \ . \tag{3}$$

In the following, we will assume a spherically symmetric Ansatz for our solutions [8]

$${\rm d}s^2 = -b(r){\rm d}t^2 + \frac{{\rm d}r^2}{f(r)} + r^2\left({\rm d}\theta^2 + \sin^2\theta{\rm d}\varphi^2\right) \ , \ \phi(t,r) = qt + F(r) \ , \tag{4}$$

i.e. the tensor part is static, while the gravity scalar has an explicit time-dependence. The non-vanishing components of the Noether current (3) then read

$$J^t = \eta q\frac{f'r + f - 1}{r^2 b} \quad , \quad J^r = \eta\phi'\frac{f\left(-b'rf - bf + b\right)}{r^2 b} \quad , \tag{5}$$

where the prime now and in the following denotes the derivative with respect to r. The norm of the Noether current is

$$J_\mu J^\mu = \eta^2\left[-q^2\frac{(f'r + f - 1)^2}{r^4 b} + \phi'^2\frac{f\left(b'rf + bf - b\right)^2}{r^4 b^2}\right] \ . \tag{6}$$

Since $f(r \ll 1) \sim 1 + f_2 r^2$ and $b(r \ll 1) \sim 1 + b_2 r^2$ with f_2, b_2 constants (see below for explicit expressions), the norm of the Noether current is finite for all $r \in [0 : \infty)$.

We want to consider a non-vanishing energy-momentum tensor that sources the tensor and scalar gravity fields. In the following, we will choose the energy-momentum tensor to be of the form

$$T^\nu_\mu = {\rm diag}(-\rho, P_r, P_t, P_t) \ , \tag{7}$$

where ρ is the energy-density, while P_r and P_t are the radial and tangential pressures, respectively. The gravity equations are then a set of coupled, non-linear ordinary differential equations that have to be solved numerically.

However, we can simplify the analysis by noting that the equation for the gravity scalar ϕ, which comes from the rr-component of (2), can be solved algebraically in terms of the other functions :

$$\eta(\phi')^2 = \frac{2r^2}{f} P_r + \frac{1-f}{bf}\eta q^2 \quad . \tag{8}$$

This allows the elimination of ϕ from the remaining equations and we are left with the equations for the metric functions which read :

$$\mathcal{F}_1 f' + \mathcal{F}_2 = 0 \quad , \quad \frac{b'}{b} = \frac{1-f}{fr} \tag{9}$$

with

$$\mathcal{F}_1 = 4\kappa br + 2br^3 P_r - 3\eta q^2 rf \tag{10}$$

and

$$\mathcal{F}_2 = 3\eta q^2 f(1-f) + 2b\left[\rho r^2(f+1) + 2fr^2 P_r + 4fr^2 P_t + 2\kappa(f-1)\right] \quad . \tag{11}$$

Note that the second equation in (9) ensures that the Noether current J^μ is covariantly conserved, i.e. $\nabla_\mu J^\mu = 0$ and is, in fact, the rt-component of the Einstein equation.

Star-like astrophysical objects are typically characterized in terms of their mass-radius relation. The gravitational mass M_G of this solution is given in terms of the asymptotic behaviour of the metric function $f(r)$:

$$f(r) \xrightarrow[r\to\infty]{} 1 - \frac{M_G}{4\pi\kappa r} + \mathcal{O}(r^{-2}) \ , \tag{12}$$

while the radius will be defined differently in the case of boson stars and neutron stars, see below. Since asymptotically, the metric function $b(r)$ becomes equal to $f(r)$ and we assume in the following that either the pressure P_r tends exponentially to zero asymptotically (in the case of boson stars) or is strictly zero (in the case of neutron stars), we observe that the mass M_G can also be read off from the behaviour of the gravity scalar at infinity. Using (8) we find that

$$(\phi')^2 \xrightarrow[r\to\infty]{} \frac{M_G}{4\pi\kappa r} q^2 \ . \tag{13}$$

In other words: $M_G q^2/(4\pi\kappa)$ constitutes the "charge" associated to the scalar field $(\phi')^2$.

3 Boson stars

In the case of boson stars, the energy-momentum content is that of a complex valued scalar field Ψ, which – in contrast to the neutron star model discussed in Section 4 – is not of perfect fluid type. The energy-momentum tensor reads :

$$T_{\mu\nu} = -g_{\mu\nu}\left[\frac{1}{2}g^{\alpha\beta}\left(\partial_\alpha\Psi^*\partial_\beta\Psi + \partial_\beta\Psi^*\partial_\alpha\Psi\right) + m^2\Psi\Psi^*\right] + \partial_\mu\Psi^*\partial_\nu\Psi + \partial_\nu\Psi^*\partial_\mu\Psi \ , \tag{14}$$

where m denotes the scalar boson mass. This model contains an additional conserved Noether current due to the internal global U(1) symmetry $\Psi \to \exp(i\chi)\psi$, where χ is a constant. This reads

$$j^\mu = -\frac{i}{2}\left(\Psi^*\nabla^\mu\Psi - \Psi\nabla^\mu\Psi^*\right) \quad , \quad \nabla_\mu j^\mu = 0 \ . \tag{15}$$

With the standard spherically symmetric Ansatz for boson stars

$$\Psi(r,t) = \exp(i\omega t)H(r) \ , \tag{16}$$

where $\omega > 0$ is a constant, the non-vanishing components of the energy-momentum tensor read

$$\rho = f(H')^2 \quad + \quad \left(m^2 + \frac{\omega^2}{b}\right)H^2 \ ,$$

$$P_r = f(H')^2 - \left(m^2 - \frac{\omega^2}{b}\right)H^2 \quad , \quad P_t = -f(H')^2 - \left(m^2 - \frac{\omega^2}{b}\right)H^2 \ . \tag{17}$$

The locally conserved current and associated globally conserved Noether charge are :

$$j^t = -\frac{\omega H^2}{b} \quad , \quad Q = -\int \mathrm{d}^3x \, \sqrt{-g} \, j^t = 4\pi\omega \int \mathrm{d}r \, r^2 \, \frac{H^2}{\sqrt{bf}} \tag{18}$$

Note that in the model with ungauged U(1) symmetry, the Noether charge Q is frequently interpreted as the number of bosonic particles of mass m that make up the boson star. Finally the field equation for Ψ reads :

$$H'' + \frac{1}{2}\left(\frac{4}{r} + \frac{f'}{f} + \frac{b'}{b}\right) H' + \frac{1}{f}\left(\frac{\omega^2}{b} - m^2\right) H = 0 \ . \tag{19}$$

The asymptotic behaviour of $H(r)$ that can be read of from (19) is :

$$H(r) \xrightarrow[r\to\infty]{} \frac{1}{r} \exp\left(-\sqrt{m^2 - \omega^2} r\right) \ , \tag{20}$$

i.e. although the scalar field making up the boson star decays fast, the star does not have a "hard surface" like the neutron star discussed below. Rather, its energy density ρ and pressures P_r and P_t, respectively, tend to zero only asymptotically. We can, however, use an estimate of the radius R of the boson star which is given as follows :

$$\langle R \rangle = \frac{1}{Q} \int \ \mathrm{d}^3x \sqrt{-g} \ r \ j^t \ = \frac{4\pi\omega}{Q} \int \mathrm{d}r \ r^3 \frac{H^2}{\sqrt{bf}} \ . \tag{21}$$

The equations (9) and (19) have to be solved with boundary conditions that guarantee the regularity of the solution at the origin and its finiteness of energy. The appropriate conditions read :

$$b'(0) = 0 \ \ , \ \ H'(0) = 0 \ \ , \ \ b(\infty) = 1 \ \ , \ \ H(\infty) = 0 \tag{22}$$

where the constant $H(0) \equiv H_0$ is an *a priori* free parameter that determines the value of ω as well as the central density of the boson star, see (17), via $\rho(0) = (m^2 + \omega^2/b(0))H_0^2$. As is well known from boson stars in GR, the parameter $H(0)$ can be increased arbitrarily such that a succession of branches of boson stars exist that end only for $H(0) \to \infty$ and $b(0) \to 0$ in this limit. This will be different for the scalar-tensor boson stars studied here. The expansion of the fields around the origin already gives hints that this should be the case. We find :

$$b(r) = b_0 \left[1 + \frac{4H_0^2(2\omega^2 - b_0 m^2)}{3(4\kappa b_0 - 3\eta q^2)} r^2 + \mathcal{O}(r^4)\right] \quad , \quad f(r) = f_0 \left[1 + \frac{1}{6}\left(m^2 - \frac{\omega^2}{b_0}\right) r^2 + \mathcal{O}(r^4)\right] \ , \tag{23}$$

where $b_0 = b(0)$ and $f_0 = f(0)$. This implies that we have to require $4\kappa b_0 - 3\eta q^2 \neq 0$. As we will demonstrate in the following, this condition is crucial in the limitation of the domain of existence of the solutions for $\eta > 0$. Note that for $\eta < 0$ another limitation exists, related to the requirement of positivity of the right hand side of (8).
The system of equations is unchanged under the following rescalings

$$r \to \frac{r}{m} \ \ , \ \ \omega \to m\omega \ \ , \ \ H \to \sqrt{\kappa} H \ \ , \ \ \eta \to \kappa\eta \ \ , \ \ \phi \to \frac{\phi}{m} \ , \tag{24}$$

which rescales the radius, mass and Noether charge of the boson star as follows :

$$\langle R \rangle \to \frac{\langle R \rangle}{m} \ \ , \ \ M_G \to \frac{M_G}{m} \ \ , \ \ Q \to \frac{\kappa}{m^2} Q \ . \tag{25}$$

In the following we will choose $\kappa = 1$, $m = 1$, $\eta = \pm 1$ without loss of generality.

3.1 Numerical results

We have solved the equations numerically using a collocation method for boundary-value differential equations using damped Newton-Raphson iterations [11]. The relative errors of the solutions are on the order of 10^{-6} –

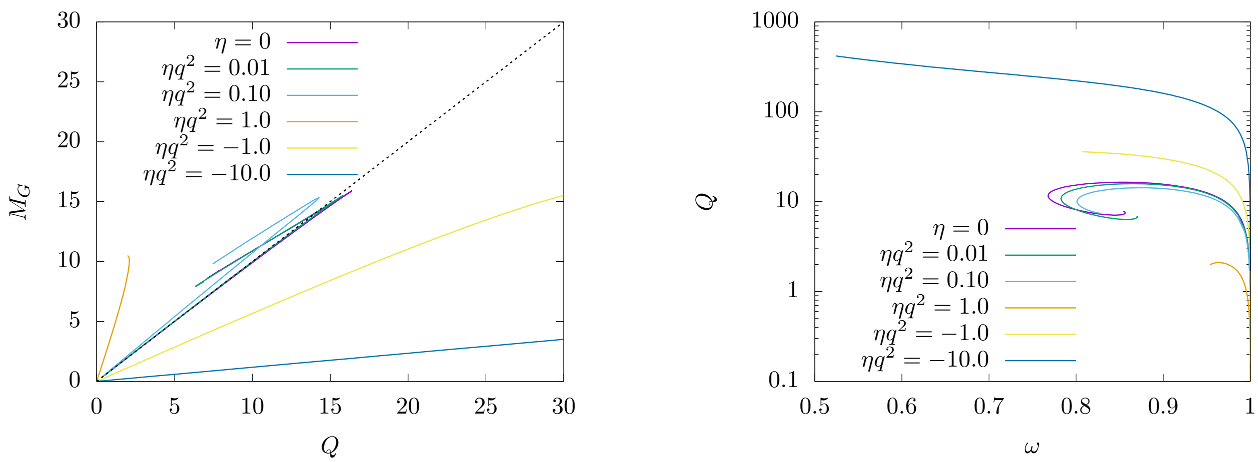

Figure 1: We show the gravitational mass M_G as function of the Noether charge Q (left) as well as the Noether charge Q as function of ω (right) for GR boson stars ($\eta = 0$) and boson stars with time-dependent scalar hair for several values of ηq^2.

10^{-10}. The constants to be varied are the combination ηq^2 as well as ω (or equivalently $H(0)$). From (20) we know that with the rescalings (24) the angular frequency is restricted by: $\omega^2 \leq 1$.

In Fig. 1 we show the relation between Noether charge Q and gravitational mass M_G (left) and the dependence of the Noether charge on ω (right), respectively, for several values of ηq^2 including the GR case $\eta = 0$. While for $\eta = 0$, we can increase the value of $H(0)$ arbitrarily, this is no longer the case in the scalar-tensor gravity model studied here. For $\eta q^2 > 0$, the curves shown in Fig. 1 are limited by the requirement discussed above which, with our choice of constant, reads : $4b_0 - 3\eta q^2 > 0$. We find that the branches of solutions stop at $4b_0 - 3\eta q^2 = 0$. For the GR case and $H(0) \to \infty$ the value of the metric function $b(r)$ at $r = 0$, b_0, tends to zero. This is obviously no longer true and hence boson stars with time-dependent scalar hair are limited in their central density of the star. For ηq^2 sufficiently large, see the curves for $\eta q^2 = 1.0$, this also leads to the observation that the Noether charge Q is strongly limited and much smaller than in the GR case. On the other hand, the mass M_G is of the same order of magnitude. Hence, scalar-tensor boson stars with time-dependent scalar fields and $\eta q^2 > 0$ are comparable in mass, but consist of an order of magnitude smaller number of

ηq^2	$M_{G,\max}$	$Q_{\max}$	$\langle R\rangle^*$	ω^*	$\rho(0)^*$	$P(0)^*$
0	15.91	16.40	3.10	0.85	0.19	0.04
0.01	15.50	15.81	3.22	0.86	0.15	0.03
0.1	15.32	14.27	3.54	0.87	0.12	0.02
1.0	10.46	2.10	6.44	0.96	$3 \cdot 10^{-3}$	$2 \cdot 10^{-4}$
-1.0	16.68	35.93	2.84	0.83	0.47	0.11
-10.0	17.90	416.81	2.50	0.79	4.13	1.00

Table 1: We give the maximal values of the mass $M_{G,\max}$ as well as the maximal value of the Noether charge $Q_{\max}$ for different values of ηq^2. Also given is the mean radius $\langle R\rangle^*$, the angular frequency ω^*, the central density $\rho(0)^*$ as well as the central pressure $P_r(0)^* = P_t(0)^* \equiv P(0)^*$ at the maximal value of the mass, i.e. at $M_{G,\max}$.

scalar bosonic particles as compared to their GR counterparts. Moreover, their central density $\rho(0)$ and central pressure $P_r(0) = P_t(0) \equiv P(0)$ is comparable to the GR case, see Table 1 as long as ηq^2 is not too large. For $\eta q^2 = 1.0$, we find that both the central density as well as the central pressure are very small.

For $\eta q^2 < 0$, we observe the exact opposite: the boson stars can contain many more scalar particles. The Noether charge increases strongly, while the mass remains of the same order of magnitude. We present some numerical values of our results in Table 1. As can be clearly seen here in combination with the data presented in Fig. 2, the mass M_G varies only slightly with ηq^2 and decreases when increasing ηq^2. The Noether charge Q on the other hand varies strongly with ηq^2. Moreover, as can be seen from Fig. 2 a gap in ηq^2 exist for which scalarized boson stars are not possible. This gap depends on the value of the frequency ω and increases when ω decreases, i.e. when ω decreases.

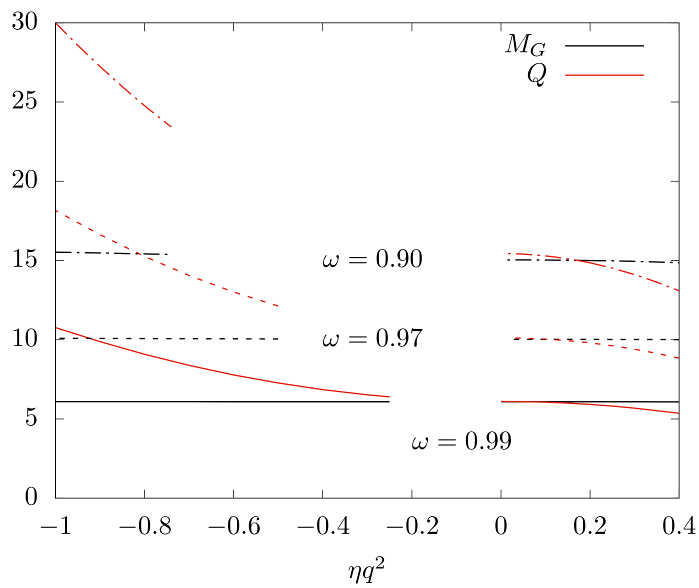

Figure 2: We show the Noether charge Q (red) and the gravitational mass M_G (black), in dependence of ηq^2 for the boson star solutions with $\omega = 0.99$ (solid), $\omega = 0.97$ (dashed) and $\omega = 0.90$ (dotted-dashed) respectively.

Finally, and since we want to compare neutron stars with scalar hair with boson stars with scalar hair in this paper, we show the mass-radius relation for the boson stars in Fig. 3 for several values of ηq^2, see also Table 1 for some values. We find that boson stars with large radius are practically not influenced by the scalar-tensor coupling, but very compact boson stars are. The radius of the boson star at maximal mass, $\langle R \rangle^*$ (see Table 1) is larger for all positive ηq^2 that we have studied, however smaller for all negative values of ηq^2.

If we use the standard argument that a boson star can be thought of as a system of a number Q of scalar particles of mass m, we can compare the actual mass M_G of the boson star and the mass of Q scalar bosons which is mQ. For $M_G < mQ$, we expect the boson star to form a bound system of these individual bosons and hence be stable with respect to the decay into those particles. Note that with our rescalings, the scalar boson mass $m \equiv 1$. Inspection of Fig. 1 demonstrates that decreasing ηq^2 from zero, the binding between the scalar particles increases, suggesting that for $\eta < 0$ the non-minimal coupling has effectively an attractive nature. On the other hand, for $\eta q^2 > 0$, we find that $M_G > Q$ for a part of the second branch of solutions (see $\eta q^2 = 0.01$) or that – for sufficiently large ηq^2 – all boson star solutions are unstable to decay into Q individual bosons (see curves for $\eta q^2 \geq 0.01$.)

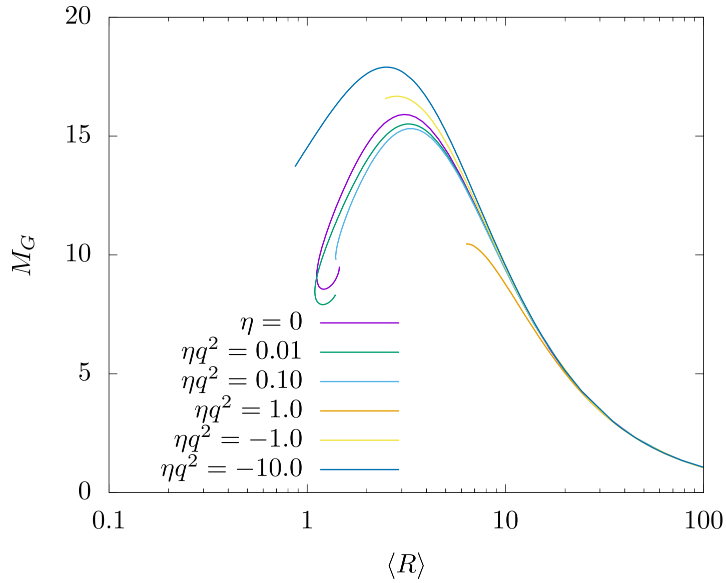

Figure 3: We show the gravitational mass M_G in function of the mean radius $\langle R \rangle$ of the boson star with time-dependent scalar hair for several values of ηq^2. For comparison we also show the mass-radius relation for the GR limit ($\eta = 0$).

4 Neutron stars

The energy-momentum tensor for a neutron star is typically assumed to be that of a perfect fluid with $P_r = P_t \equiv P$ and an equation of state (EOS) relating ρ and P. In addition to the gravity equation (2), we then also have to solve the Tolman-Oppenheimer-Volkoff (TOV) equation which reads :

$$P' = -\frac{b'}{2b}(P + \rho) \ . \tag{26}$$

In the following, we will use different equations of state to study the properties of neutron stars. It will be of the general polytropic form

$$\rho = CP + KP^{1/\Gamma} \tag{27}$$

where C and K are constants and Γ is the so-called adiabatic index. We have restricted our analysis to some specific cases :

- the first equation of state ("EOSI" in the following) has been used in [9] in the exact same context as in our work and has $C = 1$ and $\Gamma = 3/2$,
- the second equation of state ("EOSII") has $C = 1$ and $\Gamma = 5/3$,
- the third equation of state ("EOSIII") has $1/C = \Gamma - 1$ and $\Gamma = 2.34$. This has been used in [16] as a good fit to a realistic equation of state.

Note that although we use the letter K in (27) for all three equations of state, this coupling has different mass dimensions in the individual cases. Remembering that ρ and P have mass dimension -2 in natural units, the mass dimension of K is $-2 + 2/\Gamma$.

In Fig.4, we show a realistic equation of state [17] - the so-called BSk20 EOS, which is based on Hartree-Fock-Bogoliubov mass models, in comparison to the polytropic EOSI, EOSII and EOSIII, respectively. We have chosen the respective values of K corresponding to the maximal value of the gravitational mass (for details see our numerical results below). This figure demonstrates that especially the polytropic equation of state EOSIII fits the realistic BSk2 equation of state very well at high density and high pressure.

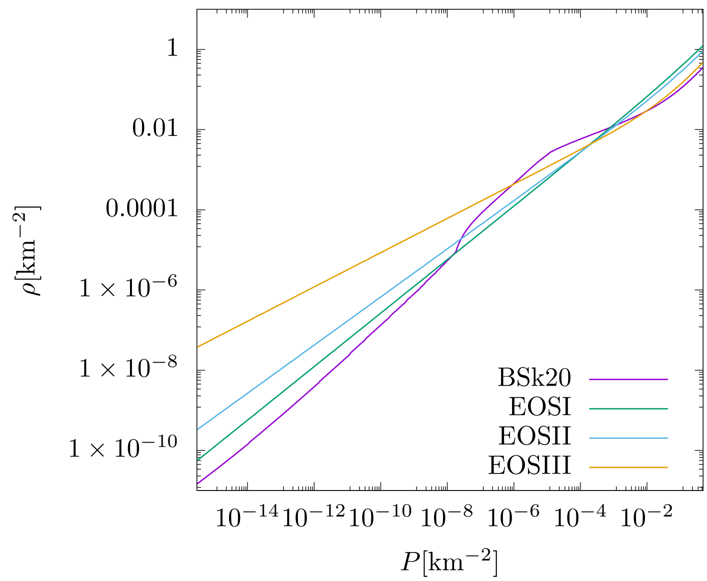

Figure 4: We show the energy density ρ as function of the pressure P in natural units ($8\pi G = c = \hbar = 1$) for the BSk20 equation of state. In comparison we show the three polytropic equations of state used in this paper, see (27). For EOSI, EOSII and EOSIII we have chosen $K = 1.5$, $K = 0.67$ and $K = 0.16$, respectively, corresponding to the maximal mass value.

The radius R of the neutron star is defined differently than that of the boson star. Here, the star has a "hard core", i.e. a surface outside of which the space-time is given by the Schwarzschild solution. The relevant conditions to impose in this case are :

$$P(R) = 0 \quad , \quad b(R) = f(R) \ . \tag{28}$$

To connect the results to physically realistic values for the mass and radius of the neutron stars, K has to be chosen accordingly. However for the purpose of our study, we note that the equations of motion are invariant under the following rescaling :

$$r \to \lambda r \quad , \quad M_G \to \lambda M_G \quad , \quad P \to \lambda^{-2} P \quad , \quad \rho \to \lambda^{-2}\rho \quad , \quad K \to \lambda^{-2+2/\Gamma} K \ . \tag{29}$$

Then, a dimensionless radius $\tilde{R}$ and a dimensionless mass $\tilde{M}_G$ of the configuration can be defined according to

$$\tilde{R} = R K^{\Gamma/(2\Gamma-2)} \quad , \quad \tilde{M}_G = M_G K^{\Gamma/(2\Gamma-2)} \ . \tag{30}$$

Note that we are using natural units here with $\hbar = c = 8\pi G \equiv 1$. Reinstalling the natural constants, we find that the mass M_G and R given in Fig. 6 are related to the dimensionful mass $M_{G,\mathrm{phys}}$ and dimensionful radius R_{phys} as follows

$$\frac{M_{G,\mathrm{phys}}\,[M_\odot]}{R_{\mathrm{phys}}\,[\mathrm{km}]} \approx 0.68 \frac{M_G}{R} = 0.68 \frac{\tilde{M}_G}{\tilde{R}} \ . \tag{31}$$

In the following, it will also be useful to define the number N_B of baryons of mass m_B that make up the neutron star. This is the equivalent of the Noether charge Q for the boson star and can be used to estimate whether the constructed solutions are stable to decay into a number of individual baryons or if they form a bound system. We follow the discussion in [15]. As argued in this latter paper, the particle number conservation follows from the energy-momentum conversation when

$$\frac{n'}{n} = \frac{\rho'}{\rho + p} , \tag{32}$$

where n is the particle number denisty $n = n(r)$. Combining this with the TOV equation (26) gives

$$n(r) = \frac{\sqrt{b}(\rho + p)}{m_B\sqrt{1 - \frac{2M_G}{R}}} , \tag{33}$$

where the integration constant has been fixed by assuming the conditions on the surface of the star to be $p(r = R) = 0$, $\rho(r = R) = \rho_0$, $n(r = R) = n_0$ and using the relation between ρ and n: $\rho_0 = m_B n_0$. The globally conserved quantity associated to the particle density current nu^μ, u^μ the 4-velocity of a particle, is the total baryon number

$$N_B = \int \sqrt{-g} n u^0 \mathrm{d}^3x = \frac{4\pi}{m_B\sqrt{1 - \frac{2M_G}{R}}} \int_0^R (\rho + p)\sqrt{\frac{b}{f}} r^2 \mathrm{d}r \tag{34}$$

where we have used that for a particle at rest $u^0 = 1/\sqrt{b}$. Comparing $m_B N_B$ with M_G will tells us whether the neutron star is stable ($m_B N_B > M_G$) or unstable ($m_B N_B < M_G$), respectively, to decay into N_B individual baryons of mass m_B.

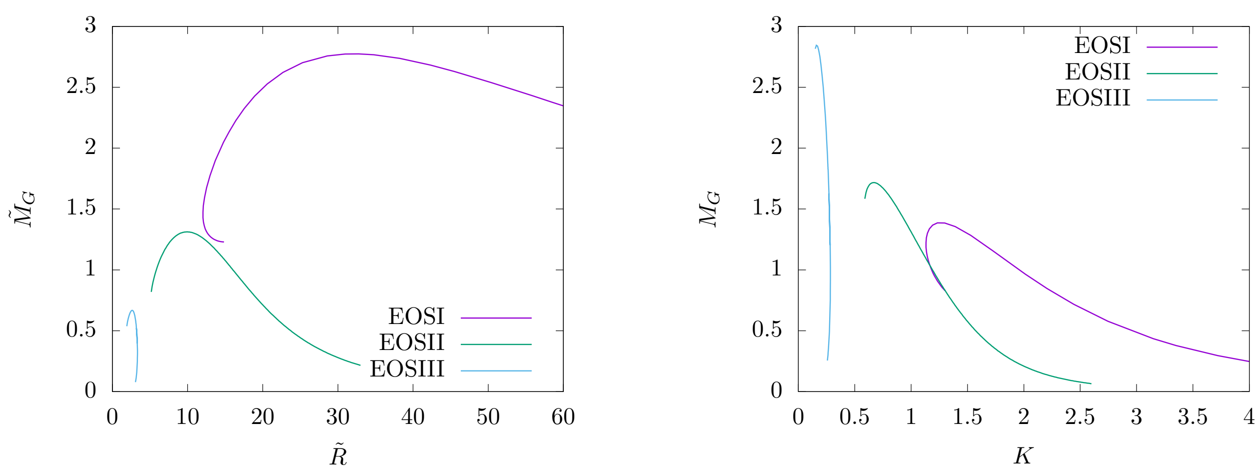

Figure 5: We show the mass $\tilde{M}_G$ as function of the radius $\tilde{R}$ of the neutron star solutions for $\eta = 0$ and three different EOS, see (27), respectively (left). We also show M_G as function of K for the same EOS and $R = 10$ (right).

4.1 Numerical results

In this first part, we will discuss and review already existing results to clarify our construction and compare the three different EOS discussed above in the GR limit. We will then turn to new scalar-tensor neutron stars using EOSII and EOSIII, respectively.

4.1.1 Neutron stars in GR

In Fig. 5 (left) we show the dimensionless quantity $\tilde{M}_G$ in function of the radius $\tilde{R}$ of the neutron star in the GR limit and for the two different equations of state. Note that using (29), the axes in this plot have to be rescaled by the same factor K in order to find the physical values of mass and radius of the neutron star. Contrary to what is presented in [9], we find that for a typical neutron star of radius $R_{\rm phys} = 10$km (corresponding to the maximum of the curve) the ratio $M/M_\odot \approx 0.6$, and not $M/M_\odot \approx 1.2$ as stated in [9]. Moreover, the qualitative relation between mass and radius is different to that in Fig. 2 of [9].

Comparing e.g. with the gravitational wave detections GW170817 from a binary neutron star merger [18] which suggests that the two neutron stars in the merger had masses between $0.86M_\odot$ and $2.26M_\odot$ and radii between 10.7 km and 11.9 km [19] (compare also very new results in [20]), we find that EOSI seems to have neutron stars of too low mass. We have hence considered EOSII and EOSIII, respectively. In Fig.5 we show the mass $\tilde{M}_G$ in function of the radius $\tilde{R}$ (left) and M_G in function of K for $R = 10$ (right) for EOSI, EOSII and EOSII. The combination of the data shown in this figure gives a maximal mass of a $R = 10$ neutron star of $M_{G,max} \approx 1.39$ at $K = 1.23$ of $M_{G,max} \approx 1.72$ at $K = 0.67$ for EOSII and $M_{G,max} \approx 2.85$ at $K = 0.16$ for EOSIII, respectively. Note that these are also the values of K used in Fig. 4. We conclude that EOSIII seems to be a good approximation to the realistic BSk20 equation of state for high pressure and high density neutron stars.

4.1.2 Scalar-tensor neutron stars

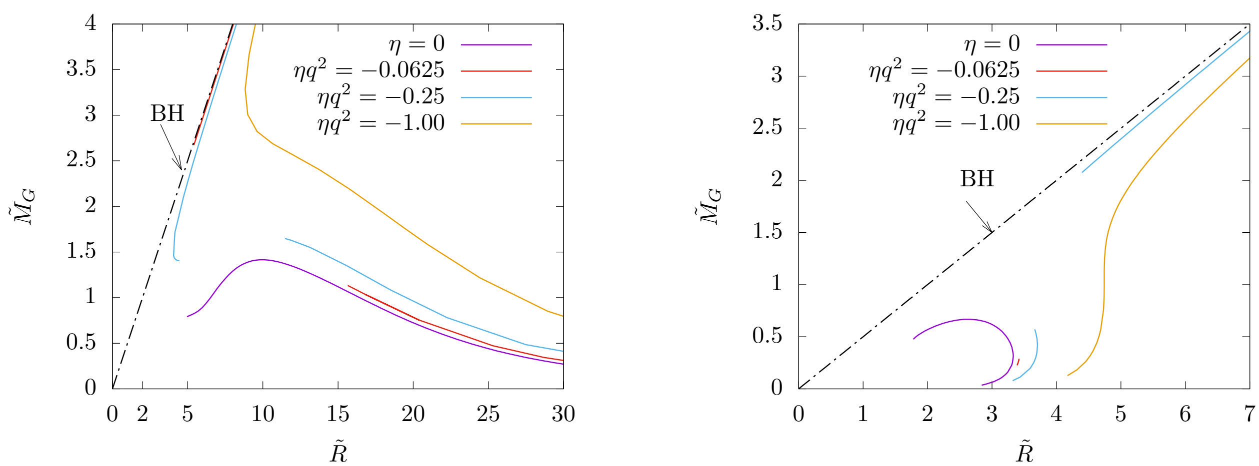

Figure 6: We show the mass $\tilde{M}_G$ as function of the radius $\tilde{R}$ of the neutron star solutions for $\Gamma = 3/5$ (left) and $\Gamma = 2.34$ (right) for several values of ηq^2. The mass-radius relation of the corresponding Schwarzschild black hole is indicated by "BH".

We now turn to the description of the influence of the non-minimal scalar-tensor coupling on the neutron star solutions constructed with EOSII and EOSIII.

We find that the existence of neutron stars – very similar to that of boson stars – is limited by the requirement of positivity of the denominator in the expansion (23) for $\eta q^2 > 0$ and by the requirement of positivity of ϕ'^2 (see (8)) for $\eta q^2 < 0$, respectively. Our results for the mass-radius relation of neutron stars for different values

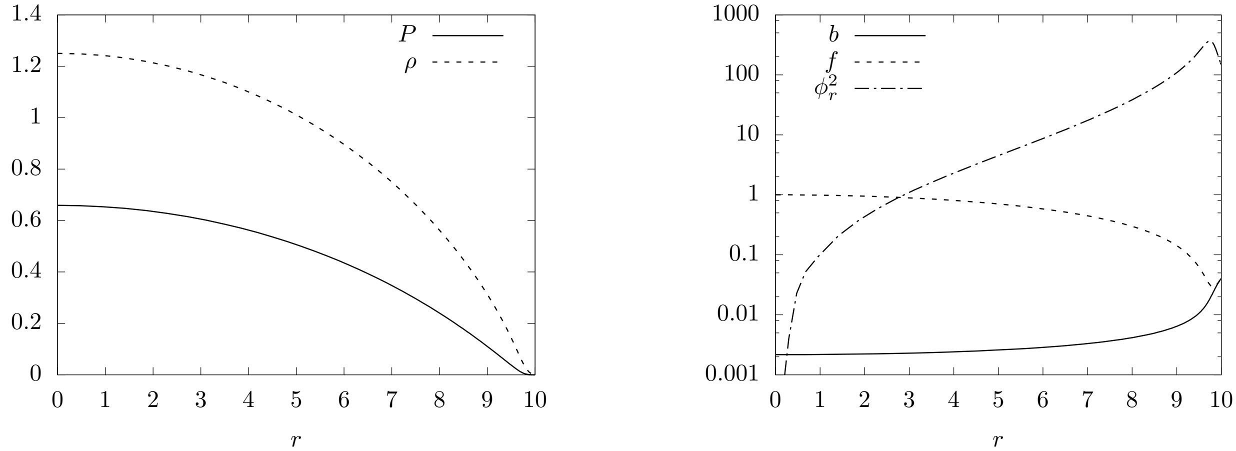

Figure 7: We show the profiles of the pressure P (solid) and the energy density ρ (dashed) of a scalar-tensor neutron star with radius $R = 10$ for $\eta q^2 = -0.25$, $P(0) = 0.66$ and EOSII (left). We also show the metric functions b (solid) and f (dashed) as well as $\phi_r^2 \equiv \phi'^2$ (dotted-dashed) for the same solution (right).

of ηq^2 are shown in Fig.6. The maximal mass $M_{G,\max}$ of the scalarized neutron stars is reached at roughly the same value of $R \approx 10$, however, when increasing ηq^2, the value of the maximal mass decreases as compared to the GR limit. When decreasing ηq^2 from zero, we find an interesting new phenomenon which appears for both EOSII and EOSIII. Let us choose the value $\eta q^2 = -0.25$for EOSII to explain this in more detail : when increasing the central pressure of the star, $P(0)$, we find a branch of solutions for $P_0 \leq 0.009$ (in our units) corresponding to $R > 11.4$. The solutions constructed for larger P_0 (and $R \leq 11.4$) have $(\phi')^2 < 0$ in some region and are therefore not acceptable, i.e. we find an interval of $P(0)$ for which no scalarized neutron stars exist. Interestingly, we observe that when increasing $P(0)$ sufficiently (in fact, $P(0) > 0.12$) a new, second branch of scalarized neutron stars for which $(\phi')^2 > 0$, exists. The reason for the existence of this new branch can be understood when considering (8) and the plot of the energy density ρ, pressure P, the metric functions $f(r)$ and $b(r)$ as well as ϕ'^2 given in Fig. 7 for neutron star corresponding to the second branch of solutions. This neutron star has $R = 10$ and $P(0) = 0.66$. Clearly, all functions are well behaved in particular $\phi'^2 \geq 0$ inside the star. The reason for the existence of these solutions then also becomes clear : since $b(r)$ is very small everywhere inside the star by inspection of (8) the value of ϕ'^2 can become positive again. The crucial point is hence the presence of the explicit time-dependence of the scalar field, i.e. the fact that $q \neq 0$. Not surprisingly, these neutron stars are very dense : as Fig. 6 demonstrates they are very close to the branch of Schwarzschild black holes. When decreasing ηq^2 further, see the curve for $\eta q^2 = -1.0$ in Fig. 6, we find that there exists a continuous branch of solutions along which the central pressure $P(0)$ increases and $(\phi')^2$ stays always positive. Hence, we find neutron stars that through a continuous deformation of the central pressure can reach mass densities that are very close to that of black holes.

We have also studied the stability of the neutron stars with respect to the decay into N_B individual baryons with mass m_B. For all solutions obtained, we observe that increasing ηq^2 from zero leads to a decrease in the binding energy and that for sufficiently large ηq^2 the neutron star becomes unstable to decay into individual baryons. This is, however, different when decreasing ηq^2 from zero. Remember that the existence of solutions in linked to the requirement of the positivity of the quantity $(\phi')^2$ and hence the domain of the parameter K for which solutions exist depends strongly on ηq^2. This is shown in Fig. 8 for some values of $\eta q^2 < 0$, where we give the ratio between M_G and $m_B N_B$. A ratio smaller than one indicates stability of the neutron star with respect to this specific decay. As is apparent from Fig. 8, the inclusion of a gravity scalar with ηq^2 negative increases the binding between the individual baryons. Moreover, when two separate branches of solutions are present, the branch closer to the BH limit has much stronger binding.

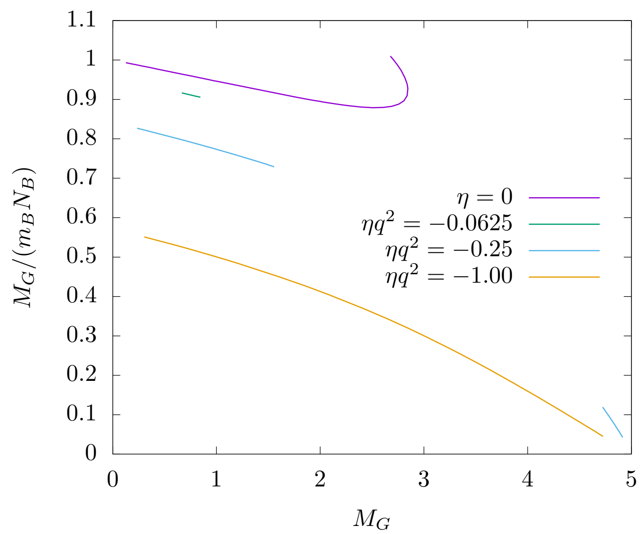

Figure 8: We show the ratio $M_G/(m_B N_B)$ between the mass M_G of the neutron star and the mass of N_B individual baryons of mass m_B in function of M_G for EOSIII and several values of ηq^2.

5 Conclusions

In this paper, we have studied the properties of boson and neutron stars in a scalar-tensor gravity models which contains an explicitly time-dependent real scalar field. The norm of the Noether current associated to the shift symmetry of the gravity scalar is finite everywhere in the space-time. We find that the explicit time-dependence does allow non-trivial scalar fields to exist in both the space-time of a boson star and neutron star, respectively. Moreover, the presence of the gravity scalar has interesting consequence for the properties of these objects. While the boson star's mass does not vary strongly when increasing or decreasing the scalar-tensor coupling from zero, it has a large effect on the number of scalar bosonic particles making up the boson star, the mean radius and central density and pressure. This means that while in the GR limit, boson stars of the type studied here, so-called "mini boson stars", have radius of a few Schwarzschild radii (see e.g. [1]), the radius of the scalar-tensor counterparts could, in fact, be much closer to the Schwarzschild radius.

For neutron stars, we have investigated two polytropic equation of states out of which one seems to be a very good fit to realistic equations of state. While neutron stars have a "hard core" outside which the pressure is strictly zero, the change of properties is comparable to that of boson stars. In particular, for negative scalar-tensor coupling and the gravity scalar changing slowly in time, we find that new branches of solutions of neutron stars exist that have a mass-radius relation very close to that of Schwarzschild black holes. Increasing the time change of the gravity scalar, we find that we can continuously deform "standard" mass neutron stars to these objects with large central pressure $P(0)$. We observe that this phenomenon arises for both equations of state that we have investigated.

In summary, our results indicate that the presence of a gravity scalar in the case of globally regular, compact objects prevents these objects from collapsing to a black hole at the values known in GR due to an increased central pressure allowed inside the stars. Our results furthermore indicate that the scalar-tensor objects studied here are stable to decay into their individual constituents. Surely this does not mean that they are generally stable under perturbations - a study outside the scope of this paper - but it indicates that when considering extensions of GR, the formation of star-like objects that are in density close to that of black holes is a viable possibility.

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