Generalised Bargmann - Wigner classification :

Mixed-symmetry fields in Minkowski and $(A)dS_{d+1>3}$

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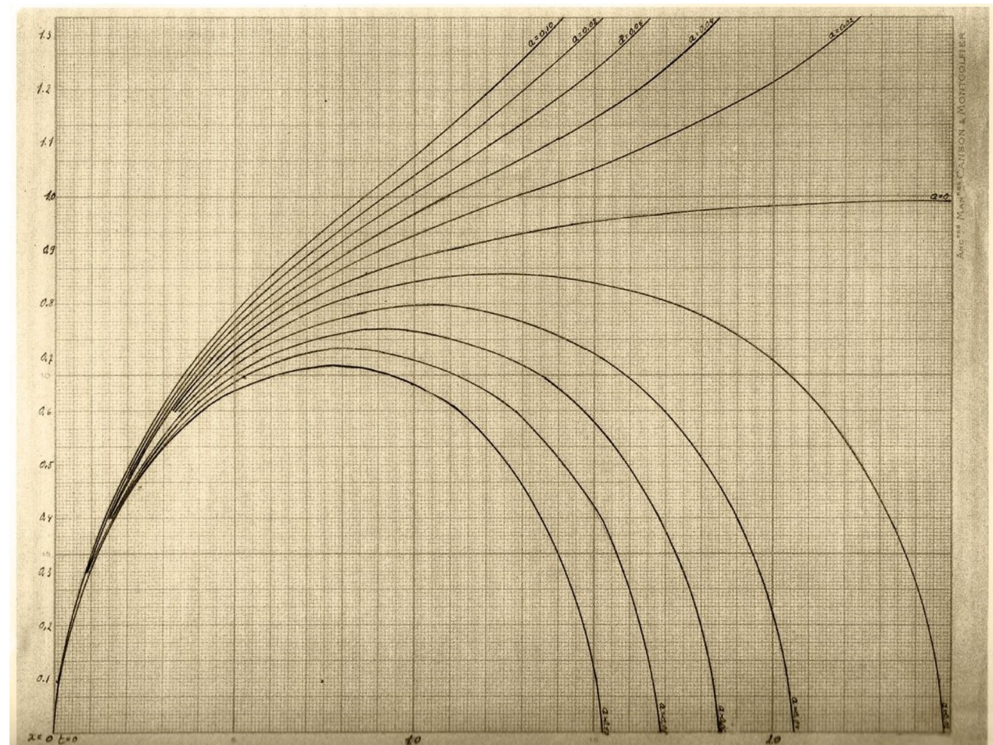
$\boxed{\Lambda = 0}$ Review with Xavier Bekaert [hep-th/0611263] SciPost Lect. Notes (2021)

$\boxed{\Lambda < 0}$ From works of Metsaev [9810231] & joint work with Carlo Iazeolla & Per Sundell [0812.3615 0812.4438] JHEP

$\boxed{\Lambda > 0}$ From [1612.08166] with Thomas Basile and Xavier Bekaert JHEP

G. Lemaître



PLAN

1. Motivation

2. Bargmann-Wigner in Minkowski space

3. Bargmann-Wigner in AdS_{d+1} space

4. UIR's of $SO(1,d+1)$ & fields in dS_{d+1}

5. Dirac singletons and Flato-Fronsdal

1 A motivation for higher-spin fields: Quantum Field Theory

- At dawn of QFT: Majorana (1932), Dirac (1936), Fierz & Pauli (1939) and ...

↳ Wigner's classification (1939) of UIRs of $ISO(1,3)$ ⟶ fundamental particles

↳ Bargmann-Wigner (1948): Relativistic & linear equations whose solutions in UIRs.

⟷ Particles in 4D Minkowski characterised by mass and spin (or helicity)

- Bargmann-Wigner program: classification of field equations ⟷ UIRs of isometry group

Rem. In massless case, also "continuous" or "infinite" spin UIRs.

See Review with X. Bekaert or [1708.01030] by X. Bekaert & E. Skvortsov

Once the HS representations have been seen to exist in the sense of UIRs of maximally-symmetric spacetime isometry group then standard **second** quantization naturally requires a covariant **Lagrangian**

Fierz-Pauli program: Associate a quadratic, local and $so(1,d)$ covariant Lagrangian to every UIR of **maximally-symmetric** spacetime isometry algebra

Initiated by Fierz-Pauli in 1939 for massive spin-2 particles in $\mathbb{R}^{1,3}$.

Then, Chang (1967), Schwinger (1970), Singh-Hagen (1974).

→ In 1978, Fronsdal & Fang ⇝ **Lagrangian** for $m=0$, helicity-s field around $\mathbb{R}^{1,3}$ and $(A)dS_4$ by taking limit $m \to 0$ of Singh-Hagen's Lagrangian.

Rem. $\mathbb{R}^{1,2}$ very interesting too. Reviewed in SciPost Lect. Notes (2021)

- The BW program in $\mathbb{R}^{1,d}$ was achieved in late 80's [W. Siegel & B. Zwiebach] and in minimal form in [Labastida 89, X. Bekaert & N.B. 2001]

- The BW program in AdS_{d+1} in the late nineties by R. Metsaev.

- The BW program in dS_{d+1} ⇝ Th. Basile, X. Bekaert & N.B. [1612.08166]

↳ establish a dictionary between UIR's of $SO(1, d+1)$ and covariant linear field equations in dS_{d+1}.

First: review $\Lambda \leqslant 0$ cases.

② Wigner's classification of UIR's of $ISO(1,d)$ $\mathcal{M}_{d+1}$

↳ One-to-one with the $SO(1,d)$ orbits $\mathcal{O}_p$ of $p \in (\mathbb{R}^{1,d})^*$

together with UIR of little group $G_p \subseteq SO(1,d)$ stabilizing p.

1) $\forall g \in G_p \qquad g \cdot p = p$

2) Given a repres. R of G_p, induce a UIR $\mathcal{T}$ of $ISO(1,d)$

on the Hilbert space of functions on $\mathcal{O}_p$ valued in R.

$$\mathcal{T}(\Lambda, a) \cdot \tilde{\varphi}(q) = \sqrt{\rho_{\Lambda^{-1}}(q)} \; e^{i(q,a)} R\left(g_q^{-1} \circ \Lambda \circ g_{\Lambda^{-1} q}\right) \cdot \tilde{\varphi}(\Lambda^{-1} q)$$

where $g_q \in SO(1,d)$ standard boost for p : $g_q \cdot p = q \in \mathcal{O}_p$

$$g_q^{-1} \circ \Lambda \circ g_{\Lambda^{-1} q} : p \xrightarrow[g_{\Lambda^{-1} q}]{} \Lambda^{-1} q \xrightarrow[\Lambda]{} q \xrightarrow[g_q^{-1}]{} p$$

The various orbits $\{\mathcal{O}_p\}$ correspond to p being

1) Timelike $\rightsquigarrow$ Massive particle $p^\mu = (m, 0, \dots, 0)$

$p^2 = -m^2$, $G_p \simeq SO(d)$ $[\,E := p^0$ & $\eta = \mathrm{diag}(-,+,\dots,+)\,]$

2) Light-like $\rightsquigarrow$ Massless particle

$p^2 = 0$ & $p \neq 0$ $\quad p_\mu = (-E, 0, \dots, 0, E)$

In Light frame $\quad x^\pm := \dfrac{x^d \pm x^0}{\sqrt{2}}$, $\quad p_\mu = (p_-, \overset{p_+}{0}, \overbrace{0, \dots, 0}^{p_i})$

Little group $\quad G_p \cong ISO(d-1) \cong T_{d-1} \rtimes SO(d-1)$

M_{i-} & M_{-+} rejected $\Rightarrow \{p^+ M_{i+} =: \pi_i\} \cup \{M_{ij}\}$

$\hookrightarrow$ Take π_i trivial $\rightsquigarrow$ helicity UIR's : $G_p \cong SO(d-1)$.

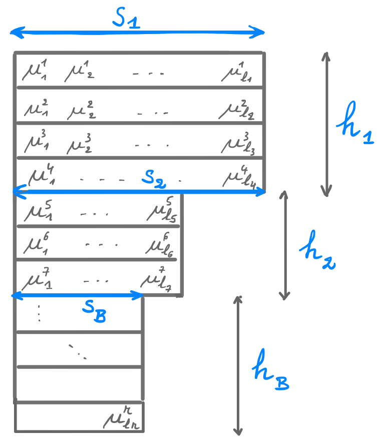
3) Spacelike $\rightsquigarrow$ tachyons : $G_p \cong SO(1, d-1)$

4) Nul $\quad p = (0, \dots, 0)$ in $(\mathbb{R}^{1,d})^*$: $G_0 \cong SO(1,d)$

As for covariant, linear wave equations in $\mathcal{M}_{d+1}$

- Take $\varphi_{\underline{\mathbb{Y}}}(x)$ valued in $GL(d+1)$ irrep $\rightsquigarrow \mathbb{Y}$

$\mathbb{Y} =$



with $\boxed{c_1 + c_2 \leq d - 1}$

$$\vec{s} = (\overbrace{\ell_1, \dots, \ell_4}^{\text{all equal to } s_1}, \overbrace{\ell_5, \dots, \ell_7}^{\text{all equal to } s_2}, \dots, \overset{s_B}{\overset{\shortparallel}{\ell_r}})$$

- Symmetrizing the indices of any row with any index of a lower row gives zero identically.

- Antisymmetrizing the indices of a column with any index of a column at its right gives zero identically.

- Define $p_B = \sum_{I=1}^{B} h_I$ the height of $\mathbb{Y}$.

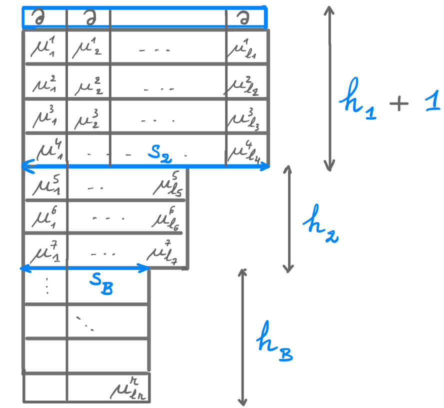
• Build the curvature

$$\boxed{K_{\overline{\mathbb{Y}}} := d^{(1)} \dots d^{(s_1)} \varphi_{\mathbb{Y}}}$$

by acting on $\varphi_{\mathbb{Y}}$ with s_1 curls

and impose the wave equation

$$\boxed{\mathrm{Tr}\, K_{\overline{\mathbb{Y}}} \approx 0}\ .$$

$\overline{\mathbb{Y}} =$ 

From Bianchi identity $d^{(i)} K_{\overline{\mathbb{Y}}} \equiv 0 \quad \forall\, i \in \{1, \dots, s_1\}$

Deduce that $d^{\dagger}_{(i)} K \approx 0 \quad \forall\, i$ where $\boxed{d^{\dagger}_{(i)} := *_i\, d^{(i)} *_i}$ divergence.

Hence $\{ d^{(i)}, d^{\dagger}_{(i)} \} K_{\overline{\mathbb{Y}}} \equiv \Box K_{\overline{\mathbb{Y}}} \approx 0 \Rightarrow K_{\overline{\mathbb{Y}}}$ massless field.

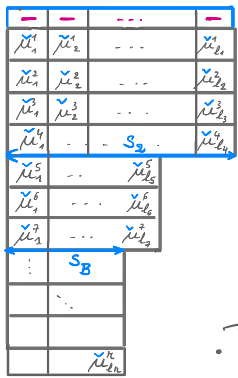
Fourier modes $\tilde{K}_{\overline{\mathbb{Y}}}(p)$ on $p^2 = 0$

light-cone : mass shell for light-like particles.

$$\boxed{\begin{aligned} [d^{(i)}, d^{\dagger}_{(j)}]_{\mathbb{Z}_2} &= \delta^i_j\, \Box \\ [d^{(i)}, d^{(j)}]_{\mathbb{Z}_2} &= 0 = [d^{\dagger}_{(i)}, d^{\dagger}_{(j)}]_{\mathbb{Z}_2} \end{aligned}}$$

Bianchi Id.

↳ $d^{(i)}K = 0 \quad \forall i$, $\boxed{p_{[-}\tilde{K}_{\mu^1_1\,\mu^2_1\,\dots\,\mu^{r_B}_1]|\dots} \equiv 0}$ $\Longleftrightarrow$ $\tilde{K}_{\bar{\mathbb{Y}}} \rightsquigarrow \mathbb{Y}$ of $GL(d,\mathbb{R})$ (*)

$\tilde{K} =$ 

$\check{\mu} \in \{+, \overbrace{1, \dots, d-1}^{\text{transverse directions}}\}$

$d^\dagger_{(i)} K \approx 0$ Divergenceless : $p^+ \tilde{K}_{+\dots} \approx 0$

$\Longrightarrow$ $\tilde{K}$ valued in $\mathbb{Y}$ of $GL(d-1)$

. Tracelessness in $so(1,d)$ $\Rightarrow$ Tracelessness in $so(d-1)$

Ccl : $\tilde{K}_{\bar{\mathbb{Y}}}$ reduces on-shell to $\tilde{K}$ in UIR $R_{\bar{\mathbb{Y}}}$ of G_p

$G_p \cong SO(d-1)$ little group for helicity particles $\xrightarrow{\text{induce}}$ $T(\Lambda, a)$ UIR of $ISO(1,d+1)$

(*)

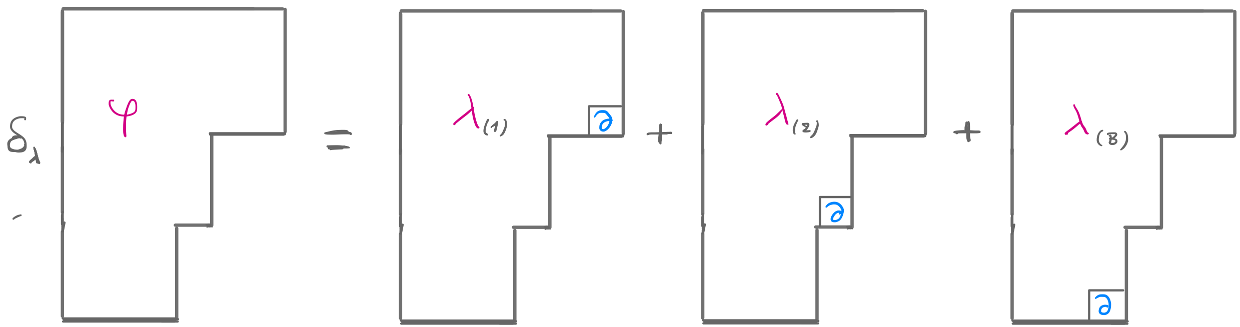
$K_{-\check{\mu}\check{\nu}|-\check{\varepsilon}}$ $\quad 0 \equiv K_{[-\check{\mu}\check{\nu}|\check{\varepsilon}]-} = 3\tilde{K}_{-[\check{\mu}\check{\nu}|\check{\varepsilon}]-} - \cancel{\tilde{K}_{\check{\mu}\check{\nu}\check{\varepsilon}|--}}^{\,0}$ $\Longleftrightarrow$ $\tilde{K}_{-[\check{\mu}\check{\nu}|\check{\varepsilon}]-} \equiv 0$

Gauge invariance

$$K_{\bar{Y}} = d^{(1)} \dots d^{(s_1)} \varphi_Y$$

Field equation $\mathrm{Tr}\, K_{\bar{Y}} \approx 0$ is PDE order s_1 for φ_{Y}

Invariant under



On-shell, fixing gauge $\tilde{K}$ reduces to $\tilde{K} \approx (p_-)^{s_1} \varphi_{i_1 \dots j \dots}$

X.B. & N.B. Partial gauge fixing of $\boxed{\mathrm{Tr}\, \tilde{K} = 0}$ $SO(d-1)$

to $\boxed{\left(\Box - \sum_{i=1}^{s_1} d^{(i)} d^\dagger_{(i)} + \frac{1}{2} \sum_{i,j=1}^{s_1} d^{(i)} d^{(j)} \mathrm{Tr}_{ij} \right) \varphi_Y \approx 0}$: Labastida 89.

3 Field Equations in AdS_{d+1}

- Conventions and notations

Lie algebra $so\left(1+\frac{1+\sigma}{2}, d+\frac{1-\sigma}{2}\right)$

with generators $M_{AB} = M_{AB}^{\dagger}$

$$(\eta^{(\sigma)}_{AB}) = \text{diag}(-\sigma, \underbrace{-, \overbrace{+, \cdots, +}^{(\delta_{ij})}}_{(\eta_{ab})}) \longrightarrow \begin{cases} \sigma = +1 & AdS_{d+1} \\ \sigma = -1 & dS_{d+1} \end{cases}$$

$A, B, \dots \in \{0_\sigma, 0, 1, \dots, d\}$ $\qquad a, b, \dots \in \{0, 1, \dots, d\}$

$$(\eta_{ab}) = \text{diag}(\underset{0}{-}, \underset{1}{+}, \dots, \underset{d}{+}) \quad \text{of} \quad SO(1,d)$$

$$\boxed{[M_{AB}, M_{CD}] = i\left(\eta^{(\sigma)}_{BC} M_{AD} - \eta^{(\sigma)}_{AC} M_{BD} - \eta^{(\sigma)}_{BD} M_{AC} + \eta^{(\sigma)}_{AD} M_{BC}\right)}$$

Rem : From $G_{\mu\nu} + \Lambda g_{\mu\nu} = 0 \Rightarrow R = \frac{2D}{D-2}\Lambda$. $(A)dS_{d+1} \rightsquigarrow R_{\mu\nu\rho\sigma} = -2\sigma\lambda^2 g_{\mu[\rho} g_{\sigma]\nu}$, $\lambda^2 := -\frac{2\sigma\Lambda}{d(d-1)}$

$D = d+1$

- $P_a := \lambda\, M_{0_\sigma a}$ transvections of $(A)dS_{d+1}$

$$\boxed{\begin{aligned} &[M_{ab}, M_{cd}] = i\,\eta_{bc} M_{ad} + \cdots \\ &[M_{ab}, P_c] = 2i\,\eta_{c[b} P_{a]} \qquad [P_a, P_b] = i\sigma\lambda^2 M_{ab} \end{aligned}}$$

- Another useful decomposition of M_{AB}, adapted to CFT :

$$D := i c_\sigma M_{0_\sigma 0} \quad , \quad P_i := M_{0i} + c_\sigma M_{0_\sigma i} \quad , \quad K_i := M_{0i} - c_\sigma M_{0_\sigma i}$$

where $c_\sigma = \begin{cases} i & \text{for } \sigma = +1 \\ 1 & \text{for } \sigma = -1 \end{cases}$, s.t. $c_\sigma^2 = -\sigma$.

$$\boxed{\begin{aligned} &[M_{ij}, M_{kl}] = i\,\delta_{jk} M_{il} + \cdots && [K_i, P_j] = 2\left(iM_{ij} + \delta_{ij} D\right) \\ &[M_{ij}, P_k] = 2i\,\delta_{k[j} P_{i]} && [M_{ij}, K_k] = 2i\,\delta_{k[j} K_{i]} \\ &[D, P_i] = P_i && [D, K_i] = -K_i \end{aligned}}$$

<u>Note</u> : $\sigma = +1$: $D = -M_{0'0} \equiv E$

- Quadratic Casimir $C_2\left[so\left(1+\frac{1+\sigma}{2}, d+\frac{1-\sigma}{2}\right)\right] = \frac{1}{2} M^{AB} M_{AB}$

Using $M_{0i} = \frac{1}{2}(P_i + K_i)$, $M_{0_\sigma i} = \frac{1}{2c_\sigma}(P_i - K_i)$

$$\frac{1}{2} M^{AB} M_{AB} = D(D-d) - P^i K_i + C_2[so(d)]$$

$\Rightarrow$ On a lowest-weight state $|\Delta, \vec{s}\rangle$ annihilated by ladder op. K_i,

s.t. $(D-\Delta)|\Delta,\vec{s}\rangle = 0$, $K_i|\Delta,\vec{s}\rangle = 0$,

one finds

$$\boxed{C_2\left[so\left(1+\frac{1+\sigma}{2}, d+\frac{1-\sigma}{2}\right)\right] = -\Delta(-\Delta+d) + \sum_{\ell=1}^{r} s_\ell(s_\ell+d-2\ell)}$$

- In the $so(1+\frac{1+\sigma}{2}, d+\frac{1-\sigma}{2})$ - covariant basis where $P_a := \lambda M_{0_\sigma a}$,

  represent $\boxed{P_a = -i\nabla_a}$ as a diff. operator, ∇ the Lorentz-covariant derivat.

$$\Rightarrow \quad C_2 = \frac{1}{2} M^{AB} M_{AB} \equiv C_2[so(1,d)] - \sigma \eta^{ab} M_{0_\sigma a} M_{0_\sigma b}$$

$$= C_2[so(1,d)] - \frac{\sigma}{\lambda^2} P^a P_a$$

- Set $$\frac{\sigma}{\lambda^2} \nabla^a \nabla_a = -\frac{\sigma}{\lambda^2} P^2 = \underbrace{\frac{1}{2} M^{AB} M_{AB}}_{\Delta(\Delta-d)+\sum_{\ell=1}^{r} s_\ell(s_\ell+d-2\ell)} - \frac{1}{2} M^{ab} M_{ab} \stackrel{!}{=} \sigma m_{\mathbb{Y}}^2 \qquad (*)$$

$\Rightarrow$ Gives a relation between field equation (linear, relativistic)

and an abstract UIR of $so(1+\frac{1+\sigma}{2}, d+\frac{1-\sigma}{2})$

$$\boxed{(\Box - \lambda^2 m_{\mathbb{Y}}^2)\varphi_{\mathbb{Y}} = 0}$$

• Demanding gauge invariance of the field equation

$$(\Box - \lambda^2 m_{\mathbb{Y}}^2)\,\varphi_{\mathbb{Y}} = 0 \quad , \quad \mathrm{Tr}\,\varphi_{\mathbb{Y}} = 0 = \nabla\cdot\varphi_{\mathbb{Y}} \quad \text{on all indices}$$

under

$$\boxed{\delta_\lambda \varphi_{\mathbb{Y}} = \sum_{I=1}^{B} (\nabla^{(I)})^t \lambda_{(I)}}$$

gives [Metsaev' 95, $t=1$] a set of possibilities for fixed block I

$$\sigma\, m_I^2 \in \Big\{ (s_I - p_I - t)(s_I - p_I + d - t) - \sum_{k=1}^{r} s_k \Big\}_{I=1,\dots,B}$$

where $$p_I := \sum_{J=1}^{I} h_J$$

together with similar conditions on the gauge para. $\lambda_{(I)}$

and the gauge-for-gauge parameters $\{\lambda^i_{(I)}\}_{i=2,\dots,p_I}$

Note: In (A)dS, at most 1 gauge parameter! Different from Minkowski!

Group theoretical description in AdS_{d+1}

- Generalized Verma module

$$so(2) \oplus so(d) \subset so(2,d)$$

$$\mathcal{V} = \left\{ P_{i_1} \dots P_{i_n} \, |e_0, \vec{s}\rangle_{j \dots k \dots} \right\}_{n = 0, 1, \dots}$$

Recall $\quad C_2[so(2,d)] = e_0(e_0 - d) + C_2[so(d)] \quad$ with

$$\begin{cases} e_0 > s_1 - h_1 + d - 1 \quad (s_1 > 0) \longrightarrow \text{Massive unitary field} \\ e_0 = e_t^I := s_I - p_I + d - t \longrightarrow \text{partially-massless (gauge) fields} \\ e_0 \geqslant \frac{d-2}{2} \quad \text{or} \quad e_0 \geqslant \frac{d-1}{2} \longrightarrow \text{Massive scalars, Rac and Di singletons} \\ e_0 \neq e_t^I \quad \& \quad e_0 < e_0^1 \longrightarrow \text{Massive non-unitary} \end{cases}$$

$$\longrightarrow \quad \sigma m_I^2 \in \left\{ e_0^I (e_0^I - d) - \sum_{k=1}^{n} s_k \right\}_{I = 1, \dots, B}$$

in accordance with $\quad \dfrac{\sigma}{\lambda^2} \Box = \dfrac{1}{2} M^{AB} M_{AB} - \dfrac{1}{2} M^{ab} M_{ab}$

$$e_0(e_0 - d) + \sum_{\ell=1}^{r} s_\ell(s_\ell + d - 2\ell) - \sum_{\ell=1}^{r} s_\ell(s_\ell + d + 1 - 2\ell)$$

$$\frac{\sigma}{\lambda^2} \Box \varphi = -e_0(-e_0 + d) - \sum_{\ell=1}^{n} s_\ell$$

Gauge invariance of Fierz-Pauli-type wave equation

reflected by

Gauge field

Irr. module

$$\mathcal{D}(e^{\mathcal{I}}_{t}, \mathbb{Y}) \cong \frac{\mathcal{V}(e^{\mathcal{I}}_{0}, \mathbb{Y})}{\mathcal{D}(\underbrace{e^{\mathcal{I}}_{0} + t}, \mathbb{Y}_{(\mathcal{I})})}$$

Generalized Verma m.

Gauge param. module, itself a quotient in general (gauge for gauge)

minimal energy of states in the module

$$\delta\varphi = \nabla^{t}\lambda_{(\mathcal{I})}$$

AdS_{d+1}	dS_{d+1}
Vacuum $so(2) \oplus so(d)$ module	Vacuum $so(1,1) \oplus so(d)$ module
$\mathbb{V}(e_0, \mathbb{Y})$	$\mathbb{V}(\Delta_c, \mathbb{Y})$
• Casimir	• Casimir
$C_2 = e_0(e_0 - d) + C_2[so(d)]$	$C_2 = \Delta_c(\Delta_c - d) + C_2[so(d)]$
• Critical mass	
$m^2_{\mathbb{Y}} = e_0(e_0 - d) - \sum_{k=1}^{r} s_k$	$m^2_{\mathbb{Y}} = -\Delta_c(\Delta_c - d) + \sum_{k=1}^{r} s_k$
• massless for $e_0 = e_t^I$	• massless for $\Delta_c = ?$
unitarity known $(L_i^-)^\dagger = L_i^+$	unitarity ?

$$\boxed{(\nabla^2 - \lambda^2 m^2_{\mathbb{Y}})\varphi_{\mathbb{Y}} = 0}$$

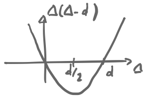
4 UIR's of $SO(1, d+1)$

- **Principal series**: $\Delta_c = \frac{d}{2} + i\rho$, $\mathbb{Y}$ & $\rho \in \mathbb{R}$ arbitrary

  [Rem: $\nabla^2\varphi_0 = (-\lambda^2)\Delta_c(\Delta_c - d)\varphi_0$ where $\Delta_c(\Delta_c-d) = (\frac{d}{2}+i\rho)(i\rho - \frac{d}{2}) = -\rho^2 - \frac{d^2}{4} \Rightarrow \nabla^2 \geq 0$ in dS_{d+1}]

- **Complementary series**: $p < \Delta_c < d-p$, $p \in \{0, 1, \dots, r-1\}$

$P = P_B$



$l_k = 0$ for $k = p+1, \dots, r$.

- **Exceptional series**: $\Delta_c = d-p$ (or $\Delta_c = p$), $p \in \{1, \dots, r-1\}$

  $l_k = 0$ for $k = p+1, \dots, r$. (no scalar)

- $(d = 2r+1)$ **Discrete series**: $\Delta_c = \frac{d}{2} + k$, $k \in \frac{\mathbb{N}}{2}$

  i.e. $\Delta_c = \frac{d-1}{2} + k'$, $k' \in \mathbb{N}$ maximal height $0 < k' \leq l_r$

[For $so(1, 2r+2)$, $\text{rank}[so(1,d+1)] = \text{rank}[so(d+1)] = r+1$.

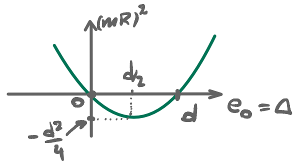
the Cartan subgroup is $SO(d+1)$, compact. The $r+1$ (commuting) generators of the Cartan subgroup are compact.

For $d = 2r$, no compact Cartan subgroup. There is a $so(1,1)$ generator among the $r+1$.]

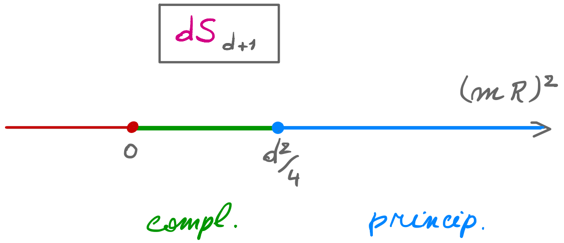
Dictionary

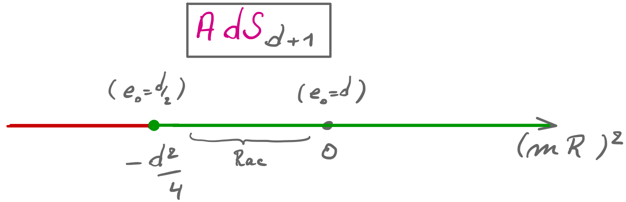
Computing $so(d+2)$ characters of Generalized Verma modules [using Bernstein – Gel'fand – Gel'fand resolution] and comparing with characters of $SO(1,d+1)$ UIR's from the math. literature, we obtained the dictionary

- Principal & complementary : Massive fields



scalars

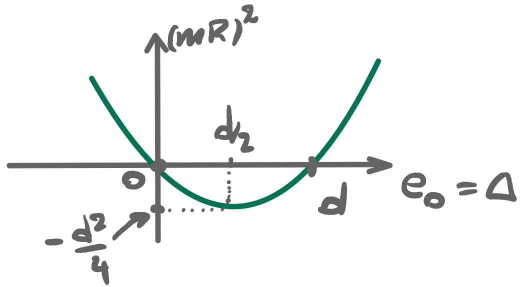




$(mR)^2 = e_0(e_0-d)$, $e_0 \neq \frac{d-2\ell}{2}$ ((higher) singletons)

$d+1=4$: Take $m^2 = -\frac{\text{RicciScal.}}{6} \xrightarrow{AdS_4} -\frac{12}{6} = -2 \rightsquigarrow \begin{cases} e_0 = 1 \\ e_0 = 2 \end{cases}$

$m^2_{Rac} = -\frac{d^2}{4}+1$



Rem : Conformally-coupled scalar field

$$S[\phi; g_{\mu\nu}] = -\frac{1}{2}\int d^D x \left[-\partial_\mu \phi \, \partial^\mu \phi + \frac{1}{4} \frac{D-2}{D-1} R \phi^2 \right]$$

$$R = \frac{2D}{D-2} \Lambda = -\sigma D(D-1) \lambda^2$$

$$\frac{\delta S}{\delta \phi} \approx 0 \quad \Longrightarrow \quad \Box \phi + \sigma \frac{\lambda^2}{4} D(D-2) \phi \approx 0$$

$\underline{D=4}$: $\boxed{\Box \phi \approx -2\sigma \lambda^2 \phi}$

$m^2 = -2\sigma \xrightarrow[AdS_4]{} m^2 = -2$

$$e_0(e_0 - 3) = -2 \quad \rightsquigarrow \quad e_0 = \begin{cases} 1 & \text{Dirichlet} \\ 2 & \text{Neuman} \end{cases} \text{B.C.}$$

$\mathcal{D}(1,0)$ & $\mathcal{D}(2,0)$

$\wr$

$\mathcal{D}(s+1, s)$

• Exceptional series : (partially) massless fields with less-than-maximal height

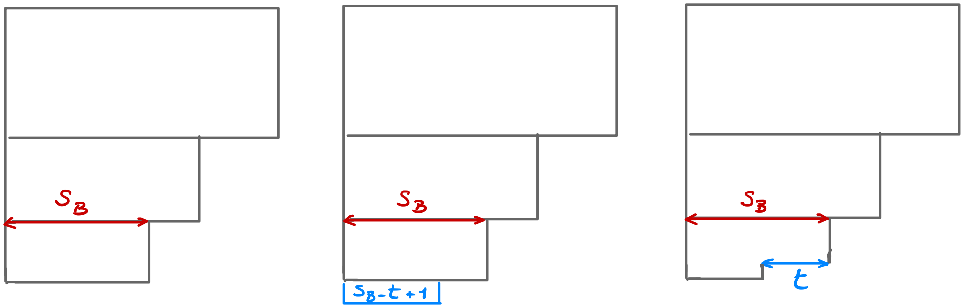
Unitarity: only the last block must be activated

$$\Delta_c = s_B - p + d - t \qquad p \equiv p_B$$

contrary to the first one in AdS.

Rem: The weights $(\Delta_c, \mathbb{Y})$ labelling the UIR $\rightsquigarrow$ curvature and not φ potential.

• Discrete series : massless field φ with maximal height



$$\Delta_c = \ell_r - r + d - t$$

$$p_B \equiv p = r$$

$$s_B \equiv \ell_r$$

φ potential $\qquad K$ curvature $\qquad \lambda$ gauge parameter

Massless cases : $t = 1$; PM : $1 < t \leq s_B$.

Summary: Unitary fields.