Interactions for partially-massless spin-2 fields

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We review the theory for a multiplet of interacting partially massless spin-2 fields around (anti-) de Sitter (A)dS$_D$ background and give new results concerning the couplings between a massless spin-1 vector field and a partially massless spin-2 field.

PACS: 04.50.-h, 11.15.-q

Introduction

Partially massless (PM) fields of spin-2 have been subject to renewed attention in recent years after the important advances made in the understanding of massive gravity [1,2]. In four-dimensional de Sitter (dS$_4$) spacetime with (positive) cosmological constant Λ , a PM graviton has a mass given by $m^2_{\rm PM} = 2\Lambda/3$ and a corresponding gauge invariance that removes the degree of freedom associated with the spin-zero mode of the particle. A consistent theory of PM gravity would be very attractive given its relevance in the context of cosmology and the fact that a PM spin-2 field is not subject to the same strict experimental constraints as a generic massive graviton, e.g. the bounds on fifth-force experiments or on dispersion of gravitational waves.

In spite of their potentially interesting phenomenology, complete and physically realistic models involving PM fields are currently lacking. A crucial hurdle one encounters when attempting to construct such a PM theory beyond linear level is the requirement of gauge invariance of the interactions. Indeed, this condition is restrictive enough to rule out theories of a single self-interacting PM spin-2 particle, see e.g. [3–5]. Such no-go results beg the question of whether a fundamental obstruction exists for the construction of non-trivial PM models. A first step in order to address this question is to precisely understand what are the assumptions that lead to the existing negative results.

In the recent work [6] we have shown that, by relaxing the requirement of classical unitarity, one can in fact construct a complete theory for a multiplet of PM gravitons around (A)dS$_4$ space, as we review in the following. This is an interesting outcome as it

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demonstrates that gauge invariance itself is not a fundamental obstacle for the existence of a nontrivial interacting PM spin-2 theory. In addition, motivated by the field content of a putative supersymmetric theory of PM gravity (as identified in [7]), we present some new results on the interactions between a massless spin-1 field and a PM spin-2 field.

1. Set-up

We will look for consistent deformations of the theory consisting of an arbitrary number of PM spin-2 fields: $h^a_{\mu\nu}$, $a = 1, \ldots, n$. The action describing this theory is given by

$$S_0[h^a_{\mu\nu}] = \int d^Dx \, \sqrt{-\bar{g}} \, k_{ab} \left(-\tfrac{1}{4} F^{a\lambda\mu\nu} F^b_{\lambda\mu\nu} + \tfrac{1}{2} F^{a\lambda} F^b_{\lambda}\right) \ , \tag{1}$$

where $\bar{g}_{\mu\nu}$ is the metric of the background which is chosen to be dS_D or AdS_D space, $F^a_{\lambda\mu\nu} := 2\nabla_{[\lambda} h^a_{\mu]\nu}$ are the field strengths of the potentials $h^a_{\mu\nu}$, $F^\lambda := \bar{g}_{\mu\nu} F^{\lambda\mu\nu}$ and k_{ab} is a metric in the internal space of the PM spin-2 fields. For simplicity one can take $(k_{ab}) = diag(+\cdots+)$ for a unitary theory, which is only possible in dS_D space, and $(k_{ab}) = diag(+\cdots+-\cdots-)$ for a non-unitary one.[1] The field strengths and thus the action are invariant under the gauge transformations

$$\delta^{(0)}_\epsilon h^a_{\mu\nu} = \nabla_\mu \nabla_\nu \epsilon^a - \tfrac{\sigma}{L^2} \, \bar{g}_{\mu\nu} \, \epsilon^a \ , \tag{2}$$

where L is the $(\mathrm{A})\mathrm{dS}_D$ radius related to the cosmological constant via $\Lambda = -\frac{(D-1)(D-2)}{2\sigma L^2}$. The parameter σ is defined to be $+1$ when the background is AdS_D and -1 when it is dS_D . In this way the commutator of Lorentz-covariant derivatives on $(\mathrm{A})\mathrm{dS}_D$ acting on a (co-)vector is given by

$$[\nabla_\mu, \nabla_\nu] \, V_\alpha = -\tfrac{2\sigma}{L^2} \, \bar{g}_{\alpha[\mu} V_{\nu]} \ . \tag{3}$$

In the BRST-BV formalism (see e.g. [9] for a review and details on the notation we use), we introduce ghosts C^a associated with the gauge parameters ϵ^a as well as the conjugate antifields and antighosts $\{\Phi^*_I\} := \{h^{*\mu\nu}_a, C^*_a\}$ canonically paired to the fields and ghosts $\{\Phi^I\} := \{h^a_{\mu\nu}, C^a\}$ via the antibracket defined on two local functionals A and B:

$$(A, B) := \frac{\delta^R A}{\delta \Phi^I} \frac{\delta^L B}{\delta \Phi^*_I} - \frac{\delta^L A}{\delta \Phi^*_I} \frac{\delta^L B}{\delta \Phi^I} \ . \tag{4}$$

The variance of the fields and antifields with respect to the color indices is by definition as displayed above up to raising or lowering indices thanks to the internal metric k_{ab} . The BV functional of this theory is written as

$$W_0 = S_0 + \int d^Dx \, \sqrt{-\bar{g}} \left[h^{*\mu\nu}_a \left(\nabla_\mu \nabla_\nu C^a - \tfrac{\sigma}{L^2} \, \bar{g}_{\mu\nu} \, C^a\right)\right] \ . \tag{5}$$

[1] We allow for the study of classically non-unitary theories. Actually, PM fields can be defined around AdS_D where they are non-unitary. This can be seen explicitly by writing the action, which is real for both signs of the cosmological constant, in the Stueckelberg formulation [8].

It satisfies the classical master equation

$$(W_0, W_0) = 0 \, . \tag{6}$$

One can define the BRST differential via the BV functional, $s\,\bullet := (W_0, \bullet)$, which splits into two differentials $s = \gamma + \delta$. The action of these differentials on the fields, as well as the Grassmann parity $|.|$, the ghost number gh, the pure ghost number pgh and the antifield number $afld$ of the fields and antifields are given in Table 1, along with $\delta h^{*\mu\nu} = \sqrt{-\bar{g}}(\nabla_\lambda F^{b\lambda(\mu\nu)} - \bar{g}^{\mu\nu}\,\nabla_\lambda F^{b\lambda} + \nabla^{(\mu}F^{\nu)b})k_{ab}$ and $\delta C^{*a} = \sqrt{-\bar{g}}\nabla_\mu\nabla_\nu h^{*\mu\nu}_a - \frac{\sigma}{L^2}h^*_a$.

	$\vert.\vert$	gh	pgh	$afld$	$\frac{1}{\sqrt{-\bar{g}}}\gamma$
$h^a_{\mu\nu}$	0	0	0	0	$\nabla_\mu\nabla_\nu C^a - \frac{\sigma}{L^2}\,\bar{g}_{\mu\nu}\,C^a$
C^a	1	1	1	0	0
$h^{*\mu\nu}_a$	1	-1	0	1	0
C^*_a	0	-2	0	2	0

Table 1. Assignement of various degrees for the fields and antifields

The cohomology of the differential γ, which will be helpful in the following, is given by

$$H(\gamma) \cong \{f\left([F^a_{\lambda\mu\nu}]\;,\;C^a\;,\;\nabla_\mu C^a\;,\;[\Phi^*_I]\right)\}\, . \tag{7}$$

2. Cubic deformations

Upon considering the perturbative expansion $W = W_0 + g\,W_1 + \dots$, the goal of this section is the classification of the first order deformation W_1 that is found by solving the master equation $(W, W) = 0$ to first order in g:

$$s\,W_1 = 0\, . \tag{8}$$

Expanding the first order BV functional according to the antifield number

$$W_1 = \int d^Dx\,\sqrt{-\bar{g}}(a_0 + a_1 + a_2)\, , \tag{9}$$

the master equation at first order is equivalent to the following descent equations[1]

$$\delta a_1 + \gamma a_0 = t.d.\, , \tag{10}$$
$$\delta a_2 + \gamma a_1 = t.d.\, , \tag{11}$$
$$\gamma a_2 = 0\, . \tag{12}$$

[1] Here $t.d. = \partial_\mu j^\mu$ for some vector j^μ. Since one can always rewrite $j^\mu = \sqrt{-\bar{g}}\,\tilde{j}^\mu$ this implies $\partial_\mu j^\mu = \sqrt{-\bar{g}}\,\nabla_\mu\tilde{j}^\mu$ and $t.d.$ represents up to a $\sqrt{-\bar{g}}$ factor a total derivative using a Lorentz-covariant derivative of the background. We will thus always write $t.d.$ although this can mean $\partial_\mu j^\mu$ or $\nabla_\mu j^\mu$ depending on the context.

2.1. **Deformations of the gauge algebra.**—We will start the classification of the deformations by listing all the possible deformations of the gauge algebra $a_2 \in H(\gamma)$ without any restriction on the number of derivatives. The complete list is given by

$$a_2^{(1)} = C_a^* \, C^b \, C^c m^a{}_{bc} \;, \qquad m^a{}_{bc} = m^a{}_{[bc]} \;, \tag{13}$$

$$a_2^{(2)} = C_a^* \, \nabla_\mu C^b \, \nabla^\mu C^c n^a{}_{bc} \;, \qquad n^a{}_{bc} = n^a{}_{[bc]} \;, \tag{14}$$

and the total deformation of the algebra that solves the last equation of the descent (12) is a linear combination of these candidates, $a_2 = \alpha_{(1)}\, a_2^{(1)} + \alpha_{(2)}\, a_2^{(2)}$.

We now have to solve the second equation of the descent (11) with the a_2 just defined. The calculation of δa_2 gives

$$\tfrac{1}{\sqrt{-\bar g}} \delta a_2^{(1)} = t.d. - \tfrac{1}{\sqrt{-\bar g}} \gamma \left[h_a^{*\mu\nu} m^a{}_{bc} h^b_{\mu\nu} C^c \right] + m^a{}_{bc} (2 h_a^{*\mu\nu} \nabla_\mu C^b \nabla_\nu C^c + \tfrac{\sigma}{L^2}\, h_a^* C^b C^c) \;, \tag{15}$$

$$\begin{aligned} \tfrac{1}{\sqrt{-\bar g}} \delta a_2^{(2)} = t.d. &- \tfrac{1}{\sqrt{-\bar g}} \gamma \left[2 n^a{}_{bc}\, h_a^{*\mu\nu} \left(\nabla_\mu h^b_{\nu\rho} \nabla^\rho C^c + h^b_{\mu\rho} \nabla_\nu \nabla^\rho C^c + \tfrac{\sigma}{L^2} h^b_{\mu\nu} C^c \right) \right] \\ &+ \tfrac{\sigma}{L^2}\, n^a{}_{bc} \left(2 h_a^{*\mu\nu} \nabla_\mu C^b \nabla_\nu C^c + \tfrac{\sigma}{L^2} h_a^* C^b C^c \right) - \tfrac{\sigma}{L^2} h_a^* n^a{}_{bc} \nabla_\mu C^b \nabla^\mu C^c \;. \end{aligned} \tag{16}$$

The obstructions $2 h_a^{*\mu\nu} m^a{}_{bc} \nabla_\mu C^b \nabla_\nu C^c + \frac{\sigma}{L^2}\, h_a^* m^a{}_{bc} C^b C^c$ and $2 h_a^{*\mu\nu} n^a{}_{bc} \nabla_\mu C^b \nabla_\nu C^c + \frac{\sigma}{L^2} h_a^* n^a{}_{bc} C^b C^c$ can be cancelled by relating $\alpha_{(2)}$ to $\alpha_{(1)}$ and $n^a{}_{bc}$ to $m^a{}_{bc}$. However, because of the obstruction $\frac{\sigma}{L^2} h_a^* n^a{}_{bc} \nabla_\mu C^b \nabla^\mu C^c$, there exists no linear combination such that all the obstructions vanish. This implies that there is no solution to the inhomogenous (with $a_2 \neq 0$) equation (11) and therefore we set $\alpha_{(1)} = 0 = \alpha_{(2)}$. In conclusion, the cubic deformations of "colored" PM fields are necessarily abelian.

2.2. **Abelian deformations of the gauge transformation.**—We can still solve the homogeneous ($a_2 = 0$) equation (11) by classifying all the possible abelian deformations of the gauge transformations $\bar a_1 \in H(\gamma)$. In terms of coefficients $f_{(i)bc}^{\;a}\,, i = 1, \ldots, 6$, there are six candidates containing up to two derivatives:

$$\bar a_1^{(1)} = h_a^{*\mu\nu}\, f_{(1)bc}^{\;a}\, \nabla_\mu F^b_\nu \, C^c \,, \qquad \bar a_1^{(2)} = h_a^{*\mu\nu}\, f_{(2)bc}^{\;a}\, F^b_\mu \, \nabla_\nu C^c \;, \tag{17}$$

$$\bar a_1^{(3)} = h_a^{*\mu\nu}\, f_{(3)bc}^{\;a}\, \nabla^\sigma F^b_{\sigma\mu\nu} \, C^c \,, \qquad \bar a_1^{(4)} = h_a^{*\mu\nu}\, f_{(4)bc}^{\;a}\, F^b_{\sigma\mu\nu} \, \nabla^\sigma C^c \;, \tag{18}$$

$$\bar a_1^{(5)} = h_a^*\, f_{(5)bc}^{\;a}\, \nabla_\sigma F^{b\sigma} \, C^c \;, \qquad \bar a_1^{(6)} = h_a^*\, f_{(6)bc}^{\;a}\, F^{b\sigma} \, \nabla_\sigma C^c \;. \tag{19}$$

An important aspect to remark is that not all combinations of these candidates are non-trivial because some combinations are δ-exact. For example, the combination $\bar a_1^{(1)} + \bar a_1^{(3)} - \bar a_1^{(5)}$ is δ-exact when $f^a_{(3)bc} = f^a_{(5)bc} = f^a_{(1)bc}$ and $k_{ad} f^a_{(1)bc} = k_{a[d} f^a_{(1)b]c}$. Those particular combinations have to be removed from the possible non-trivial deformations.

2.3. **Cubic vertices.**—We have to solve the first equation of the descent (10) with a_1 being the linear combination of all the non-trivial candidates written above. In the cases of, respectively, a single, two or three PM fields, there are 6, 48 or 162 candidates written in (17)–(19) (minus the δ-exact combinations). The resolution of the equation (10) has been done systematically by listing all the possible vertices entering in a_0 containing no more than two derivatives and then searching for a set of solutions to the equation.[1]

[1]Note that allowing for more derivatives we can trivially construct Born-Infeld type vertices; they contain at least three derivatives and do not deform the gauge transformation laws of the free theory.

We find that, *provided* $D = 4$, the general solution is given by a vertex of the type "gauge field times conserved current":

$$D = 4 : \qquad a_0 = h^a_{\mu\nu} J^{\mu\nu}_a \;, \qquad a = 1, \ldots, n \;, \tag{20}$$

where

$$J^{\mu\nu}_a := (F^{b(\mu|\rho\sigma} F^{c|\nu)}{}_{\rho\sigma} - \tfrac{1}{4}\bar{g}^{\mu\nu} F^{b\rho\sigma\lambda} F^c_{\rho\sigma\lambda} - F^{b(\mu|} F^{c|\nu)} + F^{b(\mu|\sigma|\nu)} F^c_\sigma + \tfrac{1}{2}\bar{g}^{\mu\nu} F^b_\lambda F^{c\lambda}) f_{bc,a} \;. \tag{21}$$

This vertex solves the descent equation with

$$a_1 = \bar{a}_1^{(4)} = h^{*\mu\nu}_a \, f^a{}_{b,c} \, F^b_{\sigma\mu\nu} \, \nabla^\sigma C^c \;. \tag{22}$$

We write the coefficients $f_{ab,c}$ in this way because they have to satisfy

$$f_{ab,c} = f_{(ab),c} \;. \tag{23}$$

The easiest way to see that is by noticing that the current $J^{\mu\nu}_a$ is on-shell equivalent to

$$J'^{\mu\nu}_a = (F^{b(\mu|\rho\sigma} F^{c|\nu)}{}_{\rho\sigma} - \tfrac{1}{4}\, \bar{g}^{\mu\nu} F^{b\rho\sigma\lambda} F^c_{\rho\sigma\lambda}) f_{bc,a} \approx J^{\mu\nu}_a \;, \tag{24}$$

from which we see that the constants $f_{ab,c}$ are projected on $f_{(ab),c}$. The conclusions on the existence of vertices related to the current $J'^{\mu\nu}_a$ are unchanged if we use the current $J^{\mu\nu}_a$, but it is important to use the unprimed one in order to have a two-derivative gauge transformation. We remark that this primed current was already known in the case of a single PM field in the context of couplings to a massless spin-2 field [10]. The number of deformation parameters of this solution is thus given by the number of parameters encoded in $f_{ab,c}$, that is to say, $\frac{n^2(n+1)}{2}$.

3. Quartic deformations

The goal of this section is to solve the master equation to second order in deformation:

$$s\, W_2 = -\tfrac{1}{2}(W_1, W_1) \;. \tag{25}$$

Expanding the second order BV functional according to the antifield number $W_2 = \int d^D x \sqrt{-\bar{g}}(b_0 + b_1)$, the master equation at second order is equivalent to the following descent equations:

$$\delta b_1 + \gamma b_0 = -\sqrt{-\bar{g}}\,(a_1, a_0) + t.d.\;, \tag{26}$$

$$\gamma b_1 = -\tfrac{1}{2}\sqrt{-\bar{g}}\,(a_1, a_1) + t.d.\;. \tag{27}$$

Let us start with the calculation of $-\frac{1}{2}(a_1, a_1)$ in order to solve the equation (27). We have

$$\begin{aligned} \tfrac{1}{2}(a_1, a_1) = t.d. - \tfrac{1}{\sqrt{-\bar{g}}}\gamma \left[2h^{*a\mu[\nu} \nabla^{\sigma]} C^b F^c_{\rho(\mu\nu)} h^d{}_\sigma{}^\rho f_{ae,b} f^e{}_{c,d} \right] \\ + 2h^{*a\mu[\nu} \nabla^{\sigma]} C^b \nabla_\sigma F^c_{\rho(\mu\nu)} \nabla^\rho C^d f_{ae,b} f^e{}_{c,d} + \tfrac{3\sigma}{2L^2}\, h^{*a\mu\nu} \nabla^\sigma C^b C^d F^c_{\sigma\mu\nu} f_{ae,b} f^e{}_{c,d} \;. \end{aligned} \tag{28}$$

The only way to kill the obstruction $h^{*a\mu\nu}\nabla^\sigma C^b C^d F^c_{\sigma\mu\nu} f_{ae,b} f^e{}_{c,d} \in H(\gamma)$ is by imposing the constraints

$$f_{ae,b}\, f^e{}_{c,d} = 0\ . \tag{29}$$

Let us suppose that we have a solution to these constraints. Then it implies that there is no second-order deformation of the gauge transformation ($b_1 = 0$) and we can continue to solve the equation (26). The calculation of the antibracket (a_1, a_0) gives

$$-(a_1, a_0) = (F^b_{\sigma\mu\nu}\nabla^\sigma C^c f^a{}_{b,c}) \left(J^{\mu\nu}_a + \frac{\delta J^{\rho\lambda}_d}{\delta h^a_{\mu\nu}} h^d_{\rho\lambda} \right) = J^{\mu\nu}_a F^b_{\sigma\mu\nu}\nabla^\sigma C^c f^a{}_{b,c}\ , \tag{30}$$

because the second term vanishes when the constraints (29) hold. The current $J^{\mu\nu}_a$ can be expanded as (21) and then the identities

$$F^\nu_a = -\tfrac{\sigma L^2}{D-2}\nabla_\mu \frac{\delta S_0}{\delta h^a_{\mu\nu}}\ , \tag{31}$$

$$\nabla_\sigma F^{\sigma\mu\nu}_a = \frac{\delta S_0}{\delta h^a_{\mu\nu}} - \tfrac{\sigma L^2}{D-2}\bar g^{\mu\nu}\,\nabla_\lambda\nabla_\sigma \frac{\delta S_0}{\delta h^a_{\lambda\sigma}} + \tfrac{\sigma L^2}{D-2}\nabla^\nu\nabla_\sigma \frac{\delta S_0}{\delta h^a_{\mu\sigma}}\ , \tag{32}$$

$$\nabla_\sigma F^{\mu\nu\sigma}_a = \tfrac{2\,\sigma L^2}{D-2}\nabla^{[\mu}\nabla_\sigma \frac{\delta S_0}{\delta h^a_{\nu]\sigma}}\ , \tag{33}$$

can help to rewrite terms containing a trace (F^a_μ) or a divergence ($\nabla_\mu F^{\mu\nu\sigma}_a$ or $\nabla_\sigma F^{\mu\nu\sigma}_a$) of the field strength as δ-exact terms. Those δ-exact terms actually come from a solution $\bar b_1$ of the homogeneous equation (27) $\gamma \bar b_1 = 0$. Using this trick one can write

$$-(a_1, a_0) = \tfrac{1}{\sqrt{-\bar g}}\delta \bar b_1 + F^{a\mu\rho\lambda} F^{b\nu}{}_{\rho\lambda} F^c_{\sigma\mu\nu}\nabla^\sigma C^d\, f_{ab,}{}^e f_{ec,d} \tag{34}$$

for some $\bar b_1$ whose explicit form is not needed, except for the fact that it is proportional to $f_{ab,}{}^e f_{ec,d}$ if one renames properly the summation indices. The last term is generically a non-trivial element of the cohomology of γ modulo d and modulo other δ-exact terms, and thus represents an obstruction unless we include quartic deformations of the gauge symmetry and vertices, in which case a higher-order analysis is required. We will instead get around this difficulty by imposing the second set of constraints

$$f_{ab,}{}^e f_{ec,d} = 0\ . \tag{35}$$

If one is able to solve the two sets of constraints, (29) and (35), it implies that the theory closes at cubic order and is fully consistent with respect to the gauge structure. Unfortunately, no solution exists when the internal metric k_{ab} is Euclidean, implying that some fields must carry negative energy and the theory is therefore non-unitary. Nevertheless, it is shown in [6] that the constraints still admit non-trivial solutions for non-positive-definite internal metrics, and can be classified according to the symmetries of the resulting $f_{ab,c}$: totally symmetric solutions, which are available for all $n \geq 2$; and mixed symmetric solutions, of which an explicit example has been found for $n = 3$.

4. Interactions between PM spin-2 and massless spin-1 fields

An intriguing question is whether supersymmetry could help to overcome the obstacles so far encountered when constructing interactions for PM fields. The simplest supermultiplet that contains a PM spin-2 particle is a long $\mathcal{N}_4 = 1$ multiplet that also contains a massive spin-3/2, a massless spin-3/2 and a massless spin-1 particles [7]. A tractable first step in the study of interactions for such multiplet is to restrict oneself to the bosonic sector, so that here we initiate the analysis of the problem by considering cubic interactions for a single PM spin-2 field $h_{\mu\nu}$ and a single spin-1 gauge vector A_μ.

Although the supersymmetric multiplet is valid in four dimensions, it is instructive to start the calculation in arbitrary D, so we consider the free theory

$$S_0 = -\frac{1}{4}\int d^D x\sqrt{-\bar{g}}\Big[F^{\rho\mu\nu}F_{\rho\mu\nu} - 2F^\rho F_\rho + G^{\mu\nu}G_{\mu\nu}\Big]\ , \tag{36}$$

where $G_{\mu\nu} := \nabla_\mu A_\nu - \nabla_\nu A_\mu$ is the abelian field strength of the vector. In addition to the PM gauge symmetry given above (but here restricted to a single PM field), we have the usual Maxwell gauge symmetry $\delta^{(0)}_\alpha A_\mu = \nabla_\mu \alpha$, to which we associate a ghost B in the BRST-BV procedure. The free BV functional is then given by

$$W_0 = S_0 + \int d^D x\sqrt{-\bar{g}}\Big[h^{*\mu\nu}\nabla_\mu\nabla_\nu C + \tfrac{1}{L^2}\,h^* C + A^{*\mu}\nabla_\mu B\Big]\ , \tag{37}$$

which by construction is BRST-invariant, $sW_0 = 0$, with $s = \gamma + \delta$ and

$$\gamma A_\mu = \nabla_\mu B\ , \qquad \delta A^{*\mu} = \nabla_\nu G^{\nu\mu}\ , \qquad \delta B^* = -\nabla_\mu A^{*\mu}\ , \tag{38}$$

plus the analogous relations for the PM sector already given in Table 1.

The first order deformation of the BV functional is decomposed by antifield number as $W_0 = \int d^D x\sqrt{-\bar{g}}(a_0 + a_1 + a_2)$. Without making any assumption on the number of derivatives, the most general antifield-2 term is given by

$$a_2 = \lambda C^* C B + \lambda' B^* C B\ . \tag{39}$$

When solving the descent equation, $\gamma a_1 + \delta a_2 + \text{t.d.} = 0$, we find that the second term leads to an obstruction and so we set $\lambda' = 0$ at this stage. The "lifted" solution a_1 then reads

$$a_1^{(\text{lift})} = \lambda\Big[h^{*\mu\nu}h_{\mu\nu}B - 2h^{*\mu\nu}A_\mu\nabla_\nu C - h^{*\mu\nu}\nabla_\mu A_\nu C\Big]\ . \tag{40}$$

To this we must add the solutions $\bar{a}_1$ in the cohomology $H(\gamma|\mathrm{d})$. Restricting to terms with no more than two derivatives we have

$$\begin{aligned}\bar{a}_1 = {}&\alpha_{(1)}A^{*\mu}F_\mu C + \alpha_{(2)}A^{*\mu}F_\mu B + \alpha_{(3)}A^{*\mu}\nabla^\nu G_{\mu\nu}C + \alpha_{(4)}A^{*\mu}\nabla^\nu G_{\mu\nu}B\\ &+ \alpha_{(5)}A^{*\mu}G_{\mu\nu}\nabla^\nu C + \alpha_{(6)}h^*\nabla^\rho F_\rho B + \alpha_{(7)}h^{*\mu\nu}\nabla^\rho F_{\rho\mu\nu}B\ .\end{aligned} \tag{41}$$

Note that this Ansatz omits γ-exact terms as well as terms of the pure PM type, as we know that the latter are necessarily obstructed at quartic order.

Finally we inspect the descent $\gamma a_0 + \delta a_1 + \text{t.d.} = 0$, with $a_1 = a_1^{(\text{lift})} + \bar{a}_1$. After a calculation we find that the obstructions can be canceled if and only if all the coefficients in a_1 are zero with the exception of $\alpha_{(5)} =: \alpha$. Moreover, this non-trivial solution is only available when $D = 4$.

In conclusion, only a single solution to the first-order deformation of the master equation exists, and again only in $D = 4$ dimensions:

$$a_2 = 0\,, \quad a_1 = \alpha\, A^{*\mu} G_{\mu\nu} \nabla^\nu C\,, \quad a_0 = \alpha\, \left(G^{\rho\mu} G_\rho{}^\nu h_{\mu\nu} - \tfrac{1}{4} G^{\mu\nu} G_{\mu\nu} h \right)\,. \qquad (42)$$

The consistency of this deformation at quartic order will be examined in a forthcoming work.

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