Gravity and higher spins in three-dimensional flat space

Andrea Campoleoni

Physique de l'Univers, Champs et Gravitations

University of Mons & FNRS

A.C., H.A. González, B. Oblak and M. Riegler, arXiv:1603.03812

A.C., D. Francia and C. Heissenberg, arXiv:1703.01351 &1712.09591

work in progress with L. Ciambelli, Ch. Marteau, P.M. Petropoulos, K. Siampos

Higher Symmetries and Quantum Gravity, AEI Potsdam, 4/12/2018

Gravity in 3D: an old love story with Λ

- Key developments that stimulated research on 3D gravity
  - **1984**: *Deser, Jackiw, 't Hooft*; particle dynamics in 3D 500+
  - **1986**: *Achucarro, Townsend*; Chern-Simons formulation of AdS sugra 500+
  - **1988**: *Witten*; Chern-Simons formulation of gravity for any Λ 1000+

Most of these papers discussed both flat space and (A)dS

Gravity in 3D: an old love story with Λ

- Key developments that stimulated research on 3D gravity
  - **1984**: *Deser, Jackiw, 't Hooft*; particle dynamics in 3D 500+
  - **1986**: *Achucarro, Townsend*; Chern-Simons formulation of AdS sugra 500+
  - **1988**: *Witten*; Chern-Simons formulation of gravity for any Λ 1000+

  Most of these papers discussed both flat space and (A)dS

- Then the BTZ black hole and AdS/CFT appeared…
  - **1992**: BTZ black holes when $\Lambda < 0$ 1000+
  - **1986→1998**: Brown-Henneaux asymptotic symmetries & AdS/CFT 1000+

  and flat space disappeared from hep-th!

Why coming back to flat space?

- Strong opinions against flat space until recently…

see, however, also the book by Carlip (1998)

  - For $\Lambda = 0$, above three dimensions there is a precise observable in quantum gravity, the S-matrix. However, in the three-dimensional case, there is no S-matrix in the usual sense […]. There are no gravitons in three dimensions, and there are also no black holes unless $\Lambda < 0$. **So again, we do not have a clear picture of what we would aim for to solve three-dimensional gravity with zero cosmological constant.** (Witten, *Three-dimensional gravity reconsidered*, 2007)

Why coming back to flat space?

- Strong opinions against flat space until recently… see, however, also the book by Carlip (1998)
  - For $\Lambda = 0$, above three dimensions there is a precise observable in quantum gravity, the S-matrix. However, in the three-dimensional case, there is no S-matrix in the usual sense [...]. There are no gravitons in three dimensions, and there are also no black holes unless $\Lambda < 0$. **So again, we do not have a clear picture of what we would aim for to solve three-dimensional gravity with zero cosmological constant.** (Witten, *Three-dimensional gravity reconsidered*, 2007)
- and new hopes to understand what we would aim for!
  - **2010**: *Barnich, Troessaert*; BMS/CFT correspondence
  - **2014**: *Strominger*; BMS symmetries, soft theorems and memory effects
  - **2016**: *Hawking, Perry, Strominger*; soft hairs for black holes

see Dario's talk

- These developments pertain to 4D, but 3D gravity helps…

OK… but why flat space in 3D?

- Most of the recent developments in flat space are based on the **infinite-dimensional asymptotic BMS symmetry**
  - Conformal 2D subalgebra of BMS_4 ⇔ rewriting of the S-matrix in terms of CFT correlators
    Pasterski, Shao, Strominger (2017)
  - Infinite dimensional symmetry of the S-matrix ⇔ leading and subleading soft theorems as Ward identities
    Strominger (2014)
- No S-matrix in 3D, but a great opportunity to understand better **flat-space holography** (and soft hairs)
  - **The BMS algebra is a contraction of the 2D conformal algebra!**
  - Simple models for the study of soft hairs
    Afshar, Detournay, Grumiller, Merbis, Perez, Tempo, Troncoso (2016) and further developments

BMS_3 as a contraction of the Virasoro algebra

Asymptotic symmetries in AdS$_3$

Brown, Henneaux (1986)

- Fix boundary conditions: $g_{\mu\nu} \sim \begin{pmatrix} -\frac{r^2}{R^2} + \mathcal{O}(1) & \mathcal{O}(r^{-3}) & \mathcal{O}(1) \\ & \frac{R^2}{r^2} + \mathcal{O}(r^{-4}) & \mathcal{O}(r^{-3}) \\ & & r^2 + \mathcal{O}(1) \end{pmatrix}$

- Look for gauge transf. that preserve them
  - $\delta g_{\mu\nu} = \nabla_{(\mu}\xi_{\nu)} \sim g_{\mu\nu}$
  - define charges (canonical generators): $\delta_\xi F = \{Q(\xi), F\}$

- Asymptotic symmetries: Poisson brackets of charges

$$\{\mathcal{L}(\theta), \mathcal{L}(\theta')\} = -\left(\delta(\theta-\theta')\mathcal{L}'(\theta) + 2\,\delta'(\theta-\theta')\mathcal{L}(\theta)\right) - \frac{k}{4\pi}\delta'''(\theta-\theta')$$

Asymptotic symmetries in AdS$_3$

Brown, Henneaux (1986)

- Fix boundary conditions: $$g_{\mu\nu} \sim \begin{pmatrix} -\frac{r^2}{R^2} + \mathcal{O}(1) & \mathcal{O}(r^{-3}) & \mathcal{O}(1) \\ & \frac{R^2}{r^2} + \mathcal{O}(r^{-4}) & \mathcal{O}(r^{-3}) \\ & & r^2 + \mathcal{O}(1) \end{pmatrix}$$

- Look for gauge transf. that preserve them
  - $\delta g_{\mu\nu} = \nabla_{(\mu}\xi_{\nu)} \sim g_{\mu\nu}$
  - define charges (canonical generators): $\delta_\xi F = \{Q(\xi), F\}$

- Asymptotic symmetries: Poisson brackets of charges

$$\{\mathcal{L}(\theta), \mathcal{L}(\theta')\} = -\left(\delta(\theta-\theta')\mathcal{L}'(\theta) + 2\,\delta'(\theta-\theta')\mathcal{L}(\theta)\right) - \frac{k}{4\pi}\,\delta'''(\theta-\theta')$$

$$\mathcal{L}(\theta) = \frac{1}{k}\sum_{p\in\mathbb{Z}} \mathcal{L}_p\, e^{-ip\theta} - \frac{k}{4}\,\delta_{p,0} \;\to\; \boxed{i\,\{\,\mathcal{L}_p\,,\,\mathcal{L}_q\,\} = (p-q)\,\mathcal{L}_{p+q} + \frac{c}{12}\,(p^3-p)\,\delta_{p+q,0}}$$

Asymptotic symmetries in AdS$_3$

Brown, Henneaux (1986)

- Fix boundary conditions: $$g_{\mu\nu} \sim \begin{pmatrix} -\frac{r^2}{R^2} + \mathcal{O}(1) & \mathcal{O}(r^{-3}) & \mathcal{O}(1) \\ & \frac{R^2}{r^2} + \mathcal{O}(r^{-4}) & \mathcal{O}(r^{-3}) \\ & & r^2 + \mathcal{O}(1) \end{pmatrix}$$

- Look for gauge transf. that preserve them
  - $\delta g_{\mu\nu} = \nabla_{(\mu}\xi_{\nu)} \sim g_{\mu\nu}$
  - define charges (canonical generators): $\delta_\xi F = \{Q(\xi), F\}$

- Asymptotic symmetries: Poisson brackets of charges

$$\{\mathcal{L}(\theta), \mathcal{L}(\theta')\} = -\left(\delta(\theta-\theta')\mathcal{L}'(\theta) + 2\,\delta'(\theta-\theta')\mathcal{L}(\theta)\right) - \frac{k}{4\pi}\delta'''(\theta-\theta')$$

AdS/CFT

$$\mathcal{L}(\theta) = \frac{1}{k}\sum_{p\in\mathbb{Z}} \mathcal{L}_p\, e^{-ip\theta} - \frac{k}{4}\delta_{p,0} \;\rightarrow\; \boxed{i\,\{\mathcal{L}_p\,,\,\mathcal{L}_q\} = (p-q)\,\mathcal{L}_{p+q} + \frac{c}{12}(p^3-p)\,\delta_{p+q,0}}$$

Asymptotic symmetries in flat space

- Asymptotic symmetries at spatial infinity in AdS$_3$

$$
\begin{aligned}
[\mathcal{L}_m, \mathcal{L}_n] &= (m-n)\,\mathcal{L}_{m+n} + \frac{c}{12}\, m(m^2-1)\delta_{m+n,0} \\
[\bar{\mathcal{L}}_m, \bar{\mathcal{L}}_n] &= (m-n)\,\bar{\mathcal{L}}_{m+n} + \frac{\bar{c}}{12}\, m(m^2-1)\delta_{m+n,0}
\end{aligned}
$$

Asymptotic symmetries in flat space

- Asymptotic symmetries at spatial infinity in AdS3

$$[\mathcal{L}_m, \mathcal{L}_n] = (m-n)\,\mathcal{L}_{m+n} + \frac{c}{12} m(m^2-1)\delta_{m+n,0}$$
$$[\bar{\mathcal{L}}_m, \bar{\mathcal{L}}_n] = (m-n)\,\bar{\mathcal{L}}_{m+n} + \frac{\bar{c}}{12} m(m^2-1)\delta_{m+n,0}$$

- Define new generators and central charges

$$\boxed{P_m \equiv \frac{1}{\ell}\left(\mathcal{L}_m + \bar{\mathcal{L}}_{-m}\right), \qquad J_m \equiv \mathcal{L}_m - \bar{\mathcal{L}}_{-m}}$$

$$c_1 = c - \bar{c}$$
$$c_2 = \frac{c + \bar{c}}{\ell}$$

Asymptotic symmetries in flat space

- Asymptotic symmetries at spatial infinity in AdS_3

$$
\begin{aligned}
[J_m, J_n] &= (m-n)J_{m+n} + \frac{c_1}{12}\, m(m^2-1)\,\delta_{m+n,0}\,, \\
[J_m, P_n] &= (m-n)P_{m+n} + \frac{c_2}{12}\, m(m^2-1)\,\delta_{m+n,0}\,, \\
[P_m, P_n] &= \ell^{-2}\,(\cdots)
\end{aligned}
$$

- Define new generators and central charges

$$
\boxed{P_m \equiv \frac{1}{\ell}\left(\mathcal{L}_m + \bar{\mathcal{L}}_{-m}\right)\,, \qquad J_m \equiv \mathcal{L}_m - \bar{\mathcal{L}}_{-m}}
$$

$$
c_1 = c - \bar{c}
$$

$$
c_2 = \frac{c+\bar{c}}{\ell}
$$

Asymptotic symmetries in flat space

- Asymptotic symmetries at spatial infinity in AdS3

$$\begin{aligned}
[J_m, J_n] &= (m-n)J_{m+n} + \frac{c_1}{12}\, m(m^2-1)\, \delta_{m+n,0}\,, \\
[J_m, P_n] &= (m-n)P_{m+n} + \frac{c_2}{12}\, m(m^2-1)\, \delta_{m+n,0}\,, \\
[P_m, P_n] &= \ell^{-2}\,(\cdots)
\end{aligned}$$

$\ell \to \infty$

- Define new generators and central charges

$$\boxed{P_m \equiv \frac{1}{\ell}\left(\mathcal{L}_m + \bar{\mathcal{L}}_{-m}\right), \qquad J_m \equiv \mathcal{L}_m - \bar{\mathcal{L}}_{-m}}$$

$$c_1 = c - \bar{c}$$

$$c_2 = \frac{c + \bar{c}}{\ell}$$

Asymptotic symmetries in flat space

- Asymptotic symmetries at null infinity in Minkowski$_3$

$$\begin{aligned}
[J_m, J_n] &= (m-n)J_{m+n} + \frac{c_1}{12}\, m(m^2-1)\,\delta_{m+n,0}\,, \\
[J_m, P_n] &= (m-n)P_{m+n} + \frac{c_2}{12}\, m(m^2-1)\,\delta_{m+n,0}\,, \\
[P_m, P_n] &= 0\,,
\end{aligned}$$

$\ell \to \infty$

Asymptotic symmetries in flat space

- Asymptotic symmetries at null infinity in Minkowski$_3$

$$
\begin{aligned}
[J_m, J_n] &= (m-n)J_{m+n} + \frac{c_1}{12}\, m(m^2-1)\,\delta_{m+n,0}\,, \\
[J_m, P_n] &= (m-n)P_{m+n} + \frac{c_2}{12}\, m(m^2-1)\,\delta_{m+n,0}\,, \\
[P_m, P_n] &= 0\,,
\end{aligned}
$$

$\ell \to \infty$

- Same result directly from flat gravity

Barnich, Compere (2007)

- Everything extends to higher spins
  - AdS$_3$ — Henneaux, Rey; A.C., Pfenninger, Fredenhagen, Theisen (2010)
  - flat — Afshar, Bagchi, Fareghbal, Grumiller, Rosseel; Gonzalez, Matulich, Pino, Troncoso (2013)

Higher spins in 3D flat space

The road to higher spins in D = 2+1

- Einstein-Hilbert action

$$I = \frac{1}{16\pi G}\int \mathrm{tr}\left(e\wedge R + \frac{1}{3l^2}\, e\wedge e\wedge e\right) \quad \text{with} \quad \begin{cases} e = e^a J_a \\ \omega = \omega^a J_a \end{cases}$$

- Tricks used to arrive here…
  - $\omega_\mu{}^a = \frac{1}{2}\,\epsilon^a{}_{bc}\,\omega_\mu{}^{b,c}$ so(2,1) ≃ sl(2,R): $[J_a, J_b] = \epsilon_{abc} J^c$

The road to higher spins in D = 2+1

- Einstein-Hilbert action

$$I = \frac{1}{16\pi G}\int \mathrm{tr}\left(e\wedge R + \frac{1}{3l^2}\, e\wedge e\wedge e\right) \quad \text{with} \quad \begin{cases} e = e^a J_a \\ \omega = \omega^a J_a \end{cases}$$

- Tricks used to arrive here…
  - $\omega_\mu{}^a = \frac{1}{2}\,\epsilon^a{}_{bc}\,\omega_\mu{}^{b,c}$ so(2,1) ≃ sl(2,R): $[J_a, J_b] = \epsilon_{abc} J^c$

- Yet another trick: *Einstein-Hilbert* ↔ *Chern-Simons*
  - Dreibein & spin-connection: $e = \frac{\ell}{2}(A - \bar{A})$ $\omega = \frac{1}{2}(A + \bar{A})$
  - Rewriting of the action: $\boxed{I = S_{CS}[A] - S_{CS}[\bar{A}]}$

The road to higher spins in D = 2+1

- Einstein-Hilbert action

$$I = \frac{1}{16\pi G} \int \mathrm{tr} \left(e \wedge R + \frac{1}{3l^2} e \wedge e \wedge e \right) \quad \text{with} \quad \begin{cases} e = e^a J_a \\ \omega = \omega^a J_a \end{cases}$$

- Chern-Simons action

$$S_{CS}[A] = \frac{k}{4\pi} \int \mathrm{Tr} \left(A \wedge dA + \frac{2}{3} A \wedge A \wedge A \right)$$

- Yet another trick: *Einstein-Hilbert* ↔ *Chern-Simons*
  - Dreibein & spin-connection: $e = \frac{\ell}{2}(A - \bar{A}) \qquad \omega = \frac{1}{2}(A + \bar{A})$
  - Rewriting of the action: $\boxed{I = S_{CS}[A] - S_{CS}[\bar{A}]}$

“Higher spins” in D = 2+1

- What happens using other gauge algebras in I_{grav}? sl(3,R)?

$$I = \frac{1}{16\pi G}\int \mathrm{tr}\left(e\wedge R + \frac{1}{3\ell^2}\, e\wedge e\wedge e\right)$$

$$\begin{cases} e = \left(e_\mu{}^a J_a + e_\mu{}^{ab} T_{ab}\right) dx^\mu \\ \omega = \left(\omega_\mu{}^a J_a + \omega_\mu{}^{ab} T_{ab}\right) dx^\mu \end{cases}$$

sl(3,R) algebra:

$$[J_a, J_b] = \epsilon_{abc} J^c$$

$$[J_a, T_{bc}] = \epsilon^m{}_{a(b} T_{c)m}$$

$$[T_{ab}, T_{cd}] = -\left(\eta_{a(c}\epsilon_{d)bm} + \eta_{b(c}\epsilon_{d)am}\right) J^m$$

“Higher spins” in D = 2+1

- What happens using other gauge algebras in I_{grav}? sl(3,R)?

$$I = \frac{1}{16\pi G}\int \mathrm{tr}\left(e\wedge R + \frac{1}{3\ell^2}\, e\wedge e\wedge e\right)$$

$$\begin{cases} e = \left(e_\mu{}^a J_a + e_\mu{}^{ab} T_{ab}\right) dx^\mu \\ \omega = \left(\omega_\mu{}^a J_a + \omega_\mu{}^{ab} T_{ab}\right) dx^\mu \end{cases}$$

no problems in defining the flat limit

sl(3,R) algebra:

$$[J_a, J_b] = \epsilon_{abc} J^c$$

$$[J_a, T_{bc}] = \epsilon^m{}_{a(b} T_{c)m}$$

$$[T_{ab}, T_{cd}] = -\left(\eta_{a(c}\epsilon_{d)bm} + \eta_{b(c}\epsilon_{d)am}\right) J^m$$

“Higher spins” in D = 2+1

- What happens using other gauge algebras in I_{grav}? sl(3,R)?

$$I = \frac{1}{16\pi G}\int \mathrm{tr}\left(e\wedge R + \frac{1}{3\ell^2}\, e\wedge e\wedge e\right) \qquad \begin{cases} e = \left(e_\mu{}^a J_a + e_\mu{}^{ab}\, T_{ab}\right) dx^\mu \\ \omega = \left(\omega_\mu{}^a J_a + \omega_\mu{}^{ab}\, T_{ab}\right) dx^\mu \end{cases}$$

- *Einstein-Hilbert ↔ Chern-Simons*
  - AdS: so(2,2) ≃ sl(2,R) ⊕ sl(2,R) Chern-Simons action   Achúcarro, Townsend (1986)

“Higher spins” in D = 2+1

- What happens using other gauge algebras in I_{grav}? sl(3,R)?

$$I = \frac{1}{16\pi G}\int \mathrm{tr}\left(e\wedge R + \frac{1}{3\ell^2}\, e\wedge e\wedge e\right) \qquad \begin{cases} e = \left(e_\mu{}^a J_a + e_\mu{}^{ab}\, T_{ab}\right) dx^\mu \\ \omega = \left(\omega_\mu{}^a J_a + \omega_\mu{}^{ab}\, T_{ab}\right) dx^\mu \end{cases}$$

- *Einstein-Hilbert ↔ Chern-Simons*
  - AdS: so(2,2) ≃ sl(2,R) ⊕ sl(2,R) Chern-Simons action Achúcarro, Townsend (1986)
  - Flat space: iso(2,1) = sl(2,R) $\oplus$ sl(2,R)$_{\mathrm{Ab}}$ Chern-Simons action Witten (1988)

“Higher spins” in D = 2+1

- What happens using other gauge algebras in I_{grav}? sl(3,R)?

$$I = \frac{1}{16\pi G}\int \mathrm{tr}\left(e\wedge R + \frac{1}{3\ell^2} e\wedge e\wedge e\right) \qquad \begin{cases} e = \left(e_\mu{}^a J_a + e_\mu{}^{ab} T_{ab}\right) dx^\mu \\ \omega = \left(\omega_\mu{}^a J_a + \omega_\mu{}^{ab} T_{ab}\right) dx^\mu \end{cases}$$

- *Einstein-Hilbert ↔ Chern-Simons*
  - AdS: so(2,2) ≃ sl(2,R) ⊕ sl(2,R) Chern-Simons action Achúcarro, Townsend (1986)
  - Flat space: iso(2,1) = sl(2,R) $\oplus$ $\mathrm{sl(2,R)}_{\mathrm{Ab}}$ Chern-Simons action Witten (1988)
  - Higher spins: sl(N,R) $\oplus$ sl(N,R) Chern-Simons theories Blencowe (1989)

A quick recap

- Gravity in flat space looks cool even if you like AdS/CFT!
- In 4D the infinite-dimensional BMS asymptotic symmetry isn't just a curiosity: it has interesting consequences
- Not easy to control the flat limit of AdS/CFT
- In 3D the BMS symmetry emerges as a **contraction** of the asymptotic 2D conformal symmetry
- In 3D the limiting procedure can also involve **higher spins** (more structures → more control)

A quick recap

- Gravity in flat space looks cool even if you like AdS/CFT!
- In 4D the infinite-dimensional BMS asymptotic symmetry isn't just a curiosity: it has interesting consequences
- Not easy to control the flat limit of AdS/CFT

see, however, Ciambelli, Marteau, Petkou, Petropoulos, Siampos (2018)

- In 3D the BMS symmetry emerges as a **contraction** of the asymptotic 2D conformal symmetry
- In 3D the limiting procedure can also involve **higher spins** (more structures → more control)

More on the $\Lambda \to 0$ limit: representation theory

The bms$_3$ algebra

- The centrally extended bms$_3$ algebra $(m \in \mathbb{Z})$

$$
\begin{aligned}
[J_m, J_n] &= (m-n)J_{m+n} + \frac{c_1}{12} m(m^2-1)\,\delta_{m+n,0} \\
[J_m, P_n] &= (m-n)P_{m+n} + \frac{c_2}{12} m(m^2-1)\,\delta_{m+n,0} \\
[P_m, P_n] &= 0
\end{aligned}
$$

- c_2 plays an important role in representation theory and doesn't vanish in gravity: $c_2 = \frac{3}{G}$

The bms$_3$ algebra

- The Poincaré subalgebra $(m=-1,0,1)$

$$
\begin{aligned}
[J_m, J_n] &= (m-n)J_{m+n} \qquad \leftarrow \text{Lorentz}\\
[J_m, P_n] &= (m-n)P_{m+n}\\
[P_m, P_n] &= 0
\end{aligned}
$$

- P_m → translations; J_1 and J_{-1} → boosts; J_0 → rotations

Poincaré unitary irreps in a nutshell

- Irreps of Poincaré *group* classified by orbits of momenta
  - all p^μ that satisfy $p^2 = -M^2$ for some mass M
- P_0 gives the energy and P_1,P_{-1} commute with it
  - build a basis of eigenstates of momentum: $|p^\mu, s\rangle$
- All plane waves can be obtained from a given one via

$$\boxed{U(\Lambda)|p^\mu, s\rangle = e^{is\theta}|\Lambda^\mu{}_\nu p^\nu, s\rangle}$$

$U(\omega) = \exp\left[i\left(\omega J_1 + \omega^* J_{-1}\right)\right]$ is a unitary operator

Rest-frame state & Poincaré modules

- Massive representations
  - Representative for the momentum orbit $k^\mu = (M, 0, 0)$
  - The corresponding plane wave $|M, s\rangle$ satisfies

$$\boxed{P_0|M,s\rangle = M|M,s\rangle\,, \quad P_{-1}|M,s\rangle = P_1|M,s\rangle = 0\,, \quad J_0|M,s\rangle = s|M,s\rangle}$$

  - $|M, s\rangle$ is annihilated by all P_n aside P_0!

Rest-frame state & Poincaré modules

- Massive representations
  - Representative for the momentum orbit $k^\mu = (M, 0, 0)$
  - The corresponding plane wave $|M, s\rangle$ satisfies

$$\boxed{P_0|M,s\rangle = M|M,s\rangle\,,\quad P_{-1}|M,s\rangle = P_1|M,s\rangle = 0\,,\quad J_0|M,s\rangle = s|M,s\rangle}$$

  - $|M, s\rangle$ is annihilated by all P_n aside P_0!

Save the info!

Rest-frame state & Poincaré modules

- *Rest-frame state*:

$$\boxed{P_0|M,s\rangle = M|M,s\rangle\,, \quad P_{-1}|M,s\rangle = P_1|M,s\rangle = 0\,, \quad J_0|M,s\rangle = s|M,s\rangle}$$

- Irreps of the Poincaré ***algebra*** built upon $|M,s\rangle$
  - Basis of the representation space: $\boxed{|k,l\,\rangle = (J_{-1})^k (J_1)^l |M,s\rangle}$
  - *P*$_n$ and *J*$_n$ act linearly on these states
  - Irreducible? Yes, Casimirs commute with all J$_n$
  - Unitary? Change basis! $|p^\mu,s\rangle = U(\Lambda)|M,s\rangle \longrightarrow \langle\, p^\mu, s\,|\,q^\mu, s\,\rangle = \delta_\mu(p,q)$

$\mathfrak{bms}_3$ modules

- Representation theory of BMS$_3$ *group* Barnich, Oblak (2014)
  - Irreps again classified by orbits of supermomentum $p(\varphi) = \sum_{n\in\mathbb{Z}} p_n e^{in\varphi}$
  - It exists a basis $|p(\varphi), s\rangle$ of eigenstates of supermomentum
  - Orbits with a constant $p(\varphi) = M - c_2/24$ → *rest-frame state*!

$\mathfrak{bms}_3$ modules

- Representation theory of BMS_3 *group* Barnich, Oblak (2014)
  - Irreps again classified by orbits of supermomentum $p(\varphi) = \sum_{n\in\mathbb{Z}} p_n e^{in\varphi}$
  - It exists a basis $|p(\varphi), s\rangle$ of eigenstates of supermomentum
  - Orbits with a constant $p(\varphi) = M - c_2/24$ → *rest-frame state*!

- Given the rest-frame state A.C. , Gonzalez, Oblak, Riegler (2016)

$$\boxed{P_0|M,s\rangle = M|M,s\rangle\,, \quad P_m|M,s\rangle = 0 \text{ for } m\neq 0\,, \quad J_0|M,s\rangle = s|M,s\rangle}$$

one can build a representation of the $\mathfrak{bms}_3$ *algebra* on

$$\boxed{J_{n_1}J_{n_2}\cdots J_{n_N}|M,s\rangle \quad \text{with} \quad n_1 \geq n_2 \geq \ldots \geq n_N}$$

Ultrarelativistic limit

- New generators: $$\boxed{P_m \equiv \frac{1}{\ell}\left(\mathcal{L}_m + \bar{\mathcal{L}}_{-m}\right), \qquad J_m \equiv \mathcal{L}_m - \bar{\mathcal{L}}_{-m}}$$
- In the limit $\ell \to \infty$ the conformal algebra becomes bms$_3$

Ultrarelativistic limit

- New generators: $$\boxed{P_m \equiv \frac{1}{\ell}\left(\mathcal{L}_m + \bar{\mathcal{L}}_{-m}\right)\,, \qquad J_m \equiv \mathcal{L}_m - \bar{\mathcal{L}}_{-m}}$$
- In the limit $\ell \to \infty$ the conformal algebra becomes bms3
- What happens to *highest-weight representations*?
  - HW state: $\mathcal{L}_n|h,\bar{h}\rangle = 0\,, \qquad \bar{\mathcal{L}}_n|h,\bar{h}\rangle = 0 \quad \text{when } n > 0$
  - Verma module: $\mathcal{L}_{-n_1}\cdots\mathcal{L}_{-n_k}\bar{\mathcal{L}}_{-\bar{n}_1}\cdots\bar{\mathcal{L}}_{-\bar{n}_l}|h,\bar{h}\rangle$

Ultrarelativistic limit

- New generators: $$\boxed{P_m \equiv \frac{1}{\ell}\left(\mathcal{L}_m + \bar{\mathcal{L}}_{-m}\right), \qquad J_m \equiv \mathcal{L}_m - \bar{\mathcal{L}}_{-m}}$$
- In the limit $\ell \to \infty$ the conformal algebra becomes bms3
- What happens to *highest-weight representations*?
  - HW state: $\mathcal{L}_n|h,\bar{h}\rangle = 0\,, \qquad \bar{\mathcal{L}}_n|h,\bar{h}\rangle = 0 \quad \text{when } n > 0$
  - Verma module: $\mathcal{L}_{-n_1}\cdots\mathcal{L}_{-n_k}\bar{\mathcal{L}}_{-\bar{n}_1}\cdots\bar{\mathcal{L}}_{-\bar{n}_l}|h,\bar{h}\rangle$
  - New quantum numbers of the HW state: $M \equiv \dfrac{h+\bar{h}}{\ell}\,, \qquad s \equiv h - \bar{h}$
  - Rewrite $\mathcal{L}_{-n_1}\cdots\mathcal{L}_{-n_k}\bar{\mathcal{L}}_{-\bar{n}_1}\cdots\bar{\mathcal{L}}_{-\bar{n}_l}|h,\bar{h}\rangle$ in the new basis as $J_{n_1}J_{n_2}\cdots J_{n_N}|M,s\rangle$ with $n_1 \geq n_2 \geq \ldots \geq n_N$
  - J_n don't annihilate the vacuum → invertible change of basis!

Ultrarelativistic limit

- Matrix elements of P_n and J_n

A.C., Gonzalez, Oblak, Riegler (2016)

$$\boxed{\begin{aligned} P_n\,|k_1,\ldots,k_N\rangle &= \sum_{k'_i} \mathsf{P}^{(n)}_{k'_i;k_j}(M,s,\ell)\,|k'_1,\ldots,k'_N\rangle \\ J_n\,|k_1,\ldots,k_N\rangle &= \sum_{k'_i} \mathsf{J}^{(n)}_{k'_i;k_j}(M,s)\,|k'_1,\ldots,k'_N\rangle \end{aligned}}$$

  - ℓ comes from the "old" CFT HW conditions: $\left(P_{\pm n} \pm \frac{1}{\ell} J_{\pm n}\right)|h,\bar{h}\rangle = 0$
  - only negative powers of ℓ appear: limit exists!

- If $h = \dfrac{M\ell + s}{2} + \lambda + \mathcal{O}(\ell^{-1})$, $\quad \bar{h} = \dfrac{M\ell - s}{2} + \lambda + \mathcal{O}(\ell^{-1})$

the highest-weight state $|h,\bar{h}\rangle$ satisfies in the limit

$$P_0|M,s\rangle = M|M,s\rangle\,, \quad P_m|M,s\rangle = 0 \text{ for } m \neq 0\,, \quad J_0|M,s\rangle = s|M,s\rangle$$

Galilean limit

- Alternative contraction conformal → bms3

Bagchi, Gopakumar, Mandal, Miwa (2010)

$$\boxed{M_n \equiv \epsilon\left(\bar{\mathcal{L}}_n - \mathcal{L}_n\right), \qquad L_n \equiv \bar{\mathcal{L}}_n + \mathcal{L}_n}$$

$$c_L = \bar{c} + c$$
$$c_M = \epsilon\left(\bar{c} - c\right)$$

$$\begin{aligned}
[L_m, L_n] &= (m-n)\, L_{m+n} + \frac{c_L}{12}\, m(m^2-1)\, \delta_{m+n,0} \\
[L_m, M_n] &= (m-n)\, M_{m+n} + \frac{c_M}{12}\, m(m^2-1)\, \delta_{m+n,0} \\
[M_m, M_n] &= \epsilon^2\, (\cdots)
\end{aligned}$$

- What happens to highest-weight reps?

$$\Delta = \bar{h} + h\,, \qquad \xi = \epsilon\left(\bar{h} - h\right)$$

$$L_n|\Delta, \xi\rangle = 0\,, \qquad M_n|\Delta, \xi\rangle = 0\,, \quad n > 0$$

$$|\{\mathfrak{l}_i\}, \{\mathfrak{m}_j\}\rangle = L_{-\mathfrak{l}_1} \ldots L_{-\mathfrak{l}_i} M_{-\mathfrak{m}_1} \ldots M_{-\mathfrak{m}_j} |\Delta, \xi\rangle \qquad \mathfrak{m}_1 \geq \ldots \geq \mathfrak{m}_j > 0$$

Galilean limit

- Alternative contraction conformal → bms3

Bagchi, Gopakumar, Mandal, Miwa (2010)

$$\boxed{M_n \equiv \epsilon\left(\bar{\mathcal{L}}_n - \mathcal{L}_n\right), \qquad L_n \equiv \bar{\mathcal{L}}_n + \mathcal{L}_n}$$

$$c_L = \bar{c} + c$$

$$c_M = \epsilon\left(\bar{c} - c\right)$$

- What happens to highest-weight reps?

$$\Delta = \bar{h} + h\,, \qquad \xi = \epsilon\left(\bar{h} - h\right)$$

$$L_n|\Delta,\xi\rangle = 0\,, \qquad M_n|\Delta,\xi\rangle = 0\,, \quad n > 0$$

$$|\{\mathfrak{l}_i\},\{\mathfrak{m}_j\}\rangle = L_{-\mathfrak{l}_1}\ldots L_{-\mathfrak{l}_i} M_{-\mathfrak{m}_1}\ldots M_{-\mathfrak{m}_j}|\Delta,\xi\rangle$$

- HW reps are mapped into other HW reps
  - Cool! We can define a scalar product using $(M_m)^\dagger = M_{-m}$ $(L_m)^\dagger = L_{-m}$
  - These reps are typically non-unitary and reducible
  - Ok for condensed matter applications but bad for gravity!

bms_3 representation theory: key facts

- The bms_3 algebra can be recovered via **two** different contractions of the 2D conformal algebra

- The resulting algebra is the same, but the limit of highest-weight representations is very different

  - **Ultrarelativistic (Carrollian) contraction**: unitary induced representations

  - **Non-relativistic (Galilean) contraction**: non-unitary highest-weight representations

- The latter representations are "easier" to handle: basis of recent developments

  - Holographic entanglement entropy — Bagchi, Basu, Grumiller, Riegler (2014)

  - BMS bootstrap — Bagchi, Gary, Zodinmawia (2016)

  - BMS conformal blocks — Hijano; Lodato, Merbis, Zodinmawia (2018)

More on the $\Lambda \to 0$ limit: boundary conditions

Boundary conditions: AdS vs flat

- The limit $\Lambda \to 0$ cannot be taken naively in Fefferman-Graham coordinates

$$ds^2 = \frac{l^2}{\rho^2} d\rho^2 + g_{AB}(\rho, x) dx^A dx^B \to \infty \ ?$$

- The limits of a spacetime may depend on the coordinates

- To describe the limit $\Lambda \to 0$ one has to change gauge

  - **BMS gauge:** $ds^2 = e^{2\beta} \frac{V}{r} du^2 - 2e^{2\beta} du dr + r^2 (d\phi - U du)^2$

Barnich, Gomberoff, Gongalez (2012)

  - **Fluid/gravity gauge:** $ds^2 = \frac{2}{k^2} \mathbf{u} \left(dr + r A \right) + r^2 ds^2_{(2)} + \frac{8\pi G}{k^4} \mathbf{u} \left(\epsilon \, \mathbf{u} + \chi * \mathbf{u} \right)$

$$\begin{cases} \mathrm{u} = -k^2 \left(\Omega \mathrm{d}t - (b_x + \beta_x) \, \mathrm{d}x \right) + \mathrm{O}\left(k^4\right) \\ \mathrm{A} \ = \ \theta \Omega \mathrm{d}t + (\alpha_x + \delta_x) \, \mathrm{d}x + \mathrm{O}\left(k^2\right) \end{cases} \to \text{well defined flat limit}$$

Ciambelli, Marteau, Petkou, Petropoulos, Siampos (2018)

The $\Lambda \to 0$ limit for higher spins

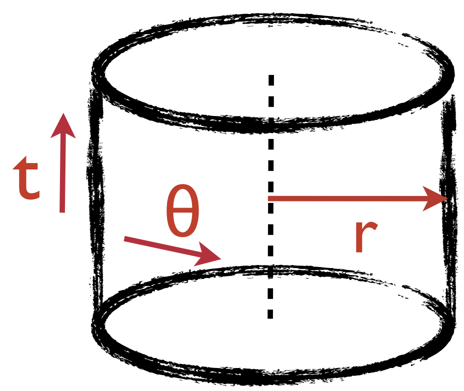
Asymptotic symmetries reloaded

Coussaert, Henneaux, van Driel (1995)

- AdS3 Chern-Simons connection: $A = b^{-1} a_+ b \, dx^+ + b^{-1} db$

Henneaux, Rey; A.C., Pfenninger, Fredenhagen, Theisen (2010)

$$a_+ = L_1 - \frac{2\pi}{k} \mathcal{L}(x^+) \, L_{-1} - \frac{\pi}{2k} \mathcal{W}(x^+) \, W_{-2}$$



$$x^\pm = \tfrac{t}{l} \pm \theta$$

sl(3,R) algebra:

$$[L_i, L_j] = (i-j) L_{i+j}$$

$$[L_i, W_m] = (2i - m) W_{i+m}$$

$$[W_m, W_n] = \frac{n-m}{3} (2m^2 + 2n^2 - mn - 8) L_{m+n}$$

-1≤i,j≤1 and -2≤m,n≤2

Asymptotic symmetries reloaded

Coussaert, Henneaux, van Driel (1995)

- AdS₃ Chern-Simons connection: $A = b^{-1} a_+ b \, dx^+ + b^{-1} db$

Henneaux, Rey; A.C., Pfenninger, Fredenhagen, Theisen (2010)

$$a_+ = L_1 - \frac{2\pi}{k} \mathcal{L}(x^+) \, L_{-1} - \frac{\pi}{2k} \mathcal{W}(x^+) \, W_{-2}$$

- Gauge transformations preserving boundary conditions
  - generated by $\Lambda = b^{-1}(\rho) \Big[\varepsilon(\theta) \, L_1 + \chi(\theta) \, W_2 + \lambda(\varepsilon, \chi) \Big] b(\rho)$

- Asymptotic symmetries

$$\delta_\varepsilon \mathcal{L} = \varepsilon \mathcal{L}' + 2 \, \varepsilon' \mathcal{L} + \frac{k}{4\pi} \varepsilon''' ,$$

$$\delta_\varepsilon \mathcal{W} = \varepsilon \mathcal{W}' + 3 \, \varepsilon' \mathcal{W} ,$$

$$\delta_\chi \mathcal{W} = \frac{\sigma}{3} \bigg(2 \chi \mathcal{L}''' + 9 \chi' \mathcal{L}'' + 15 \chi'' \mathcal{L}' + 10 \chi''' \mathcal{L} + \frac{k}{4\pi} \chi^{(5)} + \frac{64\pi}{k} \left(\chi \mathcal{L} \mathcal{L}' + \chi' \mathcal{L}^2 \right) \bigg) ,$$

Spin-3 extension of the conformal algebra

$$
\begin{aligned}
[\mathcal{L}_m,\, \mathcal{L}_n] &= (m-n)\,\mathcal{L}_{m+n} + \frac{c}{12}\,(m^3-m)\,\delta_{m+n,\,0}\,,\\
[\mathcal{L}_m,\, \mathcal{W}_n] &= (2m-n)\,\mathcal{W}_{m+n}\,,\\
[\mathcal{W}_m,\, \mathcal{W}_n] &= (m-n)(2m^2+2n^2-mn-8)\,\mathcal{L}_{m+n} + \frac{96}{c+\frac{22}{5}}\,(m-n)\,:\mathcal{L}\mathcal{L}:_{m+n}\\
&\qquad + \frac{c}{12}\,(m^2-4)(m^3-m)\,\delta_{m+n,\,0}\,,
\end{aligned}
$$

Spin-3 extension of the conformal algebra

$$\boxed{[\mathcal{L}_m,\, \mathcal{L}_n] = (m-n)\,\mathcal{L}_{m+n} + \frac{c}{12}\,(m^3-m)\,\delta_{m+n,\,0}\,,}$$

$$[\mathcal{L}_m,\, \mathcal{W}_n] = (2m-n)\,\mathcal{W}_{m+n}\,,$$

$$[\mathcal{W}_m,\, \mathcal{W}_n] = (m-n)(2m^2+2n^2-mn-8)\,\mathcal{L}_{m+n} + \frac{96}{c+\frac{22}{5}}\,(m-n)\,:\!\mathcal{L}\mathcal{L}\!:_{m+n}$$

$$+\,\frac{c}{12}\,(m^2-4)(m^3-m)\,\delta_{m+n,\,0}\,,$$

Spin-3 extension of the conformal algebra

$$[\mathcal{L}_m,\, \mathcal{L}_n] = (m-n)\,\mathcal{L}_{m+n} + \frac{c}{12}\,(m^3-m)\,\delta_{m+n,\,0}\,,$$

$$\boxed{[\mathcal{L}_m,\, \mathcal{W}_n] = (2m-n)\,\mathcal{W}_{m+n}\,,}$$

$$[\mathcal{W}_m,\, \mathcal{W}_n] = (m-n)(2m^2+2n^2-mn-8)\,\mathcal{L}_{m+n} + \frac{96}{c+\frac{22}{5}}\,(m-n)\,:\!\mathcal{L}\mathcal{L}\!:_{m+n}$$

$$+\,\frac{c}{12}\,(m^2-4)(m^3-m)\,\delta_{m+n,\,0}\,,$$

Spin-3 extension of the conformal algebra

$$[\mathcal{L}_m, \mathcal{L}_n] = (m-n)\,\mathcal{L}_{m+n} + \frac{c}{12}(m^3-m)\,\delta_{m+n,0}\,,$$

$$[\mathcal{L}_m, \mathcal{W}_n] = (2m-n)\,\mathcal{W}_{m+n}\,,$$

$$[\mathcal{W}_m, \mathcal{W}_n] = (m-n)(2m^2+2n^2-mn-8)\,\mathcal{L}_{m+n} + \frac{96}{c+\frac{22}{5}}(m-n) :\mathcal{L}\mathcal{L}:_{m+n}$$
$$+ \frac{c}{12}(m^2-4)(m^3-m)\,\delta_{m+n,0}\,,$$

- <u>Normal ordering</u> now needed:

$$:\mathcal{L}\mathcal{L}:_m = \sum_{p\geq -1}\mathcal{L}_{m-p}\mathcal{L}_p + \sum_{p<-1}\mathcal{L}_p\mathcal{L}_{m-p} - \frac{3}{10}(m+3)(m+2)\mathcal{L}_m$$

Spin-3 extension of the conformal algebra

$$[\mathcal{L}_m,\, \mathcal{L}_n] = (m-n)\,\mathcal{L}_{m+n} + \frac{c}{12}\,(m^3-m)\,\delta_{m+n,\,0}\,,$$
$$[\mathcal{L}_m,\, \mathcal{W}_n] = (2m-n)\,\mathcal{W}_{m+n}\,,$$
$$[\mathcal{W}_m,\, \mathcal{W}_n] = (m-n)(2m^2+2n^2-mn-8)\,\mathcal{L}_{m+n} + \frac{96}{c+\frac{22}{5}}\,(m-n)\,:\mathcal{L}\mathcal{L}:_{m+n}$$
$$+\frac{c}{12}\,(m^2-4)(m^3-m)\,\delta_{m+n,\,0}\,,$$

- Normal ordering now needed:

$$:\mathcal{L}\mathcal{L}:_m = \sum_{p\geq -1}\mathcal{L}_{m-p}\mathcal{L}_p + \sum_{p<-1}\mathcal{L}_p\mathcal{L}_{m-p} - \frac{3}{10}(m+3)(m+2)\mathcal{L}_m$$

- Ultrarelativistic contraction:

$$P_m \equiv \frac{1}{\ell}\left(\mathcal{L}_m + \bar{\mathcal{L}}_{-m}\right), \qquad J_m \equiv \mathcal{L}_m - \bar{\mathcal{L}}_{-m}$$
$$W_m \equiv \mathcal{W}_m - \bar{\mathcal{W}}_{-m}, \qquad Q_m \equiv \frac{1}{\ell}\left(\mathcal{W}_m + \bar{\mathcal{W}}_{-m}\right)$$

Spin-3 extension of bms$_3$

- Limit $\ell \to \infty$: bms$_3$ algebra plus…

$$
\begin{aligned}
[W_m,\, W_n] &= (m-n)(2m^2+2n^2-mn-8)J_{m+n} + \frac{96}{c_2}(m-n)\Lambda_{m+n} \\
&\qquad - \frac{96\, c_1}{c_2^2}(m-n)\Theta_{m+n} + \frac{c_1}{12}(m^2-4)(m^3-m)\,\delta_{m+n,\,0}\,, \\
[W_m,\, Q_n] &= (m-n)(2m^2+2n^2-mn-8)P_{m+n} + \frac{96}{c_2}(m-n)\Theta_{m+n} \\
&\qquad + \frac{c_2}{12}(m^2-4)(m^3-m)\,\delta_{m+n,\,0}\,, \\
[Q_m,\, Q_n] &= 0\,,
\end{aligned}
$$

- Non-linearities survive in the limit!

$$
\boxed{\Theta_m \equiv \sum_{p=-\infty}^{\infty} P_{m-p}P_p\,, \qquad \Lambda_m \equiv \sum_{p=-\infty}^{\infty} \left(P_{m-p}J_p + J_{m-p}P_p\right)}
$$

Spin-3 extension of bms$_3$

- Limit $\ell \to \infty$: bms$_3$ algebra plus…

$$
\begin{aligned}
[W_m, W_n] &= (m-n)(2m^2+2n^2-mn-8)J_{m+n} + \frac{96}{c_2}(m-n)\Lambda_{m+n} \\
&\quad - \frac{96\, c_1}{c_2^2}(m-n)\Theta_{m+n} + \frac{c_1}{12}(m^2-4)(m^3-m)\,\delta_{m+n,0}\,, \\
[W_m, Q_n] &= (m-n)(2m^2+2n^2-mn-8)P_{m+n} + \frac{96}{c_2}(m-n)\Theta_{m+n} \\
&\quad + \frac{c_2}{12}(m^2-4)(m^3-m)\,\delta_{m+n,0}\,, \\
[Q_m, Q_n] &= 0\,,
\end{aligned}
$$

- Non-linearities survive in the limit!

$$
\boxed{\Theta_m \equiv \sum_{p=-\infty}^{\infty} P_{m-p}P_p\,, \qquad \Lambda_m \equiv \sum_{p=-\infty}^{\infty} \left(P_{m-p}J_p + J_{m-p}P_p\right)}
$$

Spin-3 extension of $\mathfrak{bms}_3$

- Limit $\ell \to \infty$: bms₃ algebra plus…

$$
\begin{aligned}
[W_m,\, W_n] &= (m-n)(2m^2+2n^2-mn-8)J_{m+n} + \frac{96}{c_2}(m-n)\Lambda_{m+n} \\
&\quad - \frac{96\,c_1}{c_2^2}(m-n)\Theta_{m+n} + \frac{c_1}{12}(m^2-4)(m^3-m)\,\delta_{m+n,\,0}\,, \\
[W_m,\, Q_n] &= (m-n)(2m^2+2n^2-mn-8)P_{m+n} + \frac{96}{c_2}(m-n)\Theta_{m+n} \\
&\quad + \frac{c_2}{12}(m^2-4)(m^3-m)\,\delta_{m+n,\,0}\,, \\
[Q_m,\, Q_n] &= 0\,,
\end{aligned}
$$

Galilean limit
=
different ordering!

- Non-linearities survive in the limit!

$$
\boxed{\Theta_m \equiv \sum_{p=-\infty}^{\infty} P_{m-p}P_p\,, \qquad \Lambda_m \equiv \sum_{p=-\infty}^{\infty} (P_{m-p}J_p + J_{m-p}P_p)}
$$

Grumiller, Riegler, Rosseel (2014)

Higher-spin modules

- Representations as for bms3 and Poincaré
  - Introduce a rest-frame state $P_m|M,q_0\rangle = 0\,, \qquad Q_m|M,q_0\rangle = 0 \quad \text{for } m \neq 0$
  - Build the vector space which carries the representation as

    $$W_{k_1} \cdots W_{k_m} J_{l_1} \cdots J_{l_n}|M,q_0\rangle \qquad k_1 \geq \cdots \geq k_m \qquad l_1 \geq \cdots \geq l_n$$

  - No problems with non-linearities (construction based on the universal enveloping algebra)

- Construction compatible with normal ordering:
  - $\langle 0|\Theta_n|0\rangle = \langle 0|\Lambda_n|0\rangle = 0$
  - Not true if one uses “Galilean" highest-weight reps!

Comparison with dimension four and higher

Boundary conditions for spin 3 in D = n+2

AC, Francia, Heissenberg (2017)

- Minkowski background: $ds^2 = -\,du^2 - 2dudr + r^2\gamma_{ij}(x^k)dx^i dx^j$

- Bondi-like "gauge": $\varphi_{r\alpha\beta} = 0\,, \qquad g^{\nu\rho}\varphi_{\mu\nu\rho} = 0$

- Boundary conditions in any dimension:

$$\boxed{\begin{aligned}
\varphi_{uuu} &= \sum_{l=0}^{\left[\frac{n-4}{2}\right]} r^{-\frac{n}{2}-l} B^{(l)} + r^{1-n}\mathcal{M} + \cdots \\
\varphi_{uui} &= \sum_{l=0}^{\left[\frac{n-2}{2}\right]} r^{-\frac{n}{2}-l+1} U_i^{(l)} + r^{1-n}\mathcal{N}_i + \cdots \\
\varphi_{uij} &= \sum_{l=0}^{\left[\frac{n}{2}\right]} r^{-\frac{n}{2}-l+2} V_{ij}^{(l)} + r^{1-n}\mathcal{P}_{ij} + \cdots
\end{aligned}}$$

$$\varphi_{ijk} = r^{-\frac{n}{2}+3} C_{ijk} + \cdots$$

n = dimension of the celestial sphere

Radiation

"Coulomb"

Boundary conditions for spin 3 in D = n+2

AC, Francia, Heissenberg (2017)

- Minkowski background: $ds^2 = -\,du^2 - 2dudr + r^2\gamma_{ij}(x^k)dx^i dx^j$
- Bondi-like "gauge": $\varphi_{r\alpha\beta} = 0\,, \qquad g^{\nu\rho}\varphi_{\mu\nu\rho} = 0$

see Dario's talk

- Boundary conditions in any dimension:

$$\boxed{\begin{aligned} \varphi_{uuu} &= \sum_{l=0}^{\left[\frac{n-4}{2}\right]} r^{-\frac{n}{2}-l} B^{(l)} + r^{1-n}\mathcal{M} + \cdots \\ \varphi_{uui} &= \sum_{l=0}^{\left[\frac{n-2}{2}\right]} r^{-\frac{n}{2}-l+1} U_i^{(l)} + r^{1-n}\mathcal{N}_i + \cdots \\ \varphi_{uij} &= \sum_{l=0}^{\left[\frac{n}{2}\right]} r^{-\frac{n}{2}-l+2} V_{ij}^{(l)} + r^{1-n}\mathcal{P}_{ij} + \cdots \end{aligned}}$$

Radiation "Coulomb"

$$\varphi_{ijk} = r^{-\frac{n}{2}+3}C_{ijk} + \cdots$$

n = dimension of the celestial sphere

Higher-spin super-translations & -rotations

- Residual "gauge" symmetry

$$\epsilon^{ij} = \left[K^{ij} + \frac{u}{r}\,\mathcal{T}_2^{ij}(K) + \left(\frac{u}{r}\right)^2 \mathcal{T}_4^{ij}(K)\right] + \frac{1}{r}\left[\mathcal{U}_1^{ij}(\rho) + \frac{u}{r}\,\mathcal{U}_3^{ij}(\rho)\right] + \frac{1}{r^2}\,\mathcal{V}_2^{ij}(T)\,,$$

$$\epsilon^{ui} = \frac{u}{n+2}\left[D\cdot K^i - \frac{u\,r^{-1}}{2(n+1)}\,D^i D\cdot D\cdot K\right] - \left[\rho^i - \frac{u\,r^{-1}}{n+1}\,D^i D\cdot\rho\right] + \frac{1}{2\,r}\,D^i T\,,$$

$$\epsilon^{uu} = \frac{u^2}{(n+1)(n+2)}\,D\cdot D\cdot K - \frac{2\,u}{n+1}\,D\cdot\rho - T\,.$$

and similar expressions for ϵ^{ri}, ϵ^{ru} and ϵ^{rr}

- Asymptotic symmetries generated by T, ρ^i and K^{ij} with

$$\mathcal{K}^{ijk} \equiv D^{(i}K^{jk)} - \frac{2}{n+2}\,\gamma^{(ij}D\cdot K^{k)} = 0$$

$$\mathcal{R}^{ijk} \equiv D^{(i}D^j\rho^{k)} - \frac{2}{n+2}\left(\gamma^{(ij}\Box\rho^{k)} + \gamma^{(ij}\left\{D^{k)}, D^l\right\}\rho_l\right) = 0$$

$$\mathcal{T}^{ijk} \equiv D^{(i}D^jD^{k)}T - \frac{2}{n+2}\left(\gamma^{(ij}\Box D^{k)}T + \gamma^{(ij}\left\{D^{k)}, D^l\right\}D_l T\right) = 0 \quad \text{(for D > 4)}$$

Higher-spin super-translations & -rotations

- Residual "gauge" symmetry

$$\epsilon^{ij} = \left[K^{ij} + \frac{u}{r}\,\mathcal{T}_2^{ij}(K) + \left(\frac{u}{r}\right)^2 \mathcal{T}_4^{ij}(K)\right] + \frac{1}{r}\left[\mathcal{U}_1^{ij}(\rho) + \frac{u}{r}\,\mathcal{U}_3^{ij}(\rho)\right] + \frac{1}{r^2}\,\mathcal{V}_2^{ij}(T)\,,$$

$$\epsilon^{ui} = \frac{u}{n+2}\left[D\cdot K^i - \frac{u\,r^{-1}}{2(n+1)}\,D^i D\cdot D\cdot K\right] - \left[\rho^i - \frac{u\,r^{-1}}{n+1}\,D^i D\cdot\rho\right] + \frac{1}{2\,r}\,D^i T\,,$$

$$\epsilon^{uu} = \frac{u^2}{(n+1)(n+2)}\,D\cdot D\cdot K - \frac{2\,u}{n+1}\,D\cdot\rho - T\,.$$

and similar expressions for ϵ^{ri}, ϵ^{ru} and ϵ^{rr}

- Asymptotic symmetries generated by T, ρ^i and K^{ij} with

$$\mathcal{K}^{ijk} \equiv D^{(i}K^{jk)} - \frac{2}{n+2}\,\gamma^{(ij}D\cdot K^{k)} = 0$$

$$\mathcal{R}^{ijk} \equiv D^{(i}D^j\rho^{k)} - \frac{2}{n+2}\left(\gamma^{(ij}\Box\rho^{k)} + \gamma^{(ij}\left\{D^{k)},D^l\right\}\rho_l\right) = 0$$

$$\mathcal{T}^{ijk} \equiv D^{(i}D^jD^{k)}T - \frac{2}{n+2}\left(\gamma^{(ij}\Box D^{k)}T + \gamma^{(ij}\left\{D^{k)},D^l\right\}D_lT\right) = 0$$ (for D > 4)

Comparison with Chern-Simons

- All $B^{(l)}$, $U_i^{(l)}$ and $V_{ij}^{(l)}$ are fixed in terms of C_{ijk},
  while ***M*, *N*$_i$ and *P*$_{ij}$ are "integration constants"**

- Apparently, there are many more ingredients than in the Chern-Simons computation

- In 3D, however, "radiation" become subleading, N_i has only one independent component and P_{ij} vanishes

$$\varphi_{uuu} = \mathcal{M}(\phi) + \mathcal{O}(r^{-1})\,, \qquad \varphi_{uu\phi} = \mathcal{N}(\phi) + \frac{u}{3}\,\partial_\phi \mathcal{M}(\phi) + \mathcal{O}(r^{-1})$$

- Eventually one recovers the same boundary conditions as in the Chern-Simons theory!

Back to the “motivational” summary

- Gravity in flat space looks cool even if you like AdS/CFT!
- In 4D the infinite-dimensional BMS asymptotic symmetry isn’t just a curiosity: it has interesting consequences
- In 3D the BMS symmetry emerges as a **contraction** of the asymptotic 2D conformal symmetry
- In 3D the limiting procedure can also involve **higher spins** (more structures → more control)
- The flat limit can be controlled in the fluid/gravity setup