Observer Design for Linear Parabolic PDE Systems

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1 Introduction

The state estimation problem of parabolic PDE systems from its input and measured outputs over some finite time interval has been an active field of research over the last years (see e.g. [1], and the references therein). This work aims at designing observers for systems described by certain class of parabolic PDEs. The state estimation problem is transformed into a well-posed stabilization problem for the dynamics of the state estimation error which is formulated in terms of Linear matrix Inequalities (LMIs).

2 System Description and Formulation

Consider the following class of parabolic PDE systems:

$$x_t(z,t) = \gamma x_{zz}(z,t) - \upsilon x_z(z,t) + k(z)x(z,t) \tag{1}$$

$$\begin{aligned} \gamma x_z(0,t) &= \upsilon(x(0,t) - x_{in}(t)) \\ x_z(l,t) &= 0. \end{aligned} \tag{2}$$

The associated state estimation problem for the system (1)-(2) consists in designing a dynamical observer on the basis of its mathematical model, the measurement

$$y(t) = \int_0^l \mathbf{c}^T(z)x(z,t)dz \tag{3}$$

and the input signal $x_{in}(t)$ which produces a convergent state estimate $\hat{x}(z,t)$ such that $\lim_{t\to\infty} \|x(\cdot,t) - \hat{x}(\cdot,t)\|_2 = 0$.

2.1 Measurement sensors

The vector $\mathbf{c}(z) = \begin{bmatrix} c_1(z) & c_2(z) & \cdots & c_{n_y} \end{bmatrix}$ in (3) will lead to different forms of local measurement. For example, the choice

$$c_j(z) = \begin{cases} \frac{1}{2\varepsilon_j} & z \in [z_j - \varepsilon_j,\, z_j + \varepsilon_j] \\ 0 & \text{elsewhere} \end{cases} \quad ,\ j \in \mathbb{N}_y = \{1, 2, \ldots, n_y\}. \tag{4}$$

corresponds to piecewise measurement that produces n_y zones of piecewise uniform sensing in the interval $[z_j - \varepsilon_j,\ z_j + \varepsilon_j]$, where $0 < z_1 < z_2 < \ldots < z_{n_y} < l$. This case is ilustrated in Figure 1.

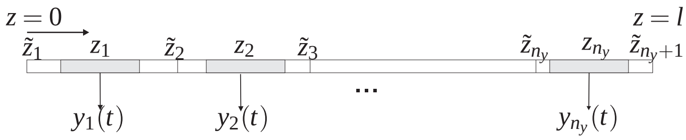

Figure 1: Distributed measurement

3 Observer design

The proposed Luenberger-type PDE observer for (1)-(2) is given by

$$\begin{aligned} \hat{x}_t(z,t) &= \gamma\hat{x}_{zz}(z,t) - \upsilon\hat{x}_z(z,t) + k_0(z)\hat{x}(z,t) \\ &\quad + \mathbf{c}(z)l_M \int_0^l \mathbf{c}^T(z)(\hat{x}(z,t) - x(z,t))dz \end{aligned} \tag{5}$$

$$\begin{aligned} \gamma\hat{x}_z(0,t) &= \upsilon(\hat{x}(0,t) - x_{in}(t)) \\ \hat{x}_z(l,t) &= 0 \quad \text{where } l_M = diag\{l_1, \ldots, l_{n_y}\}. \end{aligned} \tag{6}$$

Using Lyapunov's approach, the following theorem provides a solution to the state estimation problem.

Theorem 3.1 *Considering the PDE system (1)-(2) with a given positive constants γ, υ and the measurement output $y(t)$ of the form (3). If there exist constants l_j, $j \in \mathbb{N}_y$ satisfying the following LMIs:*

$$\begin{bmatrix} k(z) - \frac{1}{4}\pi^2\gamma\varphi_j^{-2} & \frac{1}{4}\pi^2\gamma\varphi_j^{-2} \\ \frac{1}{4}\pi^2\gamma\varphi_j^{-2} & -\frac{l_j^2}{\tilde{z}_{j+1} - \tilde{z}_j} - \frac{1}{4}\pi^2\gamma\varphi_j^{-2} \end{bmatrix} < 0 \; \forall j \in \mathbb{N}_y \tag{7}$$

with $\varphi_j^2 = max\{(z_j + \varepsilon_j - \tilde{z}_j)^2, (\tilde{z}_{j+1} - z_j + \varepsilon_j)^2\}$, $j \in \mathbb{N}_y$, then the Luenberger-type PDE observer of the form (5)-(6) ensures the exponential convergence of $\|x(\cdot,t) - \hat{x}(\cdot,t)\|_2$.

References

[1] Z. Hidayat, R. Babuska, B. De Schutter, and A. Nunez, "Observers for linear distributed-parameter systems: A survey," Proceedings of the 2011 IEEE International Symposium on Robotic and Sensors Environments (ROSE), 2011.