

Cutting tool life management in turning process: a new approach based on a stochastic wear process and the Cox model

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Abstract—In machining processes, a significant cost contribution is related to the cutting tool insert. A precocious replacement leads to lesser profitability of the cutting tool while a late replacement tends to produce more scraps due to advanced wear of the cutting tool insert. To optimize the replacement times, there is a need to develop an integrated monitoring framework to assess the wear of the cutting tool for different cutting conditions during the machining process and to predict the remaining useful life. The aim of this paper is to propose a complementary approach based on a gamma process to model the stochastic behaviour of the flank wear evolution and a Cox proportional hazard model to consider different cutting conditions. Experimental data measured in turning is used to fit a piecewise stationary gamma process on 29 cutting tool inserts using identical cutting conditions. Another set of data is used to fit a Taylor law to take into account different cutting speeds. The piecewise stationary gamma process is then adapted to simulate random flank wear paths for a defined cutting speed range. Using this model, several cutting tool lifetimes are simulated and used to feed a Cox proportional hazard model. The fitting procedure relies on a learning phase and a control phase to ensure the accuracy of the model. The results of both models are then discussed, and the robustness of the Cox Proportional Hazards Model to noise in the data is assessed.

I. INTRODUCTION

The general framework of this study is the optimization of mechanical equipment in order to diminish costs and increase availability. Proper knowledge concerning the reliability and updated Remaining Useful Life (RUL) estimates allow this improvement. In particular, this methodology is applied to the wear of cutting tools. During the machining process, cutting inserts undergo several wear mechanisms. Flank wear (see Fig. 1) appears between the flank face of the cutting tool and the machined part. This wear is mainly due to abrasive wear, and is the most commonly studied, as it is stable and may be predicted.

The importance of this wear in the maintenance of cutting inserts stems from the bad surface quality and the poor dimensional precision of parts, and thus more scraps and the associated costs, which may be crippling, in particular in high added-value production industry. The present study proposes methodologies that may help optimize the cutting inserts

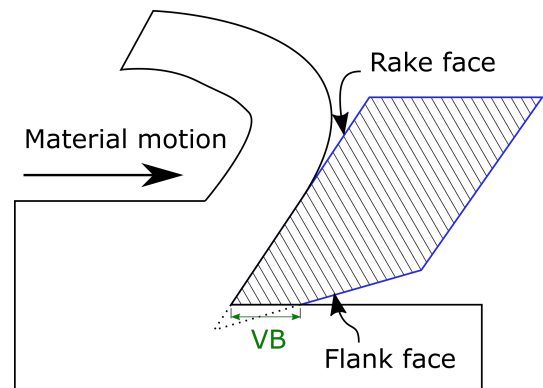


Fig. 1. Flank wear, which is a result of abrasive wear, is traditionally considered as the wear that appears in nominal machining conditions. Abrasive wear progressively eats away the cutting edge of the insert, and the material loss results in the apparition of a wear face. Its width, denoted VB , is the standardized criterion for flank wear (ISO 3685).

lifetimes, reduce costs and increase the assets availability. First, a stochastic flank wear model characterizes the flank wear evolution (see Fig. 2) for each cutting insert. This model is calibrated on experimental data and allows the production of degradation trajectories following the measured experimental behaviour. Then, a Cox Proportional Hazard Model (PHM), using the lifetimes generated by the stochastic model as learning set, is used to predict the Mean Up Time of the cutting inserts for varying cutting speeds.

The present study allows to represent the stochastic nature of the cutting tool wear and to produce updated reliability and RUL estimates at each inspection time. This estimate may then supply a more global industrial maintenance policy, providing additional relevant information about the upcoming degradation.

On an analytical point of view, [2] described the effects of adhesive, abrasive and other wear processes, and compared the contributions of several authors concerning the quantification of the tool wear rate and proposed a new mathematical model of flank wear. [3] proposed an analytical tool wear model including the different wear processes for ceramic inserts in hard turning. [4] has a more physical point of view on the

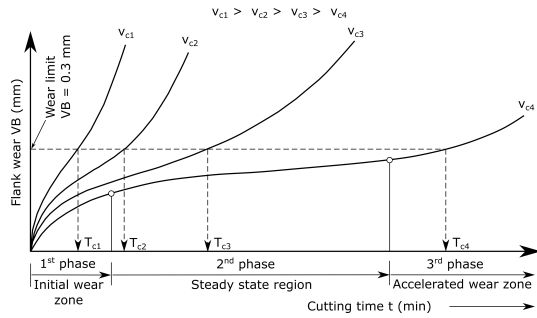


Fig. 2. Flank wear (adapted from [1]) – The evolution of Flank wear is usually divided into three phases: a first quick wear phase, followed by a longer steady-state region, and finally an accelerated wear after a given time. The VB end-of-life criterion displayed on this figure comes from the ISO 3685 standard. Regardless the cutting speed, this behaviour is seen, albeit steeper as the cutting speed increases.

wear mechanisms, and described the abrasive wear of flank face as a function of the removed material per unit of time, which also includes a tool life criterion proposition. Finally, [5] proposed a methodology for predicting the Probability Density Function (PDF) of degradation under aperiodic condition monitoring that is based on a gamma process and is close in its mathematical aspects to this study. In that study, Yan et al. used Maximum Likelihood Estimation, but this method does not allow for various cutting speed, as additional information is necessary to determine the variation of the shape parameter of the gamma process under those varying cutting conditions. This is why the present study considers the use of Taylor's Law as the corresponding additional information in the case of various cutting conditions.

The Cox PH model was first proposed by Cox [6], and was originally used for medical applications in survival analysis. Further developments later confirmed its validity as a statistical methodology applied to the general framework of machining and maintenance [7].

Tool wear modelling is a recurring subject in recent literature, because of the huge opportunity for cost reduction it represents [8]. The semi-continuous process that is machining calls for on-line Tool Condition Monitoring (TCM) [9]. Several approaches have been shown to be of use regarding TCM, such as [10], which recently presented a mathematical modelling of a linear control system based on a transfer function between the tool wear rate and cutting forces.

This work is organized as follows: first, the methodology and results of the experimental campaign that gathered the data necessary for the stochastic model fitting are presented. Second, the stochastic model highlights, fitting, and applications are presented for constant cutting speeds. It is then shown how it can be extended to varying cutting speeds. The Cox Proportional Hazards (CPH) model is then defined and presented along with its results and robustness. Finally, we conclude on the highlight of both methodologies and the perspectives of future developments.

II. MEASUREMENTS

For the experimental phase, several machining operations were conducted on a CNC SOMAB "UNIMAB 450" lathe

using a cutting tool DCLNL 2525M 12 with a tungsten carbide insert coated CNMG 1204 085B OR SAFETY SA brand. The first data set was obtained using the same cutting conditions as in [11].

A batch of 30 identical inserts was used to assess the evolution of the flank wear for determined machining conditions. The cutting conditions are: a cutting speed $v_c = 340$ m/min, a feed rate $f = 0.18$ mm/rev, a depth of cut $a_p = 1.5$ mm. The workpiece is a cylinder made of gray cast iron with lamellar graphite FGL250 that presents a hardness of 322 Hv.

Obviously, quantitatively different results are to be expected in the case of machining materials that are different in nature or in hardness, or with different cutting tools. The present study addresses only the mentioned tools, material and cutting parameters, but the methodology that is described is expected to be usable in other materials, tools and properties. These considerations however fall beyond the scope of this study.

The machining operations were designed to ensure a contact time of exactly one minute between the cutting tool and the workpiece, which correspond to the duration of one pass. The dimensions are a length of 220 mm and a diameter of 190 mm. Each minute (after one pass), the machining is stopped and the flank wear level on the insert is measured using an optical microscope LEICA MS5 type that allows a magnification of 4 times the actual size and presents a measurement error of 2%.

A second batch of insert was used to assess the influence of the cutting speed. Three different cutting speeds v_c that are 340, 390 and 440 m/min were considered, the remaining machining parameters were unchanged. The figures 3, 4 represent the evolution of the measured flank wear for the first and second data sets respectively. For the first data set, the cutting insert 18 presented another failure mode than the soft flank wear and was rejected for this study.

For the lifetime computation, we refer to the standard ISO 3685 that considers a maximum flank wear level of 0.3 mm before the failure of the cutting. Actually this criterion is not physically related to a failure of the cutting tool in the common sense, but rather relates to the quality of the workpiece (i.e. the higher the wear of the cutting tool insert, the higher the probability to produce workpieces that are beyond the tolerances).

III. STOCHASTIC FLANK WEAR MODEL

As illustrated in figure 3, the flank wear evolution is characterized by a stochastic behaviour despite the fact that identical inserts as well as machining conditions were used. In consequence, the hitting times of the failure threshold are randomly distributed which impacts the decision of replacing the cutting tool insert. In order to take into account these uncertainties, there is a need to consider a stochastic model for the flank wear evolution.

A. The gamma process

The gamma process has been widely used in various research topics involving monotonous degradation processes, reliability analyses and maintenance optimization of aging

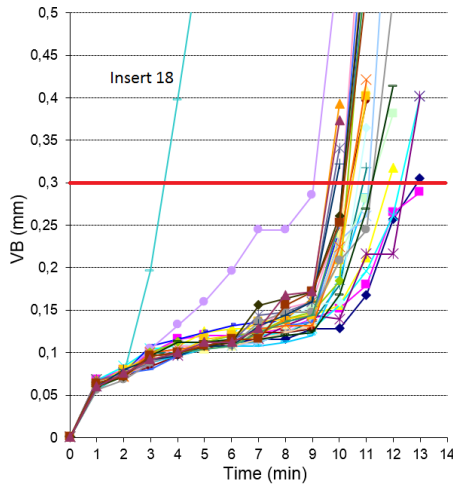


Fig. 3. First data set, gathered by repeating the experiment at a constant cutting speed (340 m/min)

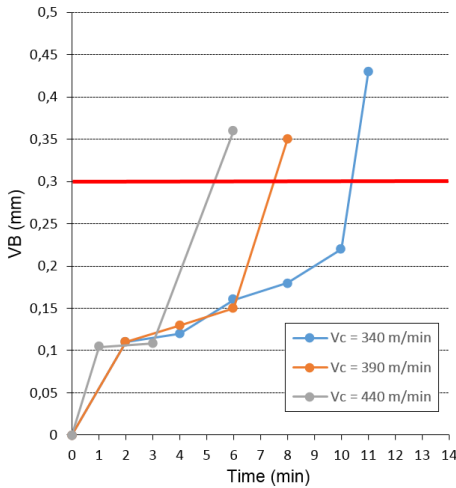


Fig. 4. Second data set, gathered by varying the cutting speed

components [12]. Due to its monotonic behaviour, this stochastic process is particularly well suited to model the evolution of some physical degradation such as wear, crack growth, corrosion, fatigue, degrading health index, creep that can only increase in proportion with the elapsed time or the number of usages. According to [12] the definition of the gamma process is the following. Let $Z(t)$ denotes the evolution of a degradation process with respect to time and $Z(t_0)$ is the initial degradation at $t = 0$:

$$Z(t) = Z(t_0) + G(t|m(t), \lambda) \quad (1)$$

G is the gamma process with the following characteristics:

- G has independent increments;
- $G(0) = 0$ with probability one;
- G is a stochastic continuous process, and for $\forall t_2, t_1$ ($t_2 \geq t_1 \geq 0$), the intensity of the jump $G(t_2) - G(t_1)$ follows a gamma distribution with shape parameter $m(t_2) - m(t_1)$ and scale parameter λ with density function:

$$f(x) = \frac{\lambda^{m(t_2) - m(t_1)}}{\Gamma(m(t_2) - m(t_1))} x^{m(t_2) - m(t_1) - 1} \exp(-\lambda x) \quad (2)$$

Γ being the gamma function defined as:

$$\Gamma(m(t_2) - m(t_1)) = \int_0^{+\infty} x^{m(t_2) - m(t_1) - 1} \exp(-\lambda x) \quad (3)$$

Since the domain of the gamma function is \mathbb{R}^+ , this stochastic process can only produce positive increments. The mathematical expectation and variance of the gamma process at a given time t are:

$$E(G(t|m(t), \lambda)) = \frac{m(t)}{\lambda} \quad (4)$$

$$V(G(t|m(t), \lambda)) = \frac{m(t)}{\lambda^2} \quad (5)$$

When the shape parameter is a linear function of time $m(t) = \alpha t$, the gamma process is said to be stationary; i.e. the increments are identically distributed. The case of non-stationary gamma process is thoroughly discussed in [12]. On figure 3, the typical evolution of the flank wear looks like a "S" shape function with an inflection point. While it is possible to consider a complex function for the shape parameter $m(t)$ for this study (see [13] for an example on the deterioration of choke valves), it was decided to consider a piecewise linear gamma process instead. The amount of available data being significant, a piecewise gamma process allows to improve the accuracy of the model since it is able to catch the transient effects observed at each inspection time.

A piecewise linear gamma process consists in fitting a stationary gamma process for each inspection interval $\Delta t_i = t_i - t_{i-1}$ on several degradation jumps Δz_{ij} observed on several j identical items. For each time interval Δt_i a linear gamma process $G_i(\alpha_i, \lambda_i)$ is obtained as illustrated on figure 5. Considering the wear of the cutting tools, this approach allows to capture the non-stationary effect that occurs at the very beginning (phase I) and just before the end of the tool life (i.e. phase I and phase III on figure 2).

The analytical mean and variance for the piecewise gamma process are:

$$E(G(t)) = \sum_{i=1}^n E(G_i(\Delta t_i | \alpha_i, \lambda_i)) = \sum_{i=1}^n \frac{\alpha_i}{\lambda_i} \Delta t_i \quad (6)$$

$$V(G(t)) = \sum_{i=1}^n V(G_i(\Delta t_i | \alpha_i, \lambda_i)) = \sum_{i=1}^n \frac{\alpha_i}{\lambda_i^2} \Delta t_i \quad (7)$$

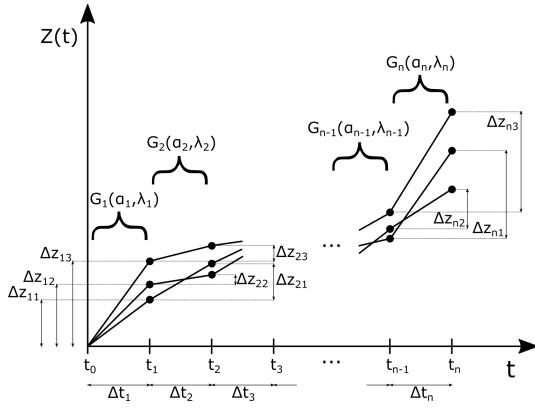


Fig. 5. Illustration of the piecewise gamma process fitted on data.

B. Fitting the gamma process

As presented in [12], there are four approaches to estimate the parameter of a gamma process to fit experimental data: the method of maximum likelihood, the method of moments, the method of Bayesian statistics and the method of expert judgement. In this study we used the method of maximum likelihood. The maximum likelihood estimation is a method of estimating the parameters of a statistical model given observations ; it consists in finding the set of parameters p that maximizes the likelihood of producing the observations again given the parameters. The likelihood function is:

$$L(p) = \prod_{i=1}^n f(t_i|p) \quad (8)$$

In the case of the stationary gamma process, the observations are different jump intensities Δz_i at given time intervals Δ_i .

$$L(\alpha, \lambda) = \prod_{i=1}^n \frac{\lambda^{\alpha \Delta t_i}}{\Gamma(\alpha \Delta t_i)} \Delta z_i^{\alpha \Delta t_i - 1} \exp(-\lambda \Delta z_i) \quad (9)$$

Using the log-likelihood $l(\alpha, \lambda) = \ln L(\alpha, \lambda)$, it becomes:

$$l(\alpha, \lambda) = \sum_{i=1}^n \ln \left(\frac{\lambda^{\alpha \Delta t_i}}{\Gamma(\alpha \Delta t_i)} \Delta z_i^{\alpha \Delta t_i - 1} \exp(-\lambda \Delta z_i) \right) \quad (10)$$

$$l(\alpha, \lambda) = \sum_{i=1}^n \ln(\alpha \Delta t_i \ln \lambda - \ln(\Gamma(\alpha \Delta t_i))) + (\alpha \Delta t_i - 1) \ln(\Delta z_i) - \lambda \Delta z_i \quad (11)$$

Taking the partial derivative of equation 11 gives the maximum likelihood estimator $(\hat{\alpha}, \hat{\lambda})$. This gives equations:

$$\frac{\delta l(\alpha, \lambda)}{\delta \alpha} = \sum_{i=1}^n (\Delta t_i \ln \lambda - \Delta t_i \frac{\Gamma'(\alpha \Delta t_i)}{\Gamma(\alpha \Delta t_i)} + \Delta t_i \ln(\Delta z_i)) = 0 \quad (12)$$

TABLE I. PARAMETERS OF THE PIECEWISE LINEAR GAMMA PROCESS. THE LAST COLUMN IS THE EXPECTED DEGRADATION RATE.

$G_i(\alpha_i, \lambda_i)$	α_i	λ_i	α_i / λ_i
G_1	349.65	5583.64	0.0626
G_2	7.12	580.07	0.0123
G_3	7.62	378.36	0.0201
G_4	2.54	291.01	0.0088
G_5	2.46	307.40	0.0080
G_6	2.14	382.25	0.0056
G_7	1.19	91.27	0.0130
G_8	1.48	138.41	0.0107
G_9	2.15	210.99	0.0102
G_{10}	1.11	12.46	0.0891
G_{11}	1.59	8.49	0.1875
G_{12}	0.99	6.69	0.1487
G_{13}	1.84	18.59	0.0992

$$\frac{\delta l(\alpha, \lambda)}{\delta \lambda} = \sum_{i=1}^n \left(\frac{\alpha \Delta t_i}{\lambda} - \Delta z_i \right) = 0 \quad (13)$$

The estimator $\hat{\lambda}$ can be directly obtained from equation 13. The estimator $\hat{\alpha}$ is obtained by numerically searching for the root of equation 12 using the Newton-Raphson method for instance.

C. Application on the first data set

Since the measurement of the flank wear evolution was recorded every minute, the interval time is one minute for each of the linear gamma process G_i . For each interval time, 29 degradation jumps were observed. The gamma piecewise linear process was fitted accordingly considering the couples $(\Delta t_i, \Delta z_{i,j})$, i being the index of the current time interval ($i = 1, 2, \dots, 13$) and j the index corresponding to the current cutting tool ($j = 1, 2, \dots, 29$).

Table I gives the values of the estimated parameters for each linear gamma process. The third column indicates the average degradation rate for a given linear gamma process (i.e. for the corresponding interval time). As expected, the degradation rate is higher for the first minute (phase I) and then decreases to present a monotonic behaviour with a flank wear rate of around 0.01 mm/min (phase II) before suddenly increasing after 10 minutes (phase III).

It should be noted that for the last minutes (i.e. at times 12 and 13 minutes), only a few cutting tools were at disposal for measurements, they are the ones that were still below the failure threshold. In consequence, the adjustment of the gamma process did not consider the cutting tools that had already passed over the threshold ; leading to an inaccurate analysis since it tends to underestimate the degradation rate. For this reason, only the first eleven minutes are considered in the following analyses. If a simulated flank wear path lasts longer than 11 minutes, the linear gamma process G_{11} is used for simulation of the next degradation jumps.

Fig. 6 presents a comparison of the data obtained from simulations using the piecewise gamma process and the experimental flank wear evolutions that were recorded. The mean flank wear is represented in green for both cases.

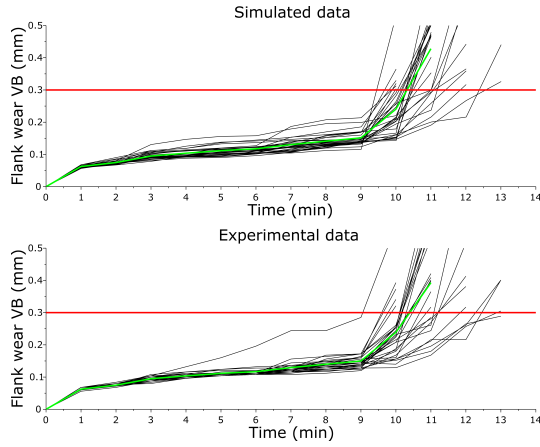


Fig. 6. Comparison of the simulated data obtained with the piecewise gamma process and the experimental data.

IV. EXTENSION TO CONSIDER VARIOUS CUTTING SPEED

A. The Taylor law

The cutting tool life assessment or various cutting conditions (e.g. mainly the cutting speed) is obtained using the Taylor's tool life equation [14]:

$$v_c T_c^n = C \quad (14)$$

T_c being the cutting tool life (min), v_c the cutting speed (m/min), n the Taylor exponent and C a constant parameter for a given cutting tool and workpiece set. Using the logarithmic operator, the Taylor law is fitted by regression on experimental tool life data obtained with different cutting speeds. The definition of tool life tests to obtain the value of the parameters n and C are specified in the standard ISO 3685 (1993) for turning operations and ISO 8688 (1989) for milling operations. Considering the reference case $v_{ref} = 340$ m/min that leads to an average tool life duration $T_{ref} = 10.6$ min (i.e. the mean failure time obtained for the first data set at constant cutting speed). The tool life T_c for any cutting speed v_c is:

$$T_c = \left(\frac{C}{v_c} \right)^{(1/n)} \quad (15)$$

Consequently, considering various cutting speed conditions in the piecewise gamma process is achieved by introducing the relative factor T_{ref}/T_c in the shape parameter so that the degradation rate for each observation is modified by the cutting speed (i.e. on table I the shape parameters α_i are multiplied by T_{ref}/T_c ; the higher the cutting speed v_c the shorter the cutting tool life T_c and so the higher the degradation rate). It was decided to not modify the first degradation jump α_1 since the first measurement is conditioned by the initial quality of the cutting tool (i.e. the model supposes that the degradation at initial time is null but this might not be the case in practice) and also because the degradation rate is significant.

B. Applications to the second dataset

Using the Taylor law on the second data set, the parameters of the Taylor law are $C = 840$ and $n = 0.383$. The figure 7 shows some simulations of the piecewise gamma process for different cutting speed values. On figure 7 The comparison

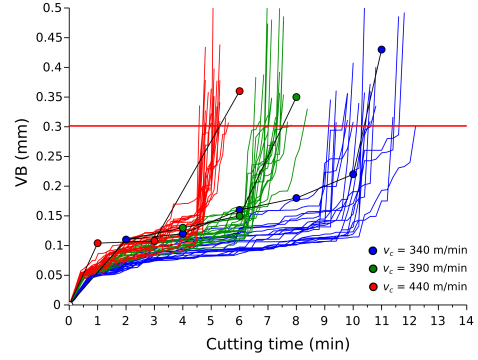


Fig. 7. Simulated data for different cutting speeds using the piecewise gamma process.

with experimental data shows that the model successfully reproduces the behaviour at different cutting speeds.

V. CUTTING TOOL MONITORING AND RUL

Given the fitted degradation model, the next step is to assess the remaining useful life of a cutting tool during machining operations for given cutting conditions. For the sake of illustration, the methodology is applied on one of the cutting tools that were used at the experimental step (e.g. the cutting tool number 4). The flank wear VB of this cutting tool is monitored every minute. At each inspection, the piecewise stochastic gamma process is used to simulate degradation trajectories by Monte Carlo simulations (considering 500 simulations for each inspection). The figure 8 shows an illustration of 5 simulated degradation paths at inspection times 5 min and 10 min. With these simulations, the hitting times of the threshold $VB = 0.3$ mm are computed. A log-normal distribution is then fitted on the simulated failure times to obtain a parametric representation of the reliability (see figure 9). The reliability law is fitted using the regression method with the rank adjust non-parametric estimator [15]. This reliability is used to compute the RUL distribution and the mean residual life (MRL). The RUL is obtained for each inspection considering the power density function of the hitting times and the MRL is used using the following equation:

$$MRL(t_j) = E(T_c - t | T_c > t, VB(t_j) < 0.3 \text{ mm}) \quad (16)$$

$$= \int_{t_j}^{\infty} R(u | VB(t_j)) du, \quad (17)$$

with $R(u | VB(t_j))$ the conditional degradation-based reliability of the cutting tool that has survived until inspection time t_j and given the measured flank wear $VB(t_j)$. The figure 10 shows the RUL after each minute and a comparison between the mean residual life and the true residual life.

Concerning the cutting tool number 4, it is observed that the MRL is underestimated during the first inspection times. It is necessary to wait until 10 min to observe a shift in the MRL that match the true residual life. This is due to the burst in period (stage 3 accelerated wear) in the VB evolution that

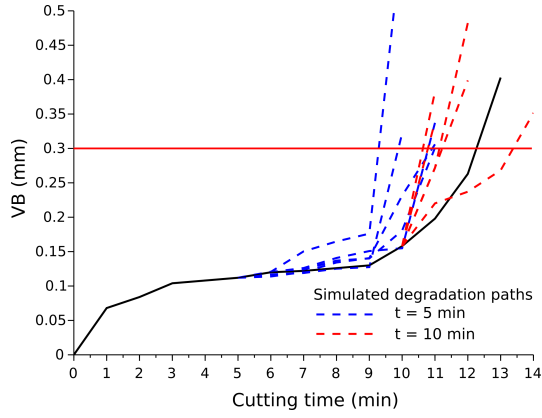


Fig. 8. Illustration for the simulated degradation trajectories using the piecewise gamma process at inspection times 5 min and 10 min on cutting tool number 4.

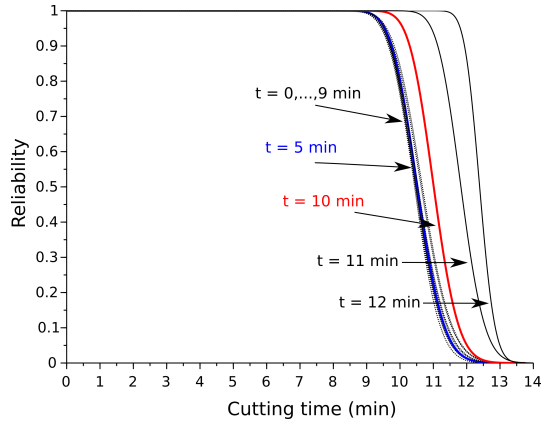


Fig. 9. Illustration of the updating of the reliability model at each inspection time given the last VB measurement.

is hard to predict in practice. Such methodology can also be performed considering a different cutting speed.

VI. COX MODEL

A. Model description

Cox's Proportional Hazards Model is the most important model in survival analysis [16]. Its assumption is that the failure rate of a system depends both on a hazard baseline,

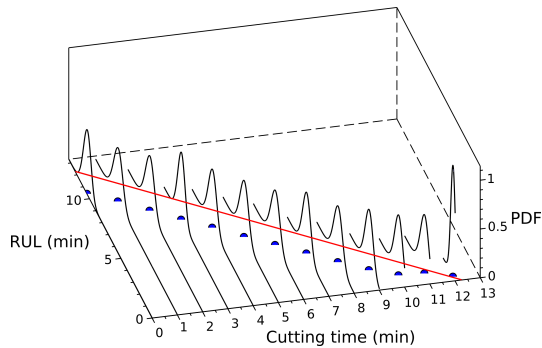


Fig. 10. RUL distribution at each inspection time and comparison of the mean residual life (blue dot) with the true residual life (continuous red line).

which may be parametric, and on covariates that describe the characteristics of the system and represent the way. As the name of the model states, one of the main hypotheses is the proportionality between the observed hazard rate and the baseline hazard rate, which is usually estimated at the mean value of the covariates. The weighing with respect to the covariate is expressed through an exponential function of a linear combination of the covariate.

In a general way, the Cox PH Model can be expressed as follows:

$$h(t) = h_0(t) \cdot \exp\left(\sum_{i=1}^p \beta_i \alpha_i\right) \quad (18)$$

with $h(t)$ being the hazard function or failure rate, $h_0(t)$ being the baseline hazard function, β_i the weighing coefficients for the p covariates and α_i the covariates.

The estimation of the weighing coefficient β and the baseline hazard curve are made through the Maximum Likelihood Method, through the computation of partial likelihoods for β and methods analogous to the Kaplan-Meier estimators for the baseline hazard curve $h_0(t)$. In the experimental phase described hereunder, the estimations are performed with the help of R's `flexsurv` package [17], which also ensure proper convergence of the results. The baseline hazard curve is fitted to a Weibull distribution, which is commonly used to describe the general behaviour of mechanical systems, as it adequately can match the typical different hazard functions of mechanical systems.

In the framework of this study, the only explanatory variable that is chosen to describe the characteristics of the system is the cutting speed. Sections II and IV showed the influence of this cutting parameter on the life expectancy of cutting inserts. Equation (18) is then written as:

$$h(t) = h_0(t) \cdot \exp(\beta v_c) \quad (19)$$

From the hazard function given by the Cox PH model, it is then possible to compute the Mean Up Time (MUT) of the cutting inserts, which are our prediction for the cutting insert lifetime:

$$MUT = \int_0^{+\infty} R(t) dt \quad (20)$$

with $R(t)$ being the reliability in the case of the 2-parameter Weibull distribution:

$$R(t) = \exp\left(-\left(\frac{t}{\eta}\right)^{\beta_w}\right) \quad (21)$$

η being the scale parameter and β_w the shape parameter of the Weibull distribution.

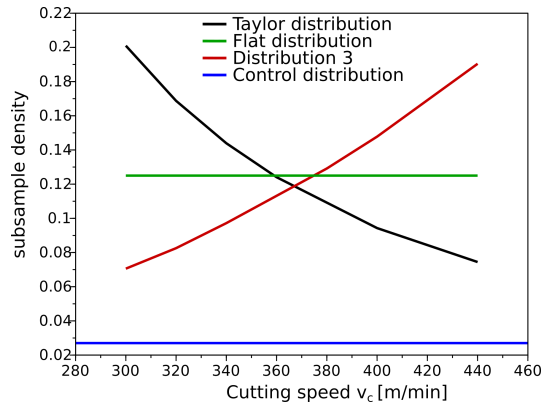


Fig. 11. Because of the non-linear relationship between failure time and cutting speed in the sample, which is due to Taylor's law linking the failure time to the cutting speed (see eq. (15)), in order to obtain a flat distribution in the failure time domain, one needs to distribute the sample data along a negative exponential with respect to v_c . This is done in the first sample, denoted "Taylor distribution" in this figure. This distribution is to be assessed by comparison with a flat distribution of the sample with respect to the cutting speed. As further comparison, a third distribution is chosen, that amplifies the unbalance toward high cutting speed. The control set distribution is, as always, uniform over the sample.

B. Numerical experiment description

The numerical experiment presented in this section follows previous work [18]. This paper introduces to the Cox PH model applied to cutting tools and in particular a sensitivity analysis to the data sample distribution with respect to the covariate value, and sensitivity to noise in the measurements. In this numerical experiment, cutting inserts lifetimes are generated through the method shown in sections III through V. The numerical experiment is divided into the following phases:

- 1) The Cox PH model will be built from data samples following specific distributions with respect to the covariate, as [18] showed a strong sensitivity of the model to this statistical aspect of the data sample. The lifetime generation method that has been presented allows to build four sets of data following specific distributions along the v_c learning interval. All numerical integrations are performed through the trapezoidal rule. A single set of 1850 generated lifetime, which is uniformly spread over the control interval, is used to assess the precision of the Cox PH models and determine the quality of the tool lifetime prediction. Previous work [18] showed important discrepancies toward the low cutting speeds. It was assumed that the non-linear relationship between the failure time and the cutting speed lead to few data points at high failure times. In order to assess this hypothesis, three distributions are tried and compared, as shows figure 11.
- 2) A Cox PH model is built from data showing the best results among the previous phase of the experiment. This time, the model is built from noised data (i.e. the observed value is added to a noise component that follows a certain distribution), with the noise following two possible distributions:
 - Continuous uniform distribution depending

- Normal distribution $\mathcal{N}\left(0, \frac{(\mu(T))^2}{400}\right)$, $\mu(T)$ being the mean observed failure time value given the covariate value.

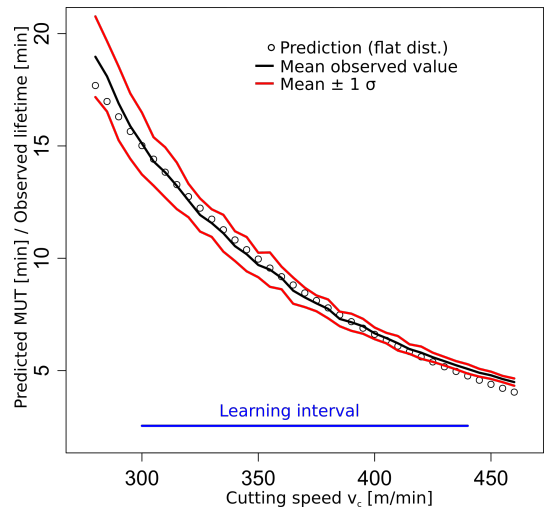


Fig. 12. The Cox PH model prediction for the lifetime of cutting inserts (circles), built on 400 data points evenly distributed with respect to the cutting speed (see figure 11) provides accurate predictions of the cutting tools within the learning interval. The control set is made of 1850 additional lifetimes. Discrepancies appear at both ends of the curve, while the prediction is very good inside the learning interval. Because of the narrowing distribution of the lifetime at higher cutting speeds, becoming quasi-deterministic, the prediction is off the 1σ interval at the higher end of cutting speeds.

- on the mean of observed value $\mathcal{U}(-0.1\mu(T), +0.1\mu(T))$, $\mu(T)$ being the mean observed failure time value given the covariate value.
- Normal distribution $\mathcal{N}\left(0, \frac{(\mu(T))^2}{400}\right)$, $\mu(T)$ being the mean observed failure time value given the covariate value.

The objective of the first experiment is to confirm or infirm the influence of the learning data sample for building a Cox PH model for cutting inserts. The second experiment aims at assessing the robustness of the Cox PH Model. A noise component is added to the observed value. In a first case, it follows a uniform distribution and generates a noise of which the width is 10 % of the mean observed value. In a second case, it follows a normal distribution and generates a noise of which the standard deviation is about 5 % of the mean observed value.

C. Results and discussion

The results of the first phase of the numerical experiment are displayed in figure 12. The output predictions of the three distributions proposed and illustrated in figure 11 are not graphically distinguishable one from another (values vary slightly).

The methodology proposed for the first experiment does not allow to witness any substantial change, regardless the distribution of the sample data. The hypothesis linking the discrepancies between the prediction and the learning set distribution appears to be false and will call for further mathematical developments and experimentations in order to determine the exact origin of the phenomenon.

Because no distribution proved better than others, the flat distribution was used during the second phase of the

Experiment	C	n	R^2
Reference value	840	0.383	-
Phase 1 - Flat dist.	749	0.335	0.996
Phase 1 - Taylor dist.	737	0.329	0.996
Phase 1 - 3rd dist.	753	0.338	0.996
Phase 2 - Uniform noise	751	0.337	0.996
Phase 2 - Normal noise	749	0.336	0.996

Fig. 13. The estimates of Taylor's constant C and Taylor's exponent n allow another point of view on the numerical experiment results. It is to be noted that the parameter estimate is worse when using a sample distribution following the Taylor law. Other estimates, on the other hand, are quite similar one to another, but still far from the reference value. Moreover, the noise on measurements does not seem to influence the quality of the estimate of the Taylor parameters.

experiment. The results of the second phase are not different than the ones of the first phase. Figure 12 shows the prediction that is achieved. No graphically distinguishable discrepancy can be noted when the data is affected by the noise of either proposed distribution. This shows how robust the Cox PH model is with respect to noise. These results are to be expected with zero-centred noise distributions. Further experiments on this subject should focus on otherwise-centred distributions.

Moreover, for all generated predictions, a least squares log-log regression allows to generate an estimate of the Taylor coefficient n and the Taylor constant parameter C . Figure 13 compares the estimated values with the reference values used for the data generation, as explained in section IV.

VII. CONCLUSION

This work presents a combination of two approaches to the cutting tool end of life prognosis in turning process. The first part features the experimental methodology and results that allowed to first build the stochastic flank wear model. In this part, the use and pertinence of the use of a gamma process for modelling the cutting tool flank wear evolution is discussed. It also features the mathematical developments that allow the fitting of the piecewise gamma process to the experimental data through the use of a maximum likelihood estimator. A comparison with the experimental data shows the quality of the proposed model for the evolution of tools flank wear. It is also shown how this gamma process may be extended to various cutting speed through the Taylor's law.

The second part of this work uses the gamma process in order to generate degradation trajectories and thus, large tool lifetime samples. In particular, it was shown how, knowing the degradation trajectory up to a given point, the gamma process model that was developed allows the projection of future degradation trajectories at the given cutting speed, and thus a MRL estimate.

Finally, it is shown how a Cox PH model may be built from the data generated by the gamma process in order to predict the MUT of cutting tools at various cutting speeds. Numerical experiments are performed to evaluate the interest of varying the distribution, with respect to the cutting speed, of the learning set in order to favour accuracy of the Cox PH model. A second phase of numerical experiments attempts to determine the impact of noise, which is applied to the learning set of the Cox model, on the quality of the prediction. In the

case studied here, it was found no impact from the learning set distribution nor from zero-centred noise distributions on the Cox prediction accuracy. Further developments on this subject may focus on the poor reproduction of the chosen Taylor parameter by the Cox PH model, and extension to cutting speeds varying over time. Moreover, in order to provide more specific decision aid concerning the tool replacement, cost models are to be developed and used in synergy with the presented MRL and degradation prognosis tools.

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