Higher Spin Extensions of the BMS group

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A.C., D. Francia, C. Heissenberg, 1703.01351, 1712.09591, 1907.05187, 2011.04420

Solvay Workshop on *'The asymptotic structure of spacetime', 23/11/2020*

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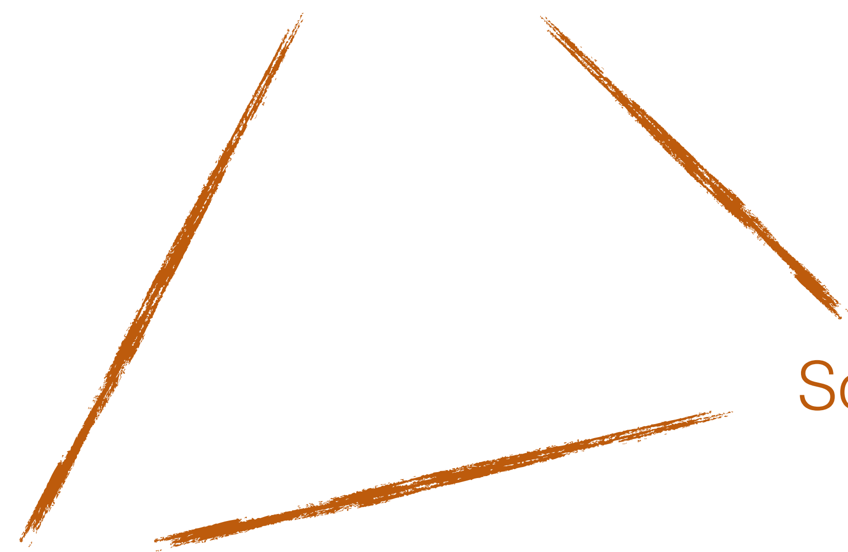
The infrared triangle (Δ)

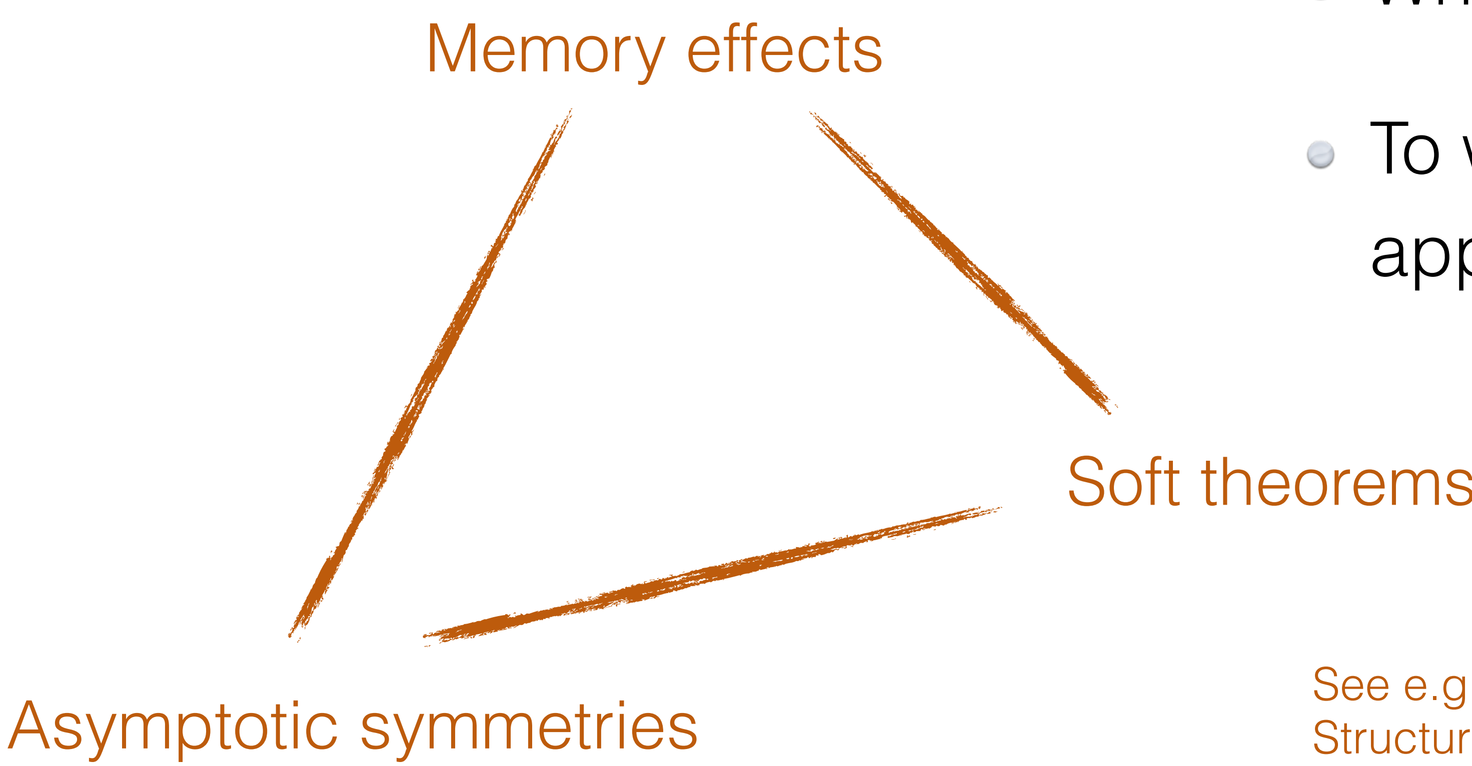


- What are the key ingredients?
- To what theories is this scheme applicable?

See e.g. A. Strominger, Lectures in the Infrared Structure of Gravity and Gauge Theory (2017)

- *Massless particles* must be involved
- Two parameters in the "universal" gauge sector: *the spacetime dimension D and the spin s*

The infrared triangle (Δ)



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See e.g. A. Strominger, Lectures in the Infrared Structure of Gravity and Gauge Theory (2017)

- First infrared triangle: ***D*=4** and ***s*=2** (2014)
- Extensions to $D=4$ and $s=0$ (2017), $s=1/2$, 1, 3/2 (2014)
- *What about higher D and higher s?*

Δ: the symmetry – soft theorem edge

Asymptotic symmetries ⟷ Soft theorems

- Observation: Weinberg's (leading) soft factorisation theorems apply to **any** *D* **and any** *s* — Weinberg (1964)
- Originally related to the Ward identities of the infinite-dimensional asymptotic symmetries of four-dimensional gravity (***BMS symmetry***)
- Problem 1: when $D > 4$ one can impose boundary conditions that kill supertranslations while still allowing for radiation
  - No "need" for BMS symmetry for $D > 4$ — Hollands, Ishibashi (2005); Tanabe, Kinoshita, Shiromizu (2011); Hollands, Ishibashi, Wald (2017)
- Problem 2: Weinberg's theorem ⇒ trivial S-matrix for $s > 2$ — Weinberg (1964)

Δ: the symmetry – soft theorem edge

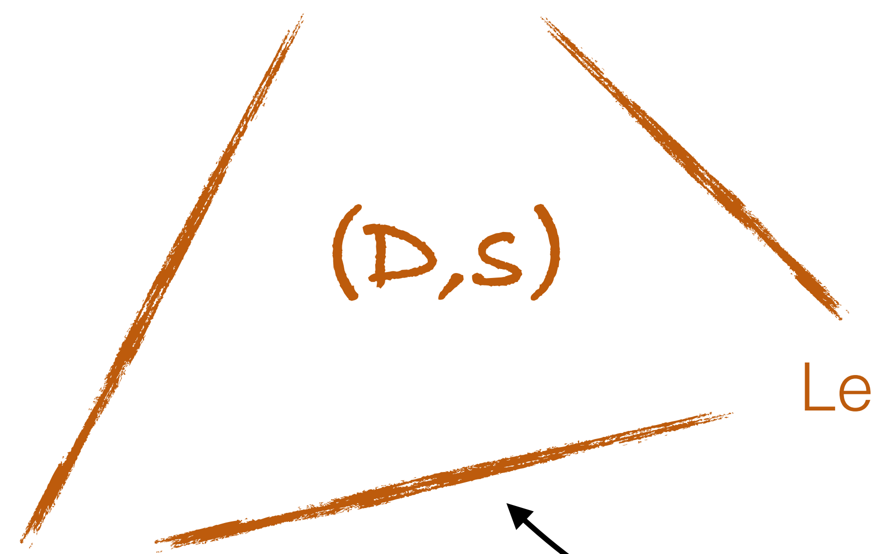
Still…

see also Aggarwal's talk

- New boundary conditions (for $s=1, 2$) ⇒ *supertranslations in $D>4$*
  - same falloffs as in $D=4$ — Kapec, Lysov, (Pasterski), Strominger (2014-2015)
  - "Lorenz" gauge in $D>4$ — Pate, Raclariu, Strominger (2017), AC, Francia, Heissenberg (2019)
- Generalisation of Bondi gauge to $s>2$ ⇒ *HS supertranslations in $D=4$*
  - HS supertranslation Ward identities ⇒ Weinberg's soft theorem — AC, Francia, Heissenberg (2017)
- Combining the previous observations ⇒ *HS supertranslations in $D>4$*
  - …and HS superrotations in $D>4$ — AC, Francia, Heissenberg (2020)

The “free" infrared triangle

Linear or ordinary memory effects

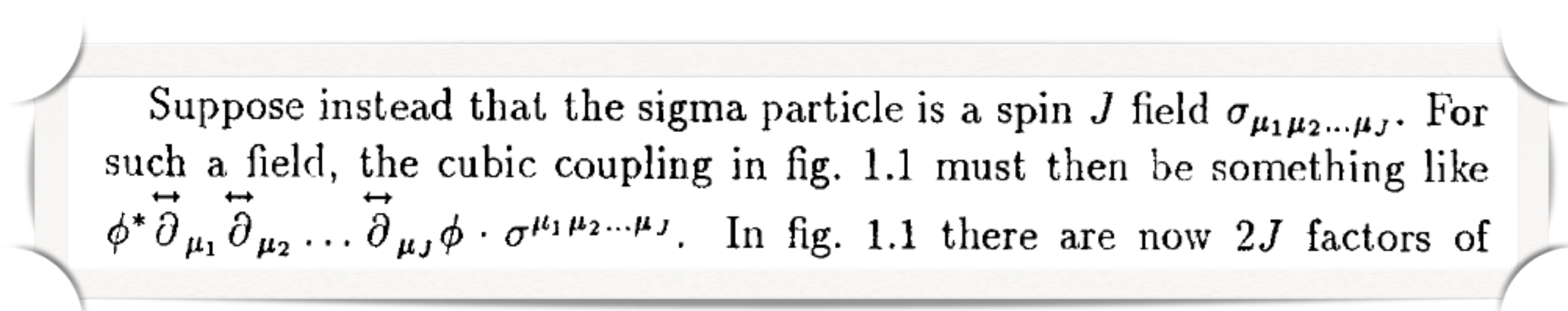


Leading soft theorems

Asymptotic symmetries
of the free theory

Valid in any (even) D and for any s!

(also requires mild assumptions
on matter coupling)

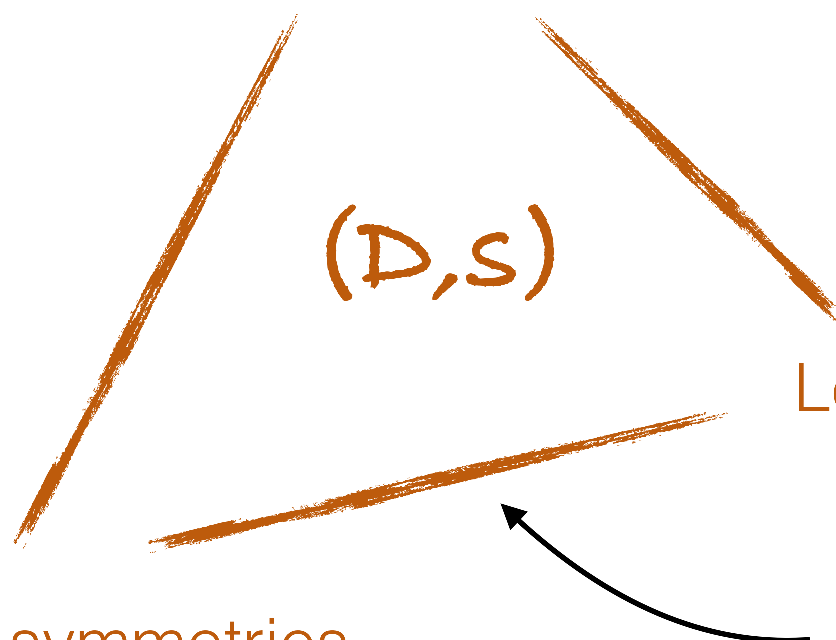


Suppose instead that the sigma particle is a spin J field $\sigma_{\mu_1\mu_2\ldots\mu_J}$. For such a field, the cubic coupling in fig. 1.1 must then be something like $\phi^* \overleftrightarrow{\partial}_{\mu_1} \overleftrightarrow{\partial}_{\mu_2} \ldots \overleftrightarrow{\partial}_{\mu_J} \phi \cdot \sigma^{\mu_1\mu_2\ldots\mu_J}$. In fig. 1.1 there are now $2J$ factors of

Credits: Green, Schwarz, Witten (1987)

The "free" infrared triangle

Linear or ordinary memory effects



Leading soft theorems

Asymptotic symmetries
of the free theory

Valid in any (even) D and for any s!

Why do you care about any *s*?

- Universality of the infrared triangle
- High-energy limit of string theory

see also the near-horizon analysis of Grumiller, Perez, Sheikh-Jabbari, Troncoso, Zwikel (2019)

Higher-spin asymptotic symmetries: the setup

- Action: $S = \int d^D x \, \varphi^{\mu_1 \cdots \mu_s} \left(\mathcal{F}_{\mu_1 \cdots \mu_s} - \frac{1}{2} \eta_{(\mu_1 \mu_2} \mathcal{F}_{\mu_3 \cdots \mu_s)\lambda}{}^{\lambda} \right)$ Fronsdal (1978)

$$\mathcal{F}_{\mu_1 \cdots \mu_s} = \Box \varphi_{\mu_1 \cdots \mu_s} - \partial_{(\mu_1} \partial \cdot \varphi_{\mu_2 \cdots \mu_s)} + \partial_{(\mu_1} \partial_{\mu_2} \varphi'_{\mu_3 \cdots \mu_{s-2})\lambda}{}^{\lambda}$$

- Gauge symmetry: $\delta \varphi_{\mu_1 \cdots \mu_s} = \partial_{(\mu_1} \epsilon_{\mu_2 \cdots \mu_s)}$ (with traceless ϵ)

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$$\mathcal{F}_{\mu_s} = \Box \varphi_{\mu_s} - \partial_\mu \partial \cdot \varphi_{\mu_{s-1}} + \partial_\mu \partial_\mu \varphi'_{\mu_{s-2}}$$

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- Retarded Bondi coordinates:

$$ds^2 = -du^2 - 2du dr + r^2 \gamma_{ij} \, dx^i dx^j$$

- *Bondi-like "gauge"* (or *part 1* of the boundary conditions)

$$\varphi_{r\mu_{s-1}} = 0 = \gamma^{ij} \varphi_{ij\mu_{s-2}}$$

AC, Francia, Heissenberg (2017 and 2020)

Boundary conds I: HS supertranslations

- Bondi-like gauge

AC, Francia, Heissenberg (2017 and 2020)

$$\varphi_{r\mu_{s-1}} = 0 = \gamma^{ij}\varphi_{ij\mu_{s-2}}$$

- Remaining field components? (part 2 of the boundary conditions)

	Falloffs	Asymptotic symmetries
D = 4	$\varphi_{u_{s-k}i_k} = \mathcal{O}(r^{k-1})$	*infinite dimensional*
Any D	$\varphi_{u_{s-k}i_k} = \mathcal{O}(r^{k+1-\frac{D}{2}})$	*only global Killing symmetries*
Any D	$\varphi_{u_{s-k}i_k} = \mathcal{O}(r^{k-1})$	*infinite dimensional*

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u-independent asymptotic symmetries

- *u*-independent residual symmetries of the Bondi-like gauge:

$$\epsilon^{u_{s-k-1}i_k} \propto r^{-k}\,\mathcal{D}^i \cdots \mathcal{D}^i T(\hat{\mathbf{x}}) + \cdots$$

(depend on an arbitrary function on the celestial sphere)

- Spin-3 example:

$$\epsilon^{uu} = T(\hat{\mathbf{x}})\,, \qquad \epsilon^{ui} = -\frac{1}{r}\,\partial^i T(\hat{\mathbf{x}})\,, \qquad \epsilon^{ij} = \frac{1}{2r^2}\left[\mathcal{D}^i\mathcal{D}^j - \frac{1}{D}\,\gamma^{ij}(\Delta-2)\right]T(\hat{\mathbf{x}})$$

- Compatible with $\varphi_{u_{s-k}i_k} = \mathcal{O}(r^{k-1})$ but not radiation falloffs!

- OK, you got infinite-dimensional symmetries… but *what is the interpretation of the terms "above radiation"*?

u-independent asymptotic symmetries

- Obs 1: on shell the overleading terms must be *pure gauge*

$$\varphi_{u_{s-k}i_k} = r^{k-1}\,\frac{k(D+k-5)!}{s(D+s-5)!}\,(\mathcal{D}\cdot)^{s-k}C_{i_k}{}^{(1-s)}(\hat{\mathbf{x}}) + \mathcal{O}\Big(r^{k+1-\frac{D}{2}}\Big)$$

(so they are perfectly fine at least for s=1)

- Obs 2: they do not contribute to surface charges

$$\begin{aligned}(-1)^{s-1}\mathcal{Q}_T(u) &= \lim_{r\to\infty} r^{D-3}\oint d\Omega_{D-2}\sum_{k=0}^{s-1}\frac{r^{-k}}{k!}\,T\left[(s-k-2)\,r\partial_r + (s-k-1)(D-k-2)\right](\mathcal{D}\cdot)^k\varphi_{u_{s-k}} \\ &= \lim_{r\to\infty} r^{D-4}\underbrace{\left(\sum_{k=1}^{s-1}\alpha_k\right)}_{0}\oint d\Omega_{D-2}\,T\,(\mathcal{D}\cdot)^s C^{(1-s)} + \mathcal{O}(r^{\frac{D-4}{2}})\,,\end{aligned}$$

u-independent asymptotic symmetries

- Obs 1: on shell the overleading terms must be *pure gauge*

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$$= \lim_{r\to\infty} r^{D-4}\underbrace{\left(\sum_{k=1}^{s-1}\alpha_k\right)}_{0}\oint d\Omega_{D-2}\,T\,(\mathcal{D}\cdot)^s C^{(1-s)} + \mathcal{O}(r^{\frac{D-4}{2}})\,,$$

Comments on surface charges

- The overleading terms do not contribute, but a divergent contribution from radiation is still present!

- A "prescription" curing this problem (and giving the "correct" Ward identities): AC, Francia, Heissenberg (2017 and 2020)

  - Assume that for $u < u_0$ the fields are stationary
  - Compute the (finite!) charge for $u < u_0$
  - Define $Q_T(u)$ as the evolution under the eom of $Q_T(-\infty)$

- Final result:

$$\mathcal{Q}_T(u) \propto \oint d\Omega_{D-2}\, T(\hat{\mathbf{x}})\, \mathcal{U}^{(0)}(u,\hat{\mathbf{x}})$$

(with $\varphi_{u\cdots u} = r^{3-D}\mathcal{U}^{(0)}(u,\hat{\mathbf{x}}) + \cdots$)

Recovering Weinberg's theorem

- "Standard" techniques to recover Weinberg's theorem apply
  - rewriting of the charge: $\mathcal{Q}_T\big|_{\mathscr{I}^+_-} = \mathcal{Q}_T\big|_{\mathscr{I}^+_+} - \int_{-\infty}^{+\infty} \frac{d\mathcal{Q}_T(u)}{du}\, du$,
  - $\mathcal{Q}_T\big|_{\mathscr{I}^+_-} = (-1)^s\,(D+s-4)\int_{-\infty}^{+\infty} du \oint d\Omega_{D-2}\, T(\hat{\mathbf{x}})\,\partial_u\,\mathcal{U}^{(0)}(u,\hat{\mathbf{x}})$,
  - eom: $\partial_u^{\frac{D-4}{2}}\mathcal{U}^{(0)} = \frac{\mathscr{D}\,(\mathcal{D}\cdot)^3\,C^{\left(\frac{D-8}{2}\right)}}{(D-1)(D-2)(D-3)}$
- *The charge is rewritten in terms of radiation data!*
- <u>Obs 3</u>: Weinberg's theorem follows by substitution in the Ward identity

$$\langle\text{out}|\left(\mathcal{Q}_{\mathscr{I}^+_-}S - S\mathcal{Q}_{\mathscr{I}^-_+}\right)|\text{in}\rangle = \sum_\ell g_\ell^{(3)} E_\ell^2\, T(\hat{\mathbf{x}}_\ell)\langle\text{out}|S|\text{in}\rangle$$

Awery, Schwab (2015)

Higher-spin superrotations

- Back to the residual symmetries of the Bondi-like gauge (spin 3)

$$\epsilon_{rr} = f\,,$$

$$\epsilon_{ri} = r^2 v_i + \frac{r}{2}\,\partial_i f\,,$$

$$\epsilon_{ij} = r^4 K_{ij} + r^3 \left(\mathcal{D}_{(i}v_{j)} - \frac{2}{D-1}\,\gamma_{ij}\mathcal{D}\cdot v\right) + \frac{r^2}{2}\left(\mathcal{D}_i\mathcal{D}_j - \frac{1}{D}\,\gamma_{ij}\,(\Delta-2)\right) f$$

with

$$K_{ij} = K_{ij}(\hat{\mathbf{x}})\,,$$

$$v_i = \rho_i(\hat{\mathbf{x}}) - \frac{u}{D}\,\mathcal{D}\cdot K_i(\hat{\mathbf{x}})\,,$$

$$f = T(\hat{\mathbf{x}}) - \frac{2u}{D-1}\,\mathcal{D}\cdot\rho(\hat{\mathbf{x}}) + \frac{u^2}{D(D-1)}\,\mathcal{D}\cdot\mathcal{D}\cdot K(\hat{\mathbf{x}})\,.$$

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\end{aligned}
$$

- Induced variations of non-vanishing field components

$$
\begin{aligned}
\delta\varphi_{ijk} = r^4 &\left\{\mathcal{D}_{(i}K_{jk)} - \frac{2}{D}\,\gamma_{(ij}D\cdot K_{k)}\right\} \\
&+ r^3\left\{\mathcal{D}_{(i}\mathcal{D}_j\rho_{k)} - \frac{2}{D}\,\gamma_{(ij}\left[(\Delta + D - 3)\,\rho_{k)} + 2\,\mathcal{D}_{k)}D\cdot\rho\right]\right\} \\
&+ \frac{r^2}{2}\left\{\mathcal{D}_{(i}\mathcal{D}_j\mathcal{D}_{k)}T - \frac{2}{D}\,\gamma_{(ij}\mathcal{D}_{k)}\left(3\,\Delta + 2(D-3)\right)T\right\}
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$$

Higher-spin superrotations

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- Induced variations of non-vanishing field components

$$\delta\varphi_{ijk} = r^4\left\{\mathcal{D}_{(i}K_{jk)} - \frac{2}{D}\,\gamma_{(ij}D\cdot K_{k)}\right\} = 0$$
$$+\, r^3\left\{\mathcal{D}_{(i}\mathcal{D}_j\rho_{k)} - \frac{2}{D}\,\gamma_{(ij}\left[(\Delta + D - 3)\,\rho_{k)} + 2\,\mathcal{D}_{k)}D\cdot\rho\right]\right\} = 0$$
$$+\, \frac{r^2}{2}\left\{\mathcal{D}_{(i}\mathcal{D}_j\mathcal{D}_{k)}T - \frac{2}{D}\,\gamma_{(ij}\mathcal{D}_{k)}\,(3\,\Delta + 2(D-3))\,T\right\} = 0$$

only global Killing symmetries

Higher-spin superrotations

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- Induced variations of non-vanishing field components

$$\varphi_{ijk} = \mathcal{O}(r^2)$$

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\end{aligned}
$$

supertranslations
+ Lorentz (if D>4)

Higher-spin superrotations

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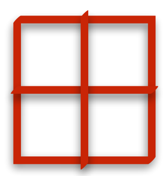
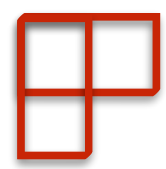
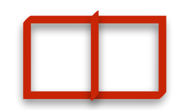
supertranslations
+ superrotations

Boundary conds II: higher-spin superrotations

- Summary: $\varphi_{u_{s-k}i_k} = \mathcal{O}(r^{s+k-2}) \;\Rightarrow$ HS superrotations

- Interpretation?
  - s = 2 $\quad \delta h_{ij} = r^2 \left(\mathcal{D}_{(i}v_{j)} - \frac{2}{D-2}\gamma_{ij}\mathcal{D}\cdot v \right) + \mathcal{O}(r)$ — Campiglia, Laddha (2014)
  - s = 3 $\quad K_{ij} \sim$  $\quad \rho_i \sim$  $\quad T \sim$  — Fradkin, Vasiliev (1986); Eastwood (2002)

- Do they make sense?
  - *Overleading terms are still pure gauge*
  - Bonus: partially massless fields on the celestial sphere
  - Charges? More problematic… — cf., however, Compère, Fiorucci, Ruzziconi (2018); Colferai, Lionetti (2020)

see also Fiorucci's talk

Superrotations & partially masslessness

- *The overleading terms are pure gauge (on shell)*
  - Consider $\varphi_{u_{s-k}i_k}(r,u,\hat{\mathbf{x}}) = \sum_n r^{-n}\, U_{i_k}{}^{(k,n)}(u,\hat{\mathbf{x}})$
  - Above radiation order, everything ∝ to φ_{i_s}: $U^{(k,n)} \propto (\mathcal{D}\cdot)^{s-k} C^{(n-s+k)}$
  - Condition imposed by the eom

$$(D-2n-2s-2)\partial_u C^{(n)} = \left[\Delta - (n-1)(D-n-2s-2) - s(D-s-2)\right] C^{(n-1)}$$
$$- \frac{D+2(s-3)}{(n+2s-2)(D-n-3)} \left(\mathcal{D}\mathcal{D}\cdot C^{(n-1)} - \frac{2}{D+2(s-3)}\,\gamma\,\mathcal{D}\cdot\mathcal{D}\cdot C^{(n-1)} \right)$$

  - Peculiar behaviour for $3-2s \leq n \leq 2-s$: consider $n \to 2-s-t$:

$$\mathcal{M}^{(s,t)} \equiv \left[\Delta - (D+s-4) + t(D+t-5)\right] C^{(1-s-t)}$$
$$- \frac{D+2(s-3)}{(s-t)(D+s+t-5)} \left(\mathcal{D}\mathcal{D}\cdot C^{(1-s-t)} - \frac{2}{D+2(s-3)}\,\gamma\,\mathcal{D}\cdot\mathcal{D}\cdot C^{(1-s-t)} \right)$$

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  - This "kinetic operator" admit a gauge symmetry:

$$\delta C^{(1-s-t)} = \mathcal{D}^{s-t}\lambda^{(t)} \quad \text{with} \quad \mathcal{D}\cdot\lambda^{(t)} = \lambda^{(t)\,\prime} = 0$$

Drew, Gegenberg (1980)
Skvortsov, Vasiliev (2007);
AC, Francia (2012)

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mass shell of a partially massless field of spin *s* and depth *t*

Summary & overview

- Boundary conditions allowing angle dependent asymptotic symmetries can be defined for any D and any s
- All contributions above radiation are (large) pure-gauge terms
- u-independent symmetries ⇒ (any-s) supertranslations
- Supertranslation Ward identities ⇒ Weinberg's soft theorems
- Even weaker falloffs ⇒ (any-s) superrotations

What's next?

- Renormalise superrotation charges
- Non-abelian HS algebras
- Remnants of these symmetries in string scattering?