Scalarized black holes in Horndeski gravity : an overview

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Plan

1. Introduction
   - Context
   - No hair theorem(s)
2. Horndeski
3. Non-minimal coupling to the Gauss-Bonnet invariant
   - Shift-symmetry ($\mathcal{F}(\phi) = \gamma_1\phi$)
   - Spontaneous scalarization ($\mathcal{F}(\phi) = \gamma_2\phi^2$)
   - Unified pattern ($\mathcal{F}(\phi) = \gamma_1\phi + \gamma_2\phi^2$)
4. Non-minimal derivative coupling to the Einstein tensor
5. Conclusions & Outlooks

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- Offer a geometrical explanation of gravitational process [elegant]
- Allow to explain many phenomenons :
  1. Mercury perihelion problem
  2. Existence and shape of gravitational waves : GW150914 (2016)
  3. Gravitational lensing : Event Horizon telescope (2019)

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Not all of them reduces to quantum correction problems !

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In GR, all the degrees of freedom are encoded in the metric $g_{\mu\nu}$.
But, formally, the equivalence principle does not rule out the possible existence of other kind of fields in the model.

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  - Cosmology
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- Important element of many models :
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  - ...
- Also experimentally motivated since the Brout-Englert-Higgs boson's discovery (CERN 2012)

Introduction : Why not considering the simplest case ?

Why not just using $\mathscr{L}_{\mathsf{EKG}} = \kappa \left(R - 2\Lambda \right) - \frac{1}{2} \nabla_\mu \phi \nabla^\mu \phi - V(\phi)$?

Introduction : Why not considering the simplest case ?

No Scalar-Hair Theorem (*Schematically*)

Consider an asymptotically flat black hole spacetime

Hypothesis 1 : (Symmetries of spacetime)

Hypothesis 2 : (Symmetries of the scalar field)

Hypothesis 3 : (Coupling condition)

Hypothesis 4 : (Energetic condition)

Then*, the scalar field must be trivial :* $\phi\left(x^{\mu}\right)=\phi_{0}, \forall x^{\mu}$.

See [Herdeiro 2015] for a review.

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No Scalar-Hair Theorem (*Example; due to Bekenstein*)

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The spacetime is stationnary
Hypothesis 2 : (Symmetries of the scalar field)
The scalar field shares the space-time symmetries.
Hypothesis 3 : (Coupling condition)

$$S = \int_{\mathcal{M}} \left[F(g_{\mu\nu}, \partial_\alpha g_{\mu\nu}, \dots) - \frac{1}{2} \nabla_\mu \phi \nabla^\mu \phi - V(\phi) \right] \sqrt{-g} \, \mathrm{d}^n x$$

Hypothesis 4 : (Energetic condition)
Ex : $\phi V'(\phi) \geq 0 \ \ \forall \phi$, with $V'(\phi) = \mathrm{d}V/\mathrm{d}\phi$, & $\phi V'(\phi) = 0$ for some discrete values of ϕ, say ϕ_i.

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***Then**, the scalar field must be trivial : $\phi(x^\mu) = \phi_0, \forall x^\mu$.*

Note : In general, the proof makes **no use** of the Einstein's equations. It just uses the scalar field equation defined thanks to hypothesis 3.

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1. Construct a positively defined integral.

$$\int_{\mathfrak{E}} \mathfrak{F}\left(\phi, \nabla_\mu \phi\right) \sqrt{-g}\, \mathrm{d}^4 x \geq 0,$$

where $\mathfrak{E}$ denotes the black-hole exterior spacetime region.

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2. Prove that, on-shell, this integral should vanish.

$$\mathcal{E}_\phi \approx 0 \Longrightarrow \int_{\mathfrak{E}} \mathfrak{F}\left(\phi, \nabla_\mu \phi\right) \sqrt{-g}\, \mathrm{d}^4 x \approx 0.$$

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3. Use the form of the integrand to conclude that the scalar field must be trivial.

$$\int_{\mathfrak{E}} \mathfrak{F}\left(\phi, \nabla_\mu \phi\right) \sqrt{-g}\, \mathrm{d}^4 x = 0 \Longrightarrow \phi(x^\mu) = \phi_0, \forall x^\mu \in \mathfrak{E}.$$

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$$S = \int_{\mathcal{M}} \mathscr{L} \sqrt{-g}\, \mathrm{d}^4 x$$

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$$\begin{aligned}\mathscr{L} =& K(\phi,\rho) - G_3(\phi,\rho)\Box\phi + G_4(\phi,\rho)R + G_{4,\rho}(\phi,\rho)\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right] \\ &+ G_5(\phi,\rho)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi \\ &- \frac{1}{6}G_{5,\rho}(\phi,\rho)\left[(\Box\phi)^3 - 3\Box\phi\,(\nabla_\mu\nabla_\nu\phi)^2 + 2\,(\nabla_\mu\nabla_\nu\phi)^3\right],\end{aligned}$$

where

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and where the functions $G_i(\phi,\rho)$ $(i \in \{3,4,5\})$ & $K(\phi,\rho)$ are **arbitrary** functions.

Horndeski

Construction : schematically

Let us briefly discuss the steps in the discovery/construction of this lagrangian density from the (more recent) point of view of Galileon theory.

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Horndeski

Construction : schematically

Horndeski had “cracked” the problem from a completely different starting point
(He directly asked the question of the most general lagrangian density [in $4D$] presenting at most second order equations for $g_{\mu\nu}$ and ϕ)

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↑ ↑ ↑ ↑ ↑

This result is non-trivial.

Even though the generalized Galileon provided the most general lagrangian density with second order field equation for ϕ on flat spacetime there was *a priori* no reasons why its covariant extension should still be the most general possibility on curved spacetime !

Horndeski

Has a final note on this construction, let us come back to the Horndeski lagrangian density and emphasize the link between the different terms.

Especially, let us emphasize which terms necessitate the introduction of an approrpiated counter term

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\begin{aligned}
\mathscr{L} =& K(\phi,\rho) - G_3(\phi,\rho)\Box\phi + G_4(\phi,\rho)R + G_{4,\rho}(\phi,\rho)\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right] \\
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\end{aligned}
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where

$$\rho = \nabla_\mu\phi\nabla^\mu\phi,$$

and where the functions $G_i(\phi,\rho)$ $(i \in \{3,4,5\})$ & $K(\phi,\rho)$ are **arbitrary** functions.

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With $G_3 = 0 = G_5, G_4 = \kappa = c^4/16\pi\mathscr{G}$ and $K(\phi,\rho) = -\frac{1}{2}\rho - V(\phi) - 2\kappa\Lambda$, one gets

$$\mathscr{L}_{\text{EKG}} = \kappa\,(R - 2\Lambda) - \frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi - V(\phi)$$

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If $G_i\left(\phi,\rho\right) = G_i\left(\rho\right), \forall i \in \{3,4,5\}$ and $K\left(\phi,\rho\right) = K\left(\rho\right)$, the system possesses an invariance under $\phi \to \phi + c$ (shift-symmetry), with $c \in \mathbb{R}$, and the EOM for ϕ reduces to a conservation law (Nœther) :

$$\nabla_\mu J^\mu = 0.$$

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Coupling to the Gauss-Bonnet invariant

A first interesting subclass of the Horndeski lagrangian is given by

$$\mathscr{L} = R - \frac{1}{2}\, g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi + \mathcal{F}(\phi)\mathscr{L}_{\mathsf{GB}}, \tag{1}$$

where

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- (1) can be obtained from the Horndeski lagrangian via specific choice of the arbitrary functions and some integrations by parts. This has been established in [Kobayashi 2011].
- In 4D, it is well known that $\mathscr{L}_{\text{GB}} = \nabla_\mu\mathcal{G}^\mu$.

Coupling to the Gauss-Bonnet invariant

→ An interesting feature of this model is that the curvature of spacetime will source the scalar field and (almost certainly) force it to be non-trivial :

$$\Box\phi = -\mathcal{F}'(\phi)\mathscr{L}_{\mathsf{GB}}.$$

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→ In the following, we will focus our review on asymptotically flat spherically symmetric black hole solutions.

Linear coupling ($\mathcal{F}(\phi) = \gamma_1\phi$)

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In this case,

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- This fixes a maximal value for the coupling constant $\gamma_1 \leq \gamma_{1,\max}$.

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- Regularity of the scalar field derivative at the event horizon, $\phi'(r_h)$, require to fix $\phi'(r_h)$ as solution of a quadratic polynomial equation. $\Rightarrow \phi'(r_h) \in \mathbb{R}$ can only be ensured if the discriminant of the equation $\Delta \geq 0$.
- This fixes a maximal value for the coupling constant $\gamma_1 \leq \gamma_{1,\text{max}}$.
- Scalarized solutions can be numerically constructed for all $\gamma_1 \in [0, \gamma_{1,\text{max}}]$.

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- Scalarized solutions can be numerically constructed for all $\gamma_1 \in [0, \gamma_{1,\max}]$.
- There are no excited solutions.
- The scalar hair is of secondary type (scalar charge $\leftrightarrow$ black hole mass).

Quadratic coupling ($\mathcal{F}(\phi) = \gamma_2\phi^2$)

Asymptotically flat, spherically symmetric black hole solutions have also been studied in [Silva 2017] under the assumption of a quadratic non-minimal coupling to the Gauss-Bonnet invariant.

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When studying the perturbative regime (on a fixed Schwarzschild background), the equation reduces to an eigen value equation

$$\hat{D}_{|\mathsf{Sch}} \delta\phi = \gamma_2 \delta\phi,$$

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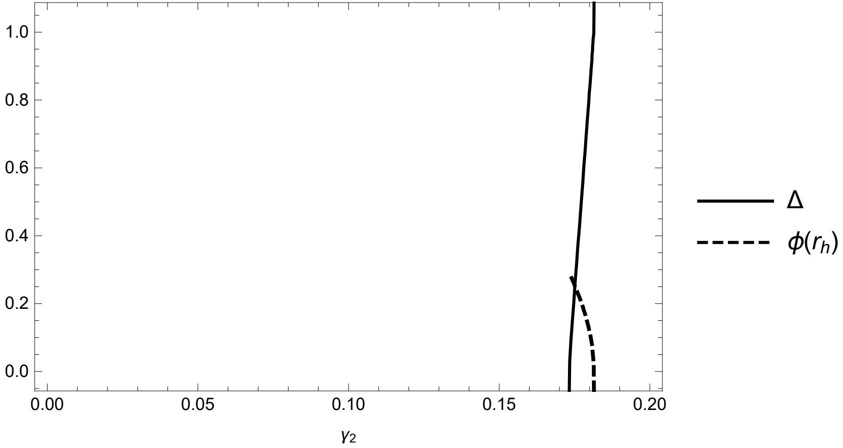
$\Longrightarrow$ In this limit, γ_2 corresponds to an eigen value of $\hat{D}_{|\mathsf{Sch}}$. This will correspond to the values of $\gamma_{2,\max}$.

Quadratic coupling ($\mathcal{F}(\phi) = \gamma_2\phi^2$)

Schematically, the existence of solution is then limited by the following pattern



Generic linear + quadratic coupling ($\mathcal{F}(\phi) = \gamma_1\phi + \gamma_2\phi^2$)

To understand the difference of pattern between the shift-symmetric and spontaneously scalarized black holes, my collaborator Y.Brihaye and I have looked at $\mathcal{F}(\phi) = \gamma_1\phi + \gamma_2\phi^2$ in [Brihaye 2018].

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- This can be seen as the most general quadratic expansion of a generic $\mathcal{F}(\phi) = \mathcal{F}(0) + \mathcal{F}'(0)\phi + \frac{\mathcal{F}''(0)}{2}\phi^2 + \mathcal{O}\left(\phi^3\right)$.
  (Remember that we can assume $\mathcal{F}(0) = 0$ without loss of generality since $\mathscr{L}_{\text{GB}} = \nabla_\mu \mathcal{G}^\mu$).

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  (Remember that we can assume $\mathcal{F}(0) = 0$ without loss of generality since $\mathscr{L}_{\text{GB}} = \nabla_\mu \mathcal{G}^\mu$).
- We obtained a pattern of spherically symmetric hairy black holes extrapolating between the shift-symmetric and spontaneously scalarized black holes.

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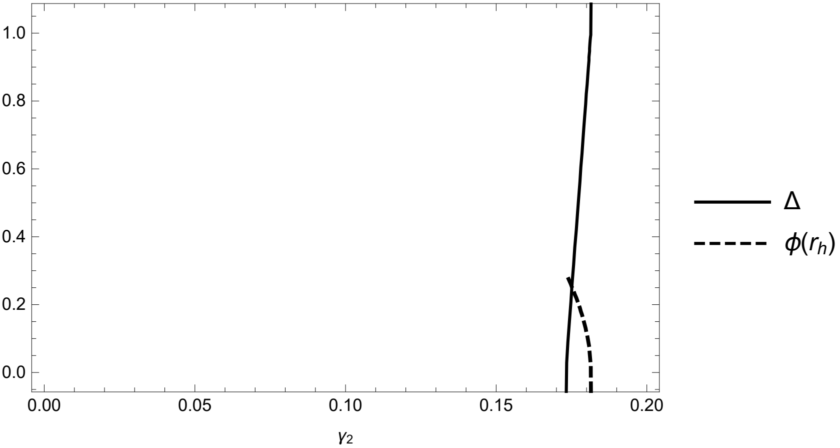

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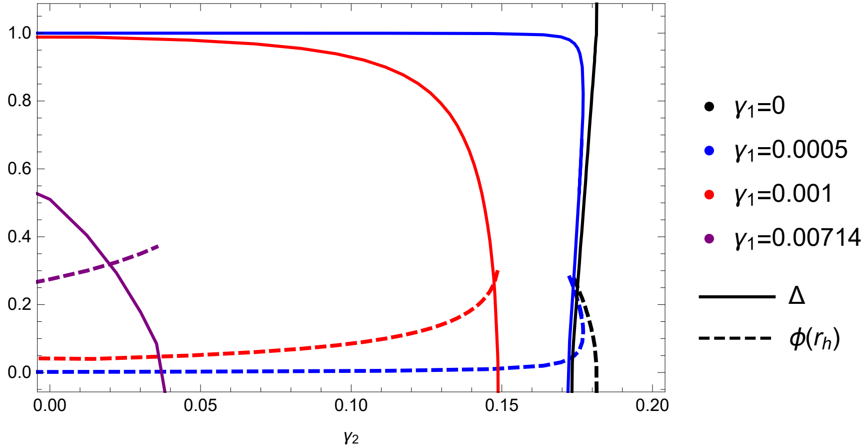

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   - Context
   - No hair theorem(s)
2. Horndeski
3. Non-minimal coupling to the Gauss-Bonnet invariant
   - Shift-symmetry ($\mathcal{F}(\phi) = \gamma_1\phi$)
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5. Conclusions & Outlooks

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Derivative coupling to the Einstein tensor

In [Babichev 2013], the authors found *exact* spherically symmetric black hole solutions for a subclass of the shift-symmetric sector of the Horndeski lagrangian

$$\mathscr{L} = \kappa \; R - \eta \; g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi + \beta \; G^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi - 2\Lambda,$$

where κ, η, β and Λ are real constants.

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An important point in the construction was to ensure the regularity of the current $J^\mu = \left(\eta g^{\mu\nu} - \beta G^{\mu\nu}\right)\nabla_\nu\phi$ (associated to the shift-symmetry) at the event horizon.

Derivative coupling to the Einstein tensor

Examples of solutions

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→ In the case $\eta = 0 = \Lambda$, the authors found a *stealth Schwarzschild solution* (*i.e.* a solution for which the spacetime metric is a Schwarzschild metric but such that the scalar field is not constant)

Derivative coupling to the Einstein tensor

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→ In the case $\eta = 0 = \Lambda$, the authors found a *stealth Schwarzschild solution* (*i.e.* a solution for which the spacetime metric is a Schwarzschild metric but such that the scalar field is not constant)

→ They also found that, if $Q^2 = (\kappa\eta + \beta\Lambda)/(\beta\eta)$, one can get a *stealth Schwarzschild-de-Sitter solution* with effective cosmological constant $\Lambda_{\text{eff}} = -\kappa\eta/\beta$.
In this case, to fit with the observational evidences on the cosmological constant, one needs a fine tuning of the parameters in the model.

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Outlooks

- This can serve as an inspiration for the study of other "scalar-to-gravity" couplings. For example, in the context of
  1. Teleparallel theories (see Sebastian Bahamonde's talk)
  2. Metric affine gravity

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- This can serve as an inspiration for the study of other "scalar-to-gravity" couplings. For example, in the context of
  1. Teleparallel theories (see Sebastian Bahamonde's talk)
  2. Metric affine gravity
- In these enhanced contexts, it would be interesting to study the behaviour of compact objects
  (black holes, boson stars, neutron stars, . . .).

Thank you for your attention !

Stay tuned for the next talk !

References (1/3)

No Scalar-Hair Theorem(s)

- **[Herdeiro 2015]** C. A. R. Herdeiro and E. Radu, Int. J. Mod. Phys. D **24** (2015) no.09, 1542014 doi:10.1142/S0218271815420146 [arXiv:1504.08209 [gr-qc]].
- **[Hui 2012]** L. Hui and A. Nicolis, Phys. Rev. Lett. **110** (2013), 241104 doi:10.1103/PhysRevLett.110.241104 [arXiv:1202.1296 [hep-th]].

References (2/3)
Horndeski Theory

- **[Nicolis 2008]** A. Nicolis, R. Rattazzi and E. Trincherini, Phys. Rev. D **79** (2009), 064036 doi:10.1103/PhysRevD.79.064036 [arXiv:0811.2197 [hep-th]].
- **[Deffayet 2011]** C. Deffayet, X. Gao, D. A. Steer and G. Zahariade, Phys. Rev. D **84** (2011), 064039 doi:10.1103/PhysRevD.84.064039 [arXiv:1103.3260 [hep-th]].
- **[Kobayashi 2011]** T. Kobayashi, M. Yamaguchi and J. Yokoyama, Prog. Theor. Phys. **126** (2011), 511-529 doi:10.1143/PTP.126.511 [arXiv:1105.5723 [hep-th]].
- **[Deffayet 2013]** C. Deffayet and D. A. Steer, Class. Quant. Grav. **30** (2013), 214006 doi:10.1088/0264-9381/30/21/214006 [arXiv:1307.2450 [hep-th]].
- **[Kobayashi 2019]** T. Kobayashi, Rept. Prog. Phys. **82** (2019) no.8, 086901 doi:10.1088/1361-6633/ab2429 [arXiv:1901.07183 [gr-qc]].

References (3/3)

Scalarized black holes in Horndeski gravity

- **[Sotiriou 2013]** T. P. Sotiriou and S. Y. Zhou, Phys. Rev. Lett. **112** (2014), 251102 doi:10.1103/PhysRevLett.112.251102 [arXiv:1312.3622 [gr-qc]].
- **[Sotiriou 2014]** T. P. Sotiriou and S. Y. Zhou, Phys. Rev. D **90** (2014), 124063 doi:10.1103/PhysRevD.90.124063 [arXiv:1408.1698 [gr-qc]].
- **[Silva 2017]** H. O. Silva, J. Sakstein, L. Gualtieri, T. P. Sotiriou and E. Berti, Phys. Rev. Lett. **120** (2018) no.13, 131104 doi:10.1103/PhysRevLett.120.131104 [arXiv:1711.02080 [gr-qc]].
- **[Brihaye 2018]** Y. Brihaye and L. Ducobu, Phys. Lett. B **795** (2019), 135-143 doi:10.1016/j.physletb.2019.06.006 [arXiv:1812.07438 [gr-qc]].
- **[Babichev 2013]** E. Babichev and C. Charmousis, JHEP **08** (2014), 106 doi:10.1007/JHEP08(2014)106 [arXiv:1312.3204 [gr-qc]].

Back up slides

$$\begin{aligned}\mathscr{L} =& K(\phi,\rho) - G_3(\phi,\rho)\Box\phi + G_4(\phi,\rho)R + G_{4,\rho}(\phi,\rho)\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right] \\ & + G_5(\phi,\rho)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi \\ & - \frac{1}{6}G_{5,\rho}(\phi,\rho)\left[(\Box\phi)^3 - 3\Box\phi\,(\nabla_\mu\nabla_\nu\phi)^2 + 2\,(\nabla_\mu\nabla_\nu\phi)^3\right],\end{aligned}$$

One can show, see [Kobayashi 2011], that, given a function $\mathcal{F}(\phi)$, the choice

$$\begin{aligned}K &= 2\mathcal{F}^{(4)}\rho^2\,(3 - \ln|\rho/2|)\,, \\ G_3 &= -2\mathcal{F}^{(3)}\rho\,(7 - 3\ln|\rho/2|)\,, \\ G_4 &= -2\mathcal{F}^{(2)}\rho\,(2 - \ln|\rho/2|)\,, \\ G_5 &= -4\mathcal{F}^{(1)}\ln|\rho/2|,\end{aligned}$$

where $\mathcal{F}^{(n)} = \mathrm{d}^n\mathcal{F}/\mathrm{d}\phi^n$, will lead, after several integrations by part, to a non-minimal coupling of the form

$$\mathcal{F}(\phi)\left(R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\lambda\sigma}R^{\mu\nu\lambda\sigma}\right).$$

No Scalar-Hair Theorem (Example)

$$\mathcal{E}_\phi = \nabla_\mu \nabla^\mu \phi - V'(\phi) \approx 0.$$

Skeleton of the proof (for the example).

1. Construct a positively defined integral.

Here, we will take $\mathfrak{F}(\phi, \nabla_\mu \phi) = \nabla_\mu \phi \nabla^\mu \phi + \phi V'(\phi)$ so that

$$\int_{\mathfrak{E}} \left(\nabla_\mu \phi \nabla^\mu \phi + \phi V'(\phi) \right) \sqrt{-g}\, \mathrm{d}^4 x \geq 0,$$

where $\mathfrak{E}$ denotes the black-hole exterior spacetime region.
This comes from the fact that, under our assumptions, each term of the integrand has a definite sign over $\mathfrak{E}$:

- → $\nabla_\mu \phi \nabla^\mu \phi \geq 0$ since the scalar field shares the spacetime symmetries,
- → $\phi V'(\phi) \geq 0$ due to our assumption on the potential.

□

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Skeleton of the proof (for the example).

2. Prove that, on-shell, this integral should vanish.

Multiplying the scalar field equation by ϕ and integrating over the black-hole exterior spacetime region $\mathfrak{E}$, one precisely gets, after integration by parts

$$\int_{\mathfrak{E}} \left(\nabla_\mu \phi \nabla^\mu \phi + \phi V'(\phi)\right) \sqrt{-g}\, \mathrm{d}^4 x \approx 0.$$

□

No Scalar-Hair Theorem (Example)

$$\mathcal{E}_\phi = \nabla_\mu \nabla^\mu \phi - V'(\phi) \approx 0.$$

Skeleton of the proof (for the example).

3. Use the form of the integrand to conclude that the scalar field must be trivial.

The only way to have

$$\int_{\mathfrak{E}} \left(\nabla_\mu \phi \nabla^\mu \phi + \phi V'(\phi) \right) \sqrt{-g}\, \mathrm{d}^4 x \approx 0,$$

is to have that, on-shell, ϕ is constant $\left(\textit{i.e. } \nabla_\mu \phi \approx 0 \Leftrightarrow \phi(x^\mu) \approx \phi_0\right)$ and correspond to a value for which $\phi_0 V'(\phi_0) = 0$.
We then get $\phi(x^\mu) \approx \phi_0$ everywhere on $\mathfrak{E}$. □