

Cosmological monopoles and non-abelian black holes

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Abstract

We discuss magnetic monopole solutions of the Einstein-Yang-Mills-Higgs equations with a positive cosmological constant. These configurations approach asymptotically the de Sitter spacetime background and exist only for a nonzero Higgs potential. We find that the total mass of the solutions within the cosmological horizon is finite. However, their mass evaluated by using the surface counterterm method outside the cosmological horizon at early/late time infinity generically diverges. Magnetic monopole solutions with finite mass and noninteger charge exist however in a truncation of the theory with a vanishing Higgs field. Both solutions with a regular origin and cosmological black holes are studied, special attention being paid to the computation of the global charges.

1 Introduction

As found some time ago by 't Hooft and Polyakov, certain gauge theories admit classical, particle-like solutions with quantised charge and finite energy [1]. The magnetic monopoles inevitably arise in grand unification theories whenever the spontaneous symmetry breakdown of this theory generates a $U(1)$ subgroup and are stabilised by a quantum number of topological origin, corresponding to their magnetic charge. Since the Standard Model of particle physics contains a $U(1)$ subgroup, monopoles with masses of the order 10^{17} GeV are predicted which leads to the so-called “monopole problem”. The accepted solution to this problem is the theory of inflation.

When coupling the Yang-Mills-Higgs (YMH) system to gravity, a branch of globally regular monopoles emerges smoothly from the corresponding flat space solutions. The nonabelian black hole solutions emerge from the globally regular configurations, when a finite regular event horizon radius is imposed [2, 3]. These solutions cease to exist beyond some maximal value of the coupling constant α , which is proportional to the ratio of the vector meson mass and Planck mass.

It has been speculated that such configurations might have played an important role in the early stages of the evolution of the Universe. Also, various analyses indicate that the monopole solutions are important in quantum theories. However, most of the investigations in the literature correspond to an asymptotically flat (AF) spacetime. Because of the physical importance of these objects, it is worthwhile to study generalisations in a different cosmological background.

As discussed in [4], the properties of magnetic monopole solutions in Anti-de Sitter (AdS) spacetime are rather similar to their asymptotically flat counterparts. Nontrivial solutions exist for any value of the cosmological constant $\Lambda < 0$; as a new feature, one finds a complicated power decay of the fields at infinity and a decrease of the maximally allowed vacuum expectation value of the Higgs field.

For a positive cosmological constant, the natural ground state of the theory corresponds to de Sitter (dS) spacetime. Solutions of Einstein equations with this type of asymptotics enjoyed recently a huge interest in

theoretical physics for a variety of reasons. First at all, the observational evidence accumulated in the last years (see, *e.g.*, ref. [5]), seems to favour the idea that the physical universe has an accelerated expansion. The most common explanation is that the expansion is driven by a small positive vacuum energy (*i.e.* a cosmological constant $\Lambda > 0$), implying the spacetime is asymptotically dS. Furthermore, dS spacetime plays a central role in the theory of inflation (the very rapid accelerated expansion in the early universe), which is supposed to solve the cosmological flatness and horizon problems.

Another motivation for studying solutions with this type of asymptotics is connected with the proposed holographic duality between quantum gravity in dS spacetime and a conformal field theory (CFT) on the boundary of dS spacetime [6]. The results in the literature suggest that the conjectured dS/CFT correspondence has a number of similarities with the AdS/CFT correspondence, although many details and interpretations remain to be clarified (see [8] for a recent review and a large set of references on this problems).

In view of these developments, an examination of the classical solutions of gravitating fields in asymptotically dS spacetimes seems appropriate, the physical relevant case of a spontaneously broken nonabelian gauge theory being particularly interesting.

Considering a static coordinate system (which generalises for $\Lambda > 0$ the usual Schwarzschild coordinates), these solutions will present a cosmological horizon for a finite value of the radial coordinate, where all curvature invariants stay finite. Similar to the well-known (electro)-vacuum solutions, there is no global timelike Killing vector, as the norm of the Killing vector $\partial/\partial t$ changes sign as it crosses the cosmological horizon. Inside the horizon, the Killing vector is timelike, and this can be used to calculate the conserved charges and action/entropy inside the cosmological horizon [9]. Outside the cosmological horizon, however, with the change of $\partial/\partial t$ to a spacelike Killing vector, the physical meaning of the conserved quantities is less clear.

Recently a method for computing conserved charges (and associated boundary stress tensors) of asymptotically dS spacetimes from data at early or late time infinity was proposed [10], in analogy with the prescription used in asymptotically AdS spacetimes [11]. This approach uses counterterms on spatial boundaries at early and late times, yielding a finite action for asymptotically dS spacetimes. The boundary stress tensor on the spacetime boundary can also be calculated, and a conserved charge (spacelike, due to its association with the Killing vector $\partial/\partial t$) – now interpreted as the mass of the solutions, can be defined.

The main goal of this paper is to present a study of the basic properties of the spherically symmetric monopole solutions in Einstein-Yang-Mills-Higgs (EYM-H) theory with a positive cosmological constant. (Solutions of the Abelian-Higgs model with $\Lambda > 0$ have been studied in [12].) Although an analytic solution to the EYM-H equations appears to be intractable, we present both analytical and numerical arguments for the existence of both solutions with a regular origin and cosmological black-hole, approaching asymptotically the dS background. As found in [13], the features of the dS solutions of a spontaneously broken nonabelian gauge theory are rather different as compared to the asymptotically flat or AdS counterparts. The most interesting feature is that the mass of dS magnetic monopoles evaluated at timelike infinity according to the counterterm prescription generically diverges, although the total mass within the cosmological horizon of these configurations is finite. Also, no solutions exist in the absence of a Higgs potential.

However, in the last years it became clear that a nonasymptotically flat metric background may allow for nonabelian magnetic monopole solutions even in the absence of a Higgs field. For example, a charge-one monopole solution has been found by Chamseddine and Volkov [14] in the context of the $\mathcal{N} = 4$ $D = 4$ Freedman-Schwarz gauged supergravity [15]. This is one of the few analytically known configurations involving both non-abelian gauge fields and gravity, the expression of the magnetic potential coinciding, in a suitable coordinate system, to that of the Prasad-Sommerfield monopole [17]. A solution with many similar properties exists also [18] in a version of $\mathcal{N} = 4$ $d = 5$ Romans' gauged supergravity model [19] with a Liouville dilaton potential.

As found in [20, 21], monopole-type solutions exist even in a simple Einstein-Yang-Mills (EYM) theory with a negative cosmological constant. These asymptotically AdS nonabelian solutions have finite mass and an arbitrary value of the gauge potential at infinity, *i.e.* an arbitrary value of the magnetic charge. There are also finite energy solutions for several intervals of the shooting parameter (the value of gauge function at the origin or at the event horizon), rather than discrete values and stable monopole solutions in which

the gauge field has no zeros.

It is natural to conjecture the existence of similar configurations for $\Lambda > 0$. Solutions with a regular origin of the EYM-SU(2) system with positive cosmological constant have been considered by several authors (see [22]-[25], and also the systematic approach in [26]). Similar to the AdS case, the asymptotic value of the gauge potential for solutions with $\Lambda > 0$ is not fixed, implying the existence of a nonvanishing magnetic charge. However, all asymptotically dS configurations are unstable. In this paper we propose to reconsider the properties of these EYM- Λ solutions, viewing them as magnetic monopoles in a model without a Higgs field. Both solutions with a regular origin and cosmological black hole configurations are discussed.

The mass and boundary stress-tensor as well as the thermodynamic quantities of both EYMH and EYM solutions discussed in this paper are computed by using the counterterm formalism proposed in [10]. We also comment on the implications of these solutions for the conjectured dS/CFT correspondence.

The paper is structured as follows: in the next Section we present the general framework and analyse the field equations and boundary conditions. In Section 3 we present our numerical results. In Section 4 we compute the solutions' mass and action, while in Section 5 we comment on the general picture in a different coordinate system. We conclude with Section 6, where our results are summarised. The Appendix generalises the construction of [27] for a positive effective cosmological constant. We show that the BPS monopole of four-dimensional YMH theory continue to be exact solutions even after the inclusion of gravitational, electromagnetic and dilatonic interactions, provided that certain non-minimal interactions are included.

2 General framework

2.1 Action principle

The action for a gravitating non-Abelian $SU(2)$ gauge field coupled to a triplet Higgs field is

$$I_B = \int_{\mathcal{M}} d^4x \sqrt{-g} \left(\frac{1}{16\pi G} (R - 2\Lambda) + \mathcal{L}_M \right) - \frac{1}{8\pi G} \int_{\partial\mathcal{M}} d^3x \sqrt{-h} K, \quad (1)$$

with Newton's constant G and

$$\mathcal{L}_M = -\frac{1}{2} \text{Tr}(F_{\mu\nu} F^{\mu\nu}) - \frac{1}{4} \text{Tr}(D_\mu \Phi D^\mu \Phi) - V(\Phi), \quad (2)$$

where $V(\Phi) = \frac{1}{4} \lambda \text{Tr}(\Phi^2 - \eta^2)^2$ is the usual Higgs potential, with η the vacuum expectation value (vev). The gauge field strength tensor is given by

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ie[A_\mu, A_\nu], \quad (3)$$

with $D_\mu = \partial_\mu + ie[A_\mu, \]$ being the covariant derivative, A_μ the gauge potential and e the Yang-Mills coupling constant.

The last term in (1) is the Hawking-Gibbons surface term [28], where K is the trace of the extrinsic curvature for the boundary $\partial\mathcal{M}$ and h is the induced metric of the boundary. Of course, this term does not affect the equations of motion but is important when computing the mass and action of solutions.

Apart from the cosmological constant, the theory contains three mass scales: the Planck mass $M_{Pl} = 1/\sqrt{G}$, the gauge boson mass $M_W = ev$ and the mass $M_H = \sqrt{\lambda}\eta$ of the Higgs field.

Varying the action (1) with respect to $g^{\mu\nu}$, A_μ and Φ we have the field equations

$$\begin{aligned} R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} &= 8\pi G (T_{\mu\nu}^{(H)} + T_{\mu\nu}^{(YM)}), \\ \frac{1}{\sqrt{-g}} D_\mu (\sqrt{-g} F^{\mu\nu}) &= \frac{1}{4} ie [\Phi, D^\nu \Phi], \\ \frac{1}{\sqrt{-g}} D_\mu (\sqrt{-g} D^\mu \Phi) + \lambda (\Phi^2 - \eta^2) \Phi &= 0, \end{aligned} \quad (4)$$

where the stress-energy tensor is

$$T_{\mu\nu}^{(H)} = \text{Tr}\left(\frac{1}{2}D_\mu\Phi D_\nu\Phi - \frac{1}{4}g_{\mu\nu}D_\alpha\Phi D^\alpha\Phi\right) - g_{\mu\nu}V(\Phi), \quad (5)$$

$$T_{\mu\nu}^{(YM)} = 2\text{Tr}(F_{\mu\alpha}F_{\nu\beta}g^{\alpha\beta} - \frac{1}{4}g_{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}). \quad (6)$$

The EYM model is found in the limit of a vanishing Higgs field $\Phi = 0$, $V(\Phi) = 0$.

2.2 The ansatz

For $\Lambda \leq 0$, the above field equations present a large variety of solutions, including configurations with axial symmetry only [29], [30]. However, here we will restrict to the spherically symmetric case.

When discussing physics in dS spacetime, there are many coordinate systems to choose among. In this work we will consider mainly the static system. The advantage of these coordinates is their obvious simplicity and the time independence. This coordinate system is also computationally more convenient, since we will deal with ordinary differential equations. The disadvantage is that the expansion of the universe is not manifest. (In Section 5 we will discuss our results in a cosmological coordinate system. Both can be used to approach the timelike infinity and will lead to different form of the boundary metric.)

Therefore we consider a line element given by

$$ds^2 = \frac{dr^2}{N(r)} + r^2(d\theta^2 + \sin^2\theta d\varphi^2) - \sigma^2(r)N(r)dt^2 \quad (7)$$

where a convenient parametrisation of the metric function $N(r)$ is

$$N(r) = 1 - \frac{2m(r)}{r} - \frac{\Lambda r^2}{3}. \quad (8)$$

The function $m(r)$ is usually interpreted as the total mass-energy within the radius r . For $m(r) = 0$, the empty dS space written in a static coordinate system with a cosmological horizon at $r = r_c = \sqrt{3/\Lambda}$ is recovered.

For the SU(2) Yang-Mills field we use the minimal spherically symmetric (purely magnetic) ansatz employed also in previous studies on asymptotically flat (or AdS) solutions, in terms of a single magnetic potential $w(r)$

$$A = \frac{1}{2e}\left\{\omega(r)\tau_1 d\theta + \left(\cos\theta\tau_3 + \omega(r)\tau_2\sin\theta\right)d\varphi\right\}, \quad (9)$$

while the Higgs field is given by the usual form

$$\Phi = \phi \tau_3, \quad (10)$$

where τ_i are the Pauli matrices.

2.3 The equations of motion

For a nonvanishing asymptotic magnitude of the Higgs field η , dimensionless quantities are obtained by considering the rescaling¹

$$r \rightarrow r/(\eta e), \quad m \rightarrow m/(\eta e), \quad \Phi \rightarrow \phi\eta, \quad \Lambda \rightarrow \Lambda(e\eta)^2. \quad (11)$$

As a result, the EYMH equations then depend only on the dimensionless parameters $\alpha = M_W/(eM_{Pl})$, $\beta = M_H/M_W$ and on Λ .

¹It can be proven that a vanishing asymptotic magnitude of the Higgs field η implies $\Phi \equiv 0$.

Within the above ansatz, the reduced EYM action can be expressed as

$$S = \int dr dt \left[\sigma m' - \alpha^2 \sigma (\omega'^2 N + \frac{(\omega^2 - 1)^2}{2r^2}) + \frac{1}{2} \phi'^2 r^2 N + \omega^2 \phi^2 + V(\phi) r^2 \right], \quad (12)$$

where the prime denotes the derivative with respect to the radial coordinate r and $V(\phi) = \beta^2(\phi^2 - 1)^2/4$, with $\beta^2 = \lambda/e^2$.

The EYM equations reduce to the following system of four non-linear differential equations

$$m' = \alpha^2 (\omega'^2 N + \frac{(\omega^2 - 1)^2}{2r^2}) + \frac{1}{2} \phi'^2 r^2 N + \omega^2 \phi^2 + V(\phi) r^2, \quad \sigma' = -\frac{2\sigma}{r} \alpha^2 (\omega'^2 + \frac{1}{2} \phi'^2 r^2), \quad (13)$$

$$(N\sigma\omega')' = \sigma\omega \left(\frac{(\omega^2 - 1)}{r^2} + \phi^2 \right), \quad (N\sigma r^2 \phi')' = \sigma (2\omega^2 \phi + r^2 \frac{\partial V}{\partial \phi}). \quad (14)$$

In the absence of the Higgs field, one defines a set of dimensionless variables by performing the following rescalings $r \rightarrow (\sqrt{4\pi G}/e)r$, $\Lambda \rightarrow (e^2/4\pi G)\Lambda$ and $m \rightarrow (eG/\sqrt{4\pi G})m$. The equations of the EYM- Λ system model then read

$$m' = \omega'^2 N + \frac{(\omega^2 - 1)^2}{2r^2}, \quad \sigma' = -\frac{2}{r} \sigma \omega'^2, \quad (N\sigma\omega')' = \sigma \frac{\omega(\omega^2 - 1)}{r^2}. \quad (15)$$

2.4 Known solutions

Several explicit solutions of the above equations are well known. The Schwarzschild-de Sitter solution corresponds to

$$\omega(r) = \pm 1, \quad \sigma(r) = 1, \quad \phi(r) = 0 \quad \text{with} \quad v = 0, \quad N(r) = 1 - \frac{2M}{r} - \frac{\Lambda r^2}{3}, \quad (16)$$

and describes a black hole inside a cosmological horizon as long as $N(r)$ has two positive zeros, *i.e.* $M < 1/3\sqrt{\Lambda}$. The Nariai solution is found for $M = 1/3\sqrt{\Lambda}$, in which case the function $N(r)$ has a double zero at $r_c = 1/\sqrt{\Lambda}$, corresponding to the position of an extremal cosmological horizon.

The second solution corresponds to an embedded U(1) configuration with

$$\omega(r) = 0, \quad \sigma(r) = 1, \quad \phi(r) = 1, \quad N(r) = 1 - \frac{2M}{r} + \frac{\alpha^2}{r^2} - \frac{\Lambda r^2}{3} \quad (17)$$

and describes a particular parametrisation of the (magnetic-) Reissner-Nordström-dS (RNdS) black hole. Since this solution proves to be important in understanding the features of the nonabelian configurations, we present here a brief review of its properties (see *e.g.* [31], [32], [33] for more details).

For $\Lambda > 0$, up to four zeros of $N(r)$ can exist. Three of the four zeros correspond to horizons since the first zero has always negative value and thus has no physical meaning. The two inner horizons r_- , r_+ with $r_- \leq r_+$ correspond to the well known Cauchy, respectively event horizon of the Reissner-Nordström solution, while the third outer horizon $r_c > r_+$ is the cosmological horizon. Extremal black hole solutions - like in the AF case - are also possible. Then, we have $r_- = r_+ = r_h$ with $N(r_h) = N'|_{r=r_h} = 0$. This leads to the equation:

$$\Lambda r_h^4 - r_h^2 + \alpha^2 = 0, \quad (18)$$

which is solved by

$$r_{h/c} = \frac{1}{\sqrt{2\Lambda}} \sqrt{1 \pm \sqrt{1 - 4\alpha^2 \Lambda}} \quad \text{for} \quad \frac{1}{4\alpha^2} \geq \Lambda > 0. \quad (19)$$

Obviously, the appearance of horizons in dS space is restricted by $\alpha^2 \leq \frac{1}{4\Lambda}$. The corresponding mass of the extremal solution is given by:

$$M_{ext} = \frac{2}{3} \frac{\alpha^2}{r_h} + \frac{r_h}{3}. \quad (20)$$

As the parameter M increases (relative to α , with $M > 0$), the outer black hole and cosmological horizons move closer together. The charged Nariai solution is obtained when these horizons coincide at $r_h = r_c$; this is the largest charged asymptotically dS black hole. For a given α , the charged Nariai black hole has the maximal mass.

For completeness, we present also another solution which is less relevant to the situation discussed in this paper. This solution is found for a particular value of the cosmological constant $\Lambda = 3/(16\pi G)$ and a vanishing Higgs field $\phi = \beta = 0$. It reads

$$ds^2 = \frac{dr^2}{1 - \frac{2}{3}\Lambda r^2} + r^2 d\Omega^2 - dt^2, \quad \omega(r) = \sqrt{1 - \frac{2}{3}\Lambda r^2}, \quad (21)$$

corresponding to an $S^3 \times R$ Einstein universe.

2.5 Boundary conditions for the EYMH model

We want the generic line element (7) to describe a nonsingular, asymptotically de Sitter spacetime outside a cosmological horizon located at $r = r_c > 0$. Here $N(r_c) = 0$ is only a coordinate singularity where all curvature invariants are finite. Outside the cosmological horizon r and t changes the character (*i.e.* r becomes a timelike coordinate for $r > r_c$). A nonsingular extension across this null surface can be found just as at the event horizon of a black hole. The regularity assumption implies that all curvature invariants at $r = r_c$ are finite. Also, all matter functions and their first derivatives extend smoothly through the cosmological horizon, *e.g.* in a similar way as the U(1) electric potential $A_t = A_0 + Q/r$ of a RNdS solution.

Similar to the case $\Lambda \leq 0$, it is natural to consider two types of configurations, corresponding in the usual terminology to *particle-like* and *black hole* solutions.

For particle-like solutions, $r = 0$ is a regular origin, and we find the following behaviour:

$$\begin{aligned} m(r) &= \frac{1}{6}\alpha^2(12b^2 + 3a^2 + 2\beta^2)r^3 + O(r^5), \quad \sigma(r) = \sigma_0 + \frac{1}{2}(a^2 + 8b^2)\sigma_0\alpha^2 r^2 + O(r^3), \\ \omega(r) &= 1 - br^2 + O(r^4), \quad \phi(r) = ar + O(r^3), \end{aligned} \quad (22)$$

where a and b are free parameters which are found by solving the field equations.

2.5.1 Expansion at the horizon

The cosmological horizon is located at some finite value of r , where $N(r_c) = 0$, $\sigma(r_c) \neq 0$, the gauge potential and the scalar field taking some constant values. The corresponding expansion as $r \rightarrow r_c$ is

$$\begin{aligned} m(r) &= \frac{r_c}{2}\left(1 - \frac{\Lambda r_c^2}{3}\right) + m_1(r - r_c) + O(r - r_c)^2, \quad \sigma(r) = \sigma_c + \sigma_1(r - r_c) + O(r - r_c)^2, \\ \omega(r) &= \omega_c + \omega_1(r - r_c) + O(r - r_c)^2, \quad \phi(r) = \phi_c + \phi_1(r - r_c) + O(r - r_c)^2, \end{aligned} \quad (23)$$

where

$$\begin{aligned} m_1 &= \alpha^2\left(\frac{1}{2r_c^2}(\omega_c^2 - 1)^2 + \omega_c^2\phi_c^2 + V(\phi_c)r_c^2\right), \quad \omega_1 = \frac{1}{N_1}\left(\frac{\omega_c(\omega_c^2 - 1)}{r_c^2} + \omega_c\phi_c^2\right), \\ \phi_1 &= \frac{1}{N_1 r_c^2}\left(2\omega_h^2\phi_h^2 + r^2\frac{\partial V}{\partial\phi}\Big|_{r_c}\right), \quad \sigma_1 = -\frac{2\sigma_c}{r_c}\alpha^2\left(\omega_1^2 + \frac{1}{2}\phi_1^2 r_h^2\right), \end{aligned} \quad (24)$$

σ_c , ω_c , ϕ_c being arbitrary parameters and $N_1 = N'(r_c) = (1 - 2m_1 - \Lambda r_c^2)/r_c$. Note that the condition $N'(r_c) < 0$ should be satisfied, which imposes the following constraint

$$\frac{1}{2r_c^2}(\omega_c^2 - 1)^2 + \omega_c^2\phi_c^2 + V(\phi_c)r_c^2 > \frac{1 - \Lambda r_c^2}{\alpha^2}. \quad (25)$$

We will consider also cosmological black hole solutions. These configurations possess an event horizon located at some intermediate value of the radial coordinate $0 < r_h < r_c$, all curvature invariants being finite as $r \rightarrow r_h$. As $r \rightarrow r_h$, the functions m, σ, ω and ϕ present an expansion very similar to (23)

$$\begin{aligned} m(r) &= \frac{r_h}{2} \left(1 - \frac{\Lambda r_h^2}{3}\right) + m'(r_h) + O(r - r_h)^2, & \sigma(r) &= \sigma_h + \sigma'(r_h)(r - r_h) + O(r - r_h)^2, \\ \omega(r) &= \omega_h + \omega'(r_h)(r - r_h) + O(r - r_h)^2, & \phi(r) &= \phi_h + \phi'(r_h)(r - r_h) + O(r - r_h)^2, \end{aligned} \quad (26)$$

where

$$\begin{aligned} m'(r_h) &= \alpha^2 \left(\frac{1}{2r_h^2} (\omega_h^2 - 1)^2 + \omega_h^2 \phi_h^2 + V(\phi_h) r_h^2 \right), & \omega'(r_h) &= \frac{1}{N'(r_h)} \left(\frac{\omega_h (\omega_h^2 - 1)}{r_h^2} + \omega_h \phi_h^2 \right), \\ \phi'(r_h) &= \frac{1}{N'(r_h) r_h^2} \left(2\omega_h^2 \phi_h^2 + r_h^2 \frac{\partial V}{\partial \phi} \Big|_{r_h} \right), & \sigma'(r_h) &= -\frac{2\sigma_h}{r_h} \alpha^2 (\omega'(r_h)^2 + \frac{1}{2} \phi'(r_h)^2 r_h^2), \end{aligned} \quad (27)$$

$\sigma_h, \omega_h, \phi_h$ being arbitrary parameters and $N'(r_h) = (1 - 2m'(r_h) - \Lambda r_h^2)/r_h$. The obvious condition $N'(r_h) > 0$ imposes the constraint

$$\frac{1}{2r_h^2} (\omega_h^2 - 1)^2 + \omega_h^2 \phi_h^2 + V(\phi_h) r_h^2 < \frac{1 - \Lambda r_h^2}{\alpha^2}. \quad (28)$$

Both the event and the cosmological horizon have their own surface gravity κ given by

$$\kappa_{h,c}^2 = -\frac{1}{4} g^{tt} g^{rr} (\partial_r g_{tt})^2 \Big|_{r=r_h, r_c},$$

the associated Hawking temperature being $T_H = |\kappa|/(2\pi)$.

2.5.2 Expansion at infinity

The analysis of the field equations as $r \rightarrow \infty$ implies a more complicated picture as compared to the AF or asymptotically AdS case, since the cosmological constant enters in a nontrivial way the solutions' expression at infinity.

We suppose that the Higgs scalar approaches asymptotically its vev, while the magnetic gauge potential $w(r)$ vanishes. This assures the absence of supplementary contributions to the cosmological vacuum energy (apart from Λ) as $r \rightarrow \infty$, while the gauge field approaches asymptotically the U(1)-Dirac monopole field. The asymptotic expression of the metric functions $m(r)$, $\sigma(r)$ and the matter functions $w(r)$ and $\phi(r)$ is found by finding an approximate solution of the field equations (13)-(14) for these boundary conditions.

Here it is instructive to work with dimensionfull variables, the corresponding expression after considering the rescaling (11) being straightforward.

A systematic analysis of the matter field equations reveals that a positive cosmological constant sets a mass bound for the gauge sector, $M_b = \sqrt{\Lambda/12}$. The expression of the gauge field as $r \rightarrow \infty$ for $M_W < M_b$, which leads to finite mass solutions is

$$w(r) \sim c_1 r^{k_1}, \quad \text{with} \quad k_1 = -\frac{1}{2} \left(1 + \sqrt{1 - M_W^2/M_b^2} \right). \quad (29)$$

This contrasts with the exponential decay found in an asymptotically flat spacetime.

For $M_W > M_b$, the large r behaviour of the gauge field is

$$w(r) \sim c_3 r^{-1/2} \sin \Psi_1(r), \quad \text{with} \quad \Psi_1(r) = (P_1 \log r + c_4), \quad P_1 = \frac{1}{2} \sqrt{M_W^2/M_b^2 - 1}, \quad (30)$$

which we will find leads to a logarithmic divergence in the asymptotic expression of the mass function $m(r)$.

The analysis of the scalar field asymptotics is standard. Strominger's mass bound [6] (*i.e.* the dS version of the Breitenlohner-Freedman bound [7]) is $M_S^2 = 3\Lambda/4$ and separates the infinite energy solutions from

solutions which may present a finite mass (this would depend also on the gauge field behaviour). For small enough values of the Higgs field mass, $M_H < M_S$ the scalar field decay leading to a finite asymptotic value of $m(r)$ is

$$\phi(r) \sim \eta + c_2 r^{k_2}, \text{ with } k_2 = -\frac{3}{2} \left(1 + \sqrt{1 - M_H^2/M_S^2} \right). \quad (31)$$

We found numerical evidence for the existence of a secondary branch of solutions decaying as

$$\phi(r) \sim \eta + c_2 r^{\tilde{k}_2}, \text{ with } \tilde{k}_2 = -\frac{3}{2} \left(1 - \sqrt{1 - M_H^2/M_S^2} \right). \quad (32)$$

However, this decay leads to a divergent value of the function $m(r)$ as $r \rightarrow \infty$.

For a Higgs mass exceeding Strominger's bound, the scalar field behaves asymptotically as

$$\phi(r) \sim \eta + c_5 r^{-3/2} \sin \Psi_2(r), \text{ with } \Psi_2(r) = (P_2 \log r + c_6), \quad P_2 = \frac{3}{2} \sqrt{M_H^2/M_S^2 - 1}, \quad (33)$$

(the constants c_i which enter the above relations are free parameters).

One may think that this bound may be circumvented by solutions with a vanishing Higgs potential. However, by rewriting the Higgs field equation in the form

$$\frac{1}{2} (N \sigma r^2 (\phi^2)')' = \sigma (N r^2 \phi'^2 + 2 w^2 \phi^2 + r^2 \phi \frac{dV}{d\phi}),$$

and integrating it between the origin and the cosmological horizon, it can easily be proven that no nontrivial solutions exist for $V(\phi) = 0$ or for a convex potential.

Once we know the asymptotics of the matter fields, the corresponding expression for the metric functions results straightforwardly from the equations (13).

For $M_H < M_S$ and $M_W < M_b$ the mass of solutions tends to a constant value as $r \rightarrow \infty$ and we find the asymptotic expressions

$$m(r) \sim 4\pi G \left(M + (\eta^2 - \frac{\Lambda}{3g^2}) \frac{c_1^2 r^{2k_1+1}}{2k_1+1} + (\frac{\lambda\eta^2}{2} - \frac{\Lambda k_2^2}{6}) \frac{c_2^2 r^{2k_2+3}}{2k_2+1} - \frac{1}{2g^2 r} \right) + \dots, \quad (34)$$

$$\log \sigma(r) \sim -8\pi G \left(\frac{k_1^2 c_1^2 r^{2k_1-2}}{2g^2(k_1-1)} + \frac{k_2 c_2^2}{4} r^{2k_2} \right) + \dots. \quad (35)$$

In the case $M_W > M_b$ the metric function $m(r)$ gets a divergent contribution from the nonabelian field, whose leading order expansion is

$$m^{YM}(r) \sim 4\pi G M_1 - \frac{\pi G c_3^2}{P_1} \left((-2\Psi_1 + \sin 2\Psi_1) \eta^2 + \frac{\Lambda}{12g^2} (8P_1 \cos^2 \Psi_1 + 2(1 + 4P_1^2) \Psi_1 + (4P_1^2 - 1) \sin 2\Psi_1) \right) + \dots, \quad (36)$$

(note the presence in this expression of a divergent term originating in the kinetic Higgs term).

For a Higgs field mass exceeding the Strominger bound $M_H > M_S$, we find a very similar asymptotic form of the mass function $m(r)$, presenting the same type of divergencies

$$m^H(r) \sim 4\pi G M_2 - \frac{c_5^2 \pi G}{4P_2} \left(\lambda \eta^2 (-2\Psi_2 + \sin 2\Psi_2) + \frac{\Lambda}{6} (24P_2 \cos^2 \Psi_2 + 2(9 + 4P_2^2) \Psi_2 + (-9 + 4P_2^2) \sin 2\Psi_2) \right) + \dots. \quad (37)$$

Since the order of the cosmological constant is believed to be much smaller than both the gauge and Higgs boson mass, our results indicate that monopoles in the universe will always have a divergent mass function

$m(r)$ as evaluated at timelike infinity. Moreover, as discussed in Section 4, we could not find numerical solutions with $M_H < M_S$ and $M_W < M_b$.

One should also study the asymptotic expression of the σ function. Supposing $M_H > M_S$ and $M_W > M_b$ one finds

$$\log \sigma(r) \sim \frac{\pi G}{3r^3} \left[\frac{c_3^3}{9 + 4P_1^2} ((1 + 4P_1^2)(9 + 4P_1^2) + 3(-3 + 4P_1^2) \cos 2\Psi_1 - 6P_1(5 + 4P_1^2) \sin 2\Psi_1) \right. \\ \left. + c_5^2(9 + 4P_2^2 - 9 \cos 2\Psi_2 - 6P_2 \sin 2\Psi_2) \right]. \quad (38)$$

The corresponding expressions for $M_H > M_S$ or $M_W > M_b$ can easily be read from (38), (35).

The solutions with $M_H = M_S$, $M_W = M_b$ saturate these bounds and lead also to infinite mass configurations. The EYMH equations lead to a matter fields expression

$$w(r) \sim \frac{c_7}{\sqrt{r}} + \frac{c_8 \log r}{\sqrt{r}}, \quad \phi(r) \sim \eta + \frac{c_9}{\sqrt{r^3}} + \frac{c_{10} \log r}{\sqrt{r^3}}, \quad (39)$$

(with c_7, \dots, c_{10} real constants), while for metric functions one can write $m(r) = m^{YM}(r) + m^H(r)$, where

$$m^{YM}(r) = 4\pi G \left(-\frac{\Lambda}{36c_8 g^2} (c_7 - 2c_8 + c_8 \log r)^3 + \eta^2 (c_7^2 \log r + c_7 c_8 \log r + \frac{c_8^3}{3} \log^3 r) \right) + \dots \quad (40)$$

$$m^H(r) = 4\pi G \left(-\frac{\Lambda}{216c_{10}} (-2c_{10} + 3c_9 + 3c_{10} \log r)^3 + \frac{\lambda}{6} \eta^2 \log r (3c_9^2 + 3c_9 c_{10} \log r + c_{10} \log^2 r) \right) + \dots,$$

and

$$\log \sigma(r) = -\frac{2\pi G}{27g^2 r^3} (9c_7 - 30c_7 c_8 + 26c_8^2 + 3c_8 \log r (6c_7 - 10c_8 + 3c_8 \log r)) \\ - \frac{\pi G}{3r^3} (2c_{10} - 6c_9 c_{10} + 9c_9^2 + 3c_{10} \log r (-2c_{10} + 6c_9 + 3c_{10} \log r)) + \dots \quad (41)$$

One can see that even for a logarithmic diverging mass function, the spacetime is still asymptotically dS, presenting the same conformal structure at infinity as the vacuum dS solution, since $g_{tt} \sim -1 + \Lambda r^2/3 + O(\log r/r)$. Also, the Ricci scalar stays finite as $r \rightarrow \infty$, $R \rightarrow 4\Lambda$.

Solutions with $\Lambda < 0$ approaching at infinity the AdS background despite the presence of a divergent ADM mass have been considered by various authors in the last years, mostly for a scalar field matter content. Restricting to the case of models with nonabelian matter fields in the bulk, we mention the SU(2) hairy black holes in [34], the family of globally regular solutions in D=4, $\mathcal{N} = 4$ gauged supergravity in [35], and the self-gravitating Yang-monopoles in [36]. The situation for $\Lambda > 0$ is less studied. However, it is natural to expect the existence of solutions with similar features for a dS background too, the EYMH monopoles being a rather complicate case.

2.6 Boundary conditions for the EYM model

The field equations imply the following behaviour for $r \rightarrow 0$ in terms of two real parameters b , σ_0

$$w(r) = 1 - br^2 + O(r^4), \quad m(r) = 2b^2 r^3 + O(r^4), \quad \sigma(r) = \sigma_0 + 4b^2 \sigma_0 r^2 + O(r^4). \quad (42)$$

The corresponding expansion near the comological horizon is

$$m(r) = \frac{r_c}{2} \left(1 - \frac{\Lambda r_c^2}{3} \right) + \frac{(\omega_c^2 - 1)^2}{2r_c^2} (r - r_c), \quad \omega(r) = \omega_c + \frac{r_c \omega_c (\omega_c^2 - 1)}{(1 - \Lambda r_c^2) r_h^2 - (\omega_h^2 - 1)^2} (r - r_c), \\ \sigma(r) = \sigma_c - \frac{2\sigma_h}{r_c} \omega'(r_h)^2 (r - r_c) + O(r - r_h)^2, \quad (43)$$

where w_c , σ_c are real parameters.

When discussing the pure EYM system with $\Lambda > 0$, there are no restrictions on the asymptotic value of the gauge potential [22]. The field equations imply the following expansion at large r

$$\begin{aligned} m(r) &= M + \left(\frac{\Lambda C_1^2}{3} - \frac{1}{2}(\omega_0^2 - 1)^2 \right) \frac{1}{r} + O\left(\frac{1}{r^2}\right), \quad \omega(r) = \omega_0 + \frac{C_1}{r} + O\left(\frac{1}{r^2}\right), \\ \sigma(r) &= 1 - \frac{C_1^2}{r^4} + O\left(\frac{1}{r^5}\right), \end{aligned} \quad (44)$$

where ω_0 , M and C_1 are constants determined by numerical calculations.

For cosmological black hole solutions having a regular event horizon at $r = r_h > 0$, we find the following expansion near the event horizon

$$\begin{aligned} m(r) &= \frac{r_h}{2} \left(1 - \frac{\Lambda r_h^2}{3} \right) + \frac{(\omega_h^2 - 1)^2}{2r_h^2} (r - r_h), \quad \omega(r) = \omega_h + \frac{r_h \omega_h (\omega_h^2 - 1)}{(1 - \Lambda r_h^2) r_h^2 - (\omega_h^2 - 1)^2} (r - r_h), \\ \sigma(r) &= \sigma_h - \frac{2\sigma_h}{r_h} \omega'(r_h)^2 (r - r_h) + O(r - r_h)^2, \end{aligned} \quad (45)$$

with w_h , σ_h real parameters.

3 Counterterm method and conserved charges

3.1 General formalism

The computation of the conserved charges including mass in an asymptotically dS spacetime is a difficult task. This is due to the absence of the spatial infinity and of a globally timelike Killing vector in such a spacetime. One prescription to compute conserved charges in asymptotically dS was developed by Abbott and Deser [37]. In this perturbative approach one considers the deviation of metric from pure dS space which is the vacuum of the theory and measure the energy of fluctuations to find the mass.

In [10], a novel prescription was proposed, the obstacles mentioned above being avoided by computing the quasilocal tensor of Brown and York (augmented by the AdS/CFT inspired counterterms), on the Euclidean surfaces at future/past timelike infinity \mathcal{I}^\pm . The conserved charge associated with the Killing vector $\partial/\partial t$ - now spacelike outside the cosmological horizon- was interpreted as the conserved mass. This allows also a discussion of the thermodynamics of the asymptotically dS solutions outside the event horizon, the boundary counterterms regularising the (tree-level) gravitation action. The efficiency of this approach has been demonstrated in a broad range of examples.

A thorough discussion of the general formalism has been given *e.g.* in [38], and so we only recapitulate it here. In this approach one starts by considering the general path integral

$$\langle g_2, \Psi_2, S_2 | g_1, \Psi_1, S_1 \rangle = \int D[g, \Psi] \exp(iI[g, \Psi]), \quad (46)$$

which represents the amplitude to go from a state with metric and matter fields $[g_1, \Psi_1]$ on a surface S_1 to a state with metric and matter fields $[g_2, \Psi_2]$ on a surface S_2 . The quantity $D[g, \Psi]$ is a measure on the space of all field configurations and $I[g, \Psi]$ is the action taken over all fields having the given values on the surfaces S_1 and S_2 . For asymptotically dS spacetimes we replace the surfaces S_1, S_2 with histories H_1, H_2 that have spacelike unit normals and are surfaces that form the timelike boundaries of a given spatial region. The amplitude (46) describes quantum correlations between differing histories $[g_1, \Psi_1]$ and $[g_2, \Psi_2]$ of metrics and matter fields, with the modulus squared of the amplitude yielding the correlation between two histories.

In this approach, the initial action (1) is supplemented by the boundary counterterm action I_{ct} depending only on geometric invariants of these spacelike surfaces. I_{ct} regularizes the gravitational action and the boundary stress tensor. In four dimensions, the counterterm expression is (in this section we do not consider rescaled quantities; also to conform with standard conventions in the literature on this subject, we note $\Lambda = 3/\ell^2$)

$$I_{ct} = -\frac{1}{8\pi G} \int_{\partial\mathcal{M}^\pm} d^3x \sqrt{h} \left(-\frac{2}{\ell} + \frac{\ell}{2} R \right) \quad (47)$$

with R the curvature of the induced metric h_{ij} and $\int_{\partial\mathcal{M}^\pm}$ indicates the sum of the integral over the early and late time boundaries. In what follows, to simplify the picture, we will consider the \mathcal{I}^+ boundary only, dropping the \pm indices (similar results hold for \mathcal{I}^-).

The boundary metric can be written, at least locally, in a ADM-like general form

$$ds^2 = h_{ij}dx^i dx^j = N_t^2 dt^2 + \sigma_{ab} (d\psi^a + N^a dt) (d\psi^b + N^b dt), \quad (48)$$

where N_t and N^a are the lapse function and the shift vector respectively and the ψ^a are the intrinsic coordinates on the closed surfaces Σ . Varying the action with respect to the boundary metric h_{ij} , we find the boundary stress-energy tensor for gravity

$$T_{ij} = \frac{2}{\sqrt{h}} \frac{\delta I}{\delta h_{ij}}, \quad (49)$$

the corresponding expression for four dimensions being [39]

$$T_{ij} = -\frac{1}{8\pi G} \left(K_{ij} - K h_{ij} - \frac{2}{l} \left[h_{ij} + \frac{\ell^2}{2} \left(R_{ij} - \frac{1}{2} R h_{ij} \right) \right] \right), \quad (50)$$

where K_{ij} and R_{ij} are the extrinsic curvature and the Ricci tensor of the boundary metric, respectively.

In this approach, the conserved quantity associated with a Killing vector ξ^i on the \mathcal{I}^+ boundary is given by

$$\mathfrak{Q}_\xi = \oint_\Sigma d^n \phi \sqrt{\sigma} n^i T_{ij} \xi^j, \quad (51)$$

where n^i is an outward-pointing unit vector, normal to surfaces of constant τ . Physically, this means that a collection of observers, on the hypersurface with the induced metric h_{ij} , would all measure the same value of \mathfrak{Q}_ξ provided this surface has an isometry generated by ξ^i .

If $\partial/\partial t$ is a Killing vector on Σ , then the conserved mass is defined to be the conserved quantity \mathfrak{M} associated with it.

A tree-level evaluation of the path integral (46) for the EYM(-H) system may be carried out along the lines described in ref. [39] for the vacuum case (see also the discussion in [33] for the case of a gravitating U(1) field). Since the action is in general negative definite near past and future infinity (outside of a cosmological horizon), we analytically continue the coordinate orthogonal to the histories to complex values by an anticlockwise $\pi/2$ -rotation of the axis normal to them. This generally imposes, from the regularity conditions, a periodicity $\tilde{\beta}$ of this coordinate, which is the analogue of the Hawking temperature outside the cosmological horizon.

This renders the action pure imaginary, yielding a convergent path integral

$$Z' = \int e^{+i\tilde{I}} \quad (52)$$

since $\hat{I} < 0$. In the semi-classical approximation this will lead to $\ln Z' = +I_{cl}$.

For a canonical ensemble with fixed temperature we can write

$$F = \mathfrak{M} - TS, \quad (53)$$

where F is the canonical potential and $T = 1/\tilde{\beta}$. For a converging partition function, we have $F = I_{cl}/\tilde{\beta}$ and thus we find for the entropy of the system

$$S = \tilde{\beta} \mathfrak{M} - I_{cl}. \quad (54)$$

3.2 Spherically symmetric EYM(-H) solutions

3.2.1 The boundary stress tensor

The application of the general formalism to spherically symmetric EYM(-H) solutions discussed in the previous sections is straightforward. Working outside the cosmological horizon, we set following [39] $r = \tau$ and rewrite the metric ansatz (7) as

$$ds^2 = -f(\tau)d\tau^2 + \frac{\sigma^2(\tau)}{f(\tau)}dt^2 + \tau^2 d\Omega^2, \quad (55)$$

where

$$f(\tau) = \left(\frac{\tau^2}{\ell^2} + \frac{2m(\tau)}{\tau} - 1 \right)^{-1}. \quad (56)$$

We choose $\partial\mathcal{M}$ to be a three surface of fixed $\tau > r_c$, which gives $n_i = 1/\sqrt{f(\tau)}\delta_{\tau i}$. The extrinsic curvature $K_{ij} = h_i^k \nabla_k n_j$ has the nonvanishing components

$$K_{\theta\theta} = -\frac{\tau}{\sqrt{f(\tau)}}, \quad K_{\varphi\varphi} = -\frac{\tau \sin^2 \theta}{\sqrt{f(\tau)}}, \quad K_{tt} = \frac{\sigma(\sigma f' - 2f\sigma')}{2f^{5/2}}, \quad (57)$$

the corresponding expression for the trace of the extrinsic curvature being (in this section we do not consider rescaled quantities; also the prime here denotes the derivative with respect to τ)

$$K = -\frac{2}{\tau\sigma} + \frac{\sigma f' - 2f\sigma'}{2f^{3/2}\sigma}. \quad (58)$$

For EYM solutions or EYMH configurations with $M_H < M_S$ and $M_W < M_b$ we find from (50) that the nonvanishing components of the boundary stress-tensor are

$$T_\theta^\theta = T_\varphi^\varphi = \frac{1}{8\pi G} \frac{\ell M}{\tau^3} + O\left(\frac{1}{\tau^4}\right), \quad T_t^t = -\frac{1}{4\pi G} \frac{M\ell}{\tau^3} + O\left(\frac{1}{\tau^4}\right).$$

The mass of these solutions measured at the far future boundary of dS space, as computed from (51), is

$$\mathfrak{M} = -M, \quad (59)$$

where M is the parameter entering the asymptotic expansions (34), (44).

As discussed in the next section, all EYM solutions we found have $M > 0$ (both black holes and particle like solutions). (Note that we didn't find any EYMH solutions with finite mass.) Thus \mathfrak{M} is negative, consistent with the expectation [10] that pure dS spacetime has the largest mass for a singularity-free spacetime $\mathfrak{M} = 0$. If there is a CFT dual to a magnetic monopole, this mass translates into the energy of the dual CFT living on a Euclidean cylinder $R \times S^2$.

The boundary stress-tensor of the EYMH solutions with $M_H > M_S$ or $M_W > M_b$ (the only configurations we could find numerically is)

$$T_\theta^\theta = T_\varphi^\varphi = \frac{1}{8\pi G} \frac{g_1(\tau)}{\tau^3} + O\left(\frac{\log \tau}{\tau^4}\right), \quad T_t^t = -\frac{1}{4\pi G} \frac{g_1(\tau)}{\tau^3} + O\left(\frac{\log \tau}{\tau^4}\right),$$

where $g_1(\tau)$ can be read from (37), (36) presenting a logarithmic divergence in τ . This implies a divergent mass as computed from (51).

3.2.2 The magnetic charge

The computation of the magnetic charge in the absence of a spacelike infinity is also difficult. However, as similar problems appear already in the abelian case, we may use the solution proposed in [33], which generalises the methods of ref. [40] to dS case. Working again outside the cosmological horizon, one defines the usual 't Hooft field strength tensor

$$\mathcal{F}_{\mu\nu} = Tr\{\hat{\Phi}F_{\mu\nu} - \frac{i}{2}D_\mu\hat{\Phi}D_\nu\hat{\Phi}\}, \quad (60)$$

where $\hat{\Phi} = \Phi/|\Phi|$. The induced metric $h_{\mu\nu} = g_{\mu\nu} + u_\mu u_\nu$ projects the electromagnetic field on a specific slice of the foliation, where the vector $u^\nu = \frac{1}{\sqrt{|f(\tau)|}}\delta_t^\nu$ is normal to the induced metric $h_{\mu\nu}$ and $n^i = \sqrt{|f(\tau)|}\delta_\tau^i$ is the (timelike) unit vector normal to hypersurface Σ . The magnetic field with respect to a slice $\tau = const.$ is $B_i = \frac{1}{2}\epsilon_{ijk}\mathcal{F}_{jk}$ and the charge density at future infinity is

$$\rho_Q = \sqrt{\sigma}n^i B_i,$$

Integrating the charge density over the hypersurface Σ , we obtain the total magnetic charge at future/past infinity

$$\mathbf{Q}_m = \frac{1}{4\pi} \oint_\infty dS_\mu \frac{1}{2} \epsilon_{\mu\nu\alpha} Tr\{\hat{\Phi}F_{\nu\alpha}\}. \quad (61)$$

For the particular case of spherically symmetric solutions one finds $\mathbf{Q}_m = 1$ ². The interpretation of the magnetic charge is analogous to that for \mathfrak{M} noted above. \mathbf{Q}_m is the charge measured by a collection of observers following a given history; its conservation implies that observers following a different history would measure the same value of the magnetic charge³.

The magnetic charge of the EYM solutions is evaluated at future/past infinity according to (see e.g.[21])

$$\mathbf{Q}_m = \frac{1}{4\pi} \int dS_k \sqrt{-g} \tilde{F}^{kt} = Q_m \frac{\tau_3}{2}, \quad (62)$$

where $Q_m = (1 - \omega_0^2)$.

3.2.3 The entropy and action

It can also be proven that the entropy of the monopole solutions associated with the cosmological event horizon is one quarter of the cosmological event horizon area, as expected.

Here we remark that, similar to the abelian theory [42], the partition function is evaluated taking the magnetic charges as a boundary condition. Fixing the gauge potential fixes the magnetic charge directly.

By integrating the Killing identity $\nabla^a \nabla_b K_a = R_{bc} K^c$, for the Killing field $K^a = \delta_t^a$ (which gives $R_t^t = -(\sqrt{-g}g^{tt}g^{\tau\tau}g_{tt,\tau})'/\sqrt{-g}$), together with the Einstein equation $R_t^t = (R - 2\Lambda)/2 - 8\pi G T_t^t$, it is possible to isolate the bulk action contribution at $r \rightarrow \infty$ and on the cosmological event horizon. Here we use the observation that, for monopole solutions ($A_t = 0$), the term T_t^t exactly cancels the matter field lagrangean in the bulk action \mathcal{L}_M . The integration is from the cosmological horizon out to some fixed τ that will be sent to infinity. Therefore we shall work in the "upper patch" outside of the cosmological horizon. The divergent contribution given by the surface integral term at infinity in R_t^t is also cancelled by $I_{\text{surface}} + I_{ct}$ and we arrive at the simple finite expression

$$I_{cl} = \frac{\tilde{\beta}}{2}(M + r_c^3/\ell^2). \quad (63)$$

²However, we expect all AF configurations to present dS counterparts, in particular the axially symmetric monopole-antimonopole solutions, with a vanishing net magnetic charge $\mathbf{Q}_m = 0$ [41]. Such solutions exist also for $\Lambda < 0$ [30].

³This definition has nothing to do with the finiteness of the mass-energy of monopole solutions.

Since $\partial/\partial t$ is a Killing vector, we may periodically identify it and assign a value for $\tilde{\beta}$. If we analytically continue $t \rightarrow it$, we obtain a metric of signature $(-2, 2)$. The submanifold of signature $(-, -)$ described by the (t, τ) coordinates will have a conical singularity at $\tau = \tau_+$ unless the t -coordinate is periodically identified with period

$$\tilde{\beta}_H = \left| \frac{4\pi}{(-\sigma^2(r)N'(r))} \right|_{r=r_c} = \left| \frac{-\sigma^2(r)f'(\tau)}{4\pi f^2(\tau)} \right|_{\tau=r_c}^{-1} \quad (64)$$

This is the analogue of the Hawking temperature outside of the cosmological horizon. The relation $S = A_c/4$ results straightforwardly from (54).

The overall picture for black hole solutions is, however, much more complicated than in the particle-like case. This is because both black hole and dS space produce thermal radiation due to quantum effects. If these temperatures are different (which is the generic case, see Figure 6), then the energy flows from the hotter horizon to the cooler one and the black hole will gain or lose mass.

In deriving these relations we supposed that the mass function $m(\tau)$ approaches a finite value as $\tau \rightarrow \infty$. This is the case for the monopole solutions in the EYM theory. However, we could not find EYMH solutions with this feature and they are unlikely to exist. For EYMH configurations, the constant M in (63) is formally infinite, which leads to a divergent action, too.

For AdS solutions, in some cases it is still possible to obtain a finite mass by allowing the regularising counterterms to depend not only on the boundary metric but also on the matter fields on the boundary [43, 44]. We expect that this approach would apply also to dS EYMH monopoles, yielding a finite mass/action to these solutions (see *e.g.* the case of dS Goldstone solutions [45]).

Also, as remarked in [46], while the counterterms are necessary to render both the action and energy finite, the thermodynamics is not affected by changing the counterterm prescription (basically both I_{cl} and $\tilde{\beta}\mathfrak{M}$ are shifted with the same amount). Therefore we expect the entropy of the EYMH monopoles, evaluated outside the cosmological horizon to satisfy also the generic relation $S = A_c/4$.

3.3 Remarks on the boundary CFT

As in the AdS/CFT correspondence, the metric of the manifold on which the *putative* dual Euclidean CFT resides is defined by

$$\gamma_{ij} = \lim_{\tau \rightarrow \infty} \frac{\ell^2}{\tau^2} h_{ij}.$$

The dual field theory's stress tensor τ_k^i , is related to the boundary stress tensor by the rescaling [47]

$$\sqrt{\gamma}\gamma^{ij}\tau_{jk} = \lim_{\tau \rightarrow \infty} \sqrt{h}h^{ik}T_{jk}.$$

For the bulk metric ansatz (7), the geometry of the manifold on which the dual CFT resides is given by the cylinder metric

$$ds^2 = \gamma_{ij}dx^i dx^j = dt^2 + \ell^2 d\Omega^2. \quad (65)$$

The CFT's stress tensor is

$$\tau_j^i = \frac{1}{8\pi G} M(-3u^i u_j + \delta_j^i), \quad (66)$$

where $u^i = \delta_t^i$. This tensor is finite, covariantly conserved and manifestly traceless, as expected from the dS/CFT correspondence, since even dimensional bulk theories are dual to odd dimensional CFTs which have a vanishing trace anomaly.

One can see that a divergent \mathfrak{M} in the bulk would imply a divergent value for the energy of the dual CFT.

3.4 Quantities inside cosmological horizon

Following [9], one may also define a total mass \mathbf{M}_c inside the cosmological horizon. This can be done by integrating the Killing identity $\nabla^\mu \nabla_\nu K_\mu = R_{\nu\rho} K^\rho$, for the Killing field $K = \partial/\partial t$ on a spacelike hypersurface Σ from the origin to r_c to get the Smarr-type formula

$$\mathbf{M}_c \equiv \frac{1}{4\pi G} \int \nabla_\mu K_\nu d\Sigma^{\mu\nu} = \frac{1}{4\pi G} \int_{V_c} \Lambda K_\mu d\Sigma^\mu + \int_{V_c} (2T_{\mu\nu} - Tg_{\mu\nu}) K^\mu d\Sigma^\nu, \quad (67)$$

where one integrates between the origin and the cosmological horizon. It is natural to identify the left-hand side as the total mass within the cosmological horizon. The first term on the r.h.s. is the contribution of Λ to the total mass within the cosmological constant, while the last one is the contribution of the matter fields. For the metric ansatz (7), \mathbf{M}_c can also be rewritten as $\mathbf{M}_c = -\kappa_c A_c / 4\pi G = -r_c^2 \sigma(r_c) N'(r_c) / 2G$, where κ_c , A_c are the cosmological horizon surface gravity and area, respectively.

A relation similar to (67) can also be written for black hole solutions

$$\mathbf{M}_c \equiv \frac{1}{4\pi G} \int \nabla_\mu K_\nu d\Sigma^{\mu\nu} = \frac{\kappa_h A_h}{4\pi G} + \frac{1}{4\pi G} \int_{V_1} \Lambda K_\mu d\Sigma^\mu + \int_{V_1} (2T_{\mu\nu} - Tg_{\mu\nu}) K^\mu d\Sigma^\nu. \quad (68)$$

One should notice that \mathbf{M}_c in the above relations is different from \mathfrak{M} , since the former keeps track only of the mass within the cosmological horizon, whereas the latter has contributions also from outside.

Also, the EYMH monopole solutions we found have always a finite \mathbf{M}_c , while \mathfrak{M} generically diverges.

4 Numerical results

Although an analytic or approximate solution appears to be intractable, we present in this section numerical arguments that the known EYMH solutions can be extended to include a positive cosmological constant. Apart from that, we discuss also monopole solutions of the EYM model, emphasizing their properties at timelike infinity.

To integrate the field equations, we used in all cases the differential equation solver COLSYS which involves a Newton-Raphson method [48]. The case $\Lambda > 0$ leads to the occurrence of a cosmological horizon at $r = r_c$ with $N(r_c) = 0$. Similar to the U(1) case, the cosmological horizon radius r_c is a function of Λ . Numerically, we have implemented a supplementary equation for the “function” Λ with $\Lambda' = 0$. Solving the set of equations fixing r_c then gives us the solutions for a numerically determined value $\Lambda = \text{const.}$. Using this trick, we can impose two further boundary conditions for the “function” Λ , one being effectively a condition on m at r_c . Our procedure then has been to first solve the equations on the interval $[0 : r_c]$ fixing r_c and then solve the equations on the interval $[r_c : \infty)$. In all our numerical computations it then turned out that the solutions on $[0 : r_c]$ and $[r_c : \infty)$ can be combined to a continuous solution on $[0 : \infty)$.

4.1 Magnetic monopole solutions in EYMH theory

The system of equations depends in this case on three parameters (Λ , α , β) in the case of gravitating monopoles and additionally on r_h in the case of non-abelian black holes.

Using the initial conditions at the origin/event horizon and on the cosmological horizon, the equations of motion (13)-(14) were solved varying Λ for a range of values of the coupling parameter α and several values of β .

We here present our results for $\beta = 0.1$ and remark that the results are qualitatively similar for other values of β we considered.

As a general feature of both particle-like and black hole solutions, we note that while a negative cosmological constant exerts an additional pressure on configurations, causing their typical radius to become thinner [4], a positive Λ has the opposite effect, causing the typical solution radius to expand beyond the value it would have in asymptotically flat space.

Hence, for small values of Λ , the solutions in the region $r < r_c$ resemble the AF solutions, being surrounded by a cosmological horizon and approach dS geometry in the asymptotic region.

Also, as α increases, the cosmological horizon shrinks in size. The dS configurations are generally not confined inside the cosmological horizon, with all variables and their first derivatives extending smoothly through the cosmological horizon.

Note that the bounds for finite energy solutions $M_H < M_S$ and $M_W < M_b$ after the rescalings (11) read $2\beta < \sqrt{3\Lambda}$ and $\sqrt{12} < \sqrt{\Lambda}$.

4.1.1 Solutions with a regular origin

When Λ is increased from zero, while keeping α, β fixed, a branch of dS solutions emerges from the corresponding asymptotically flat configurations. This branch ends at a maximal value Λ_{max} . The value of Λ_{max} depends on the parameters α, β . For example, for solutions with $\beta = 0.1$ in a fixed dS background ($\alpha = 0$), we find $\Lambda_{max} \approx 0.035$. The value of Λ_{max} varies only little with α and β , e.g. for $\alpha = 1, \beta = 0.1$ we find $\Lambda_{max} \approx 0.033$. The profile of a typical first-branch solution is presented in Figure 1(a).

A second branch of solutions always appears at Λ_{max} (such that at Λ_{max} the two branches merge at one solution), extending backwards in Λ to a zero value of the cosmological constant. The trivial solution $\phi(r < \infty) \equiv 0, w(r < \infty) \equiv 1$ with $\phi(r = \infty) = 1, w(r = \infty) = 0$ is approached (the solution thus has an infinite slope at $r = \infty$) in this limit. This can be understood as follows: since we have rescaled $\Lambda \rightarrow \Lambda/e^2\eta^2$, the limit $\Lambda \rightarrow 0$ on the first branch corresponds to asymptotically flat space with $\eta \neq 0$. On the second branch, however, the limit $\Lambda \rightarrow 0$ corresponds to asymptotically de Sitter space with $\eta \rightarrow 0$, which then implies the trivial solution. We give the profile of a typical second-branch solution in Figure 1(b).

The main difference between the solutions on the two branches is that the matter field functions attain their asymptotic values much quicker for the solutions on the first branch.

This is shown in Figure 2 where some numerical data is given as function of Λ for the two branches. The results in this figure are obtained for $\alpha = 1, \beta = 0.1$ but they remain qualitatively the same for all gravitating solutions we have considered. The maximal values of Λ are always below the critical value found for solutions in fixed dS background, and as a result the mass of our solutions measured at future/past infinity always diverges. While we have found solutions with $2\beta < \sqrt{3\Lambda}$ (finite contribution from the Higgs field), all solutions have $\sqrt{\Lambda} < \sqrt{12}$. Thus although the contribution from the Higgs field might be finite, the contribution from the gauge field, however, leads always to an infinite mass. For small values of Λ (Figure 1a), the divergence in $m(r)$ will manifest itself only for very large values of r ; however, this becomes obvious for large enough values of Λ (see Figure 1b).

Note, though, that the mass \mathbf{M}_c within the cosmological horizon stays finite (see also Figure 1) and that the mass \mathbf{M}_c doesn't tend to zero in the limit of trivial solutions.

The existence of other disconnected branches of solutions for $\Lambda > \Lambda_{max}$ appears unlikely.

For a fixed $\Lambda < \Lambda_{max}$, our numerical analysis shows that regular solutions exist on a finite interval of the parameter α (which depends on β), i.e. $\alpha \in [0, \alpha_{max}(\Lambda)]$. Correspondingly, the cosmological horizon depends slightly on α : $r_c(\alpha = \alpha_{max}) \leq r_c \leq r_c(\alpha = 0)$. For e.g. $\Lambda = 0.003$, we find $43.6 \leq r_c \leq 44.7$, as shown in Figure 3.

When the parameter α is increased, the local minimum of the metric function $N(r)$ becomes deeper and deeper while at the same time the cosmological horizon decreases slightly. In the limit $\alpha \rightarrow \alpha_{max}$, the local minimum of N approaches zero at $r = r_h$ with $N(r = r_h) = 0$ and $N'(r)|_{r=r_h} = 0$. In fact this value is determined explicitly according to

$$r_h = \frac{1}{\sqrt{2\Lambda}} \sqrt{1 - \sqrt{1 - 4\alpha^2\Lambda}} \quad , \quad \text{for} \quad \frac{1}{4\alpha^2} > \Lambda > 0$$

These results give strong evidence that in the limit $\alpha \rightarrow \alpha_{max}(\Lambda)$ the gravitating monopole solutions bifurcate with the branch of extremal RNdS solution on the interval $[r_h, \infty]$. The gravitating monopoles separate in this limit into an interior region $r \in [0 : r_h]$ with a smooth origin and a nontrivial YM field, and an exterior region $r \in [r_h : \infty]$ where the solution corresponds to an extremal RNdS solution with $w = 0, \phi = 1$ and thus unit charge. As a consequence for r close to and larger than the cosmological horizon r_c the metric functions $\sigma(r), N(r)$ are identical to ones of the RNdS solution. The mass value m_∞ can then

be determined explicitly by imposing the condition $N(r_c) = 0$. We also observe that in the limit $\alpha \rightarrow 0$ the value $\sigma(0)$ decreases considerably, while the value r_c tends to a finite (but Λ -dependent) value. The solution corresponds to an hyperbolic monopole.

As stressed above, the value of the cosmological horizon depends only slightly on α . This is shown in Figure 2. We found also that M_c and $\sigma(0)$ depend rather weakly on Λ .

4.1.2 Black hole solutions

Similarly to the globally regular solutions, non-abelian black holes solutions can be constructed with $\Lambda > 0$. These solutions correspond to non-abelian black holes sitting inside the core of de Sitter monopoles. They have a double horizon: an event horizon at $r = r_h$ and a cosmological horizon at $r = r_c$. The function $N(r)$ is zero at these two points. A generic de Sitter black hole solution is presented in Figure 4 for $\alpha = 0.5$, $\Lambda = 0.0006$ and $r_h = 0.1$. The event horizon is fixed by hand while the cosmological horizon depends on the choice of α , Λ . r_c is then determined numerically.

For fixed r_h , and sufficiently large α the metric function $N(r)$ develops a local minimum at $r = r_{min}$, $r_h < r_{min} < r_c$ and the value $N(r_{min})$ decreases and tends to zero for a maximal value of α . This is seen in Figure 5 for $\Lambda = 0.003$ and $r_h = 0.3$. Very similarly to the regular case, in this critical limit, we end up with a solution presenting three horizons. The intermediate one corresponds a double zero of the metric function $N(r)$ with $N(r = r_{min}) = 0$, $N'(r)|_{r=r_{min}} = 0$. On the interval $[r_{min}, \infty)$ the solution approaches an extremal RNdS solution, while on the interval $[r_h, r_{min}]$ the non-abelian black hole solution remains non trivial.

This phenomenon is illustrated in Figure 5 for $\Lambda = 0.003$ and $r_h = 0.3$.

At each horizon of these black holes solutions, the surface gravity (determining the entropy) can be computed. In terms of the spherically symmetric ansatz, it is given by

$$\kappa^2 = \frac{1}{2} N'(r) \sigma(r) \Big|_{r=r_c, r_h}.$$

The various quantities characterising the black hole are shown in Figure 6. In particular, we see that the surface gravity at the inner event horizon decreases considerably when increasing α and becomes very small (along with $\sigma(r_h)$) in the limit $\alpha \rightarrow \alpha_{max}$. In contrast, the surface gravity at the cosmic horizon varies only little with α (along with $\sigma(r_c) = 1$).

Similar to the case of solutions with regular origin, the mass function $m(r)$ of the black hole solutions also diverges at infinity, while the mass inside the cosmological horizon stays finite. In order to demonstrate the peculiar behaviour of the Higgs function $\phi(r)$ for $r \rightarrow \infty$, we have to construct solutions for large values of Λ . In Figure 7, we show our results for $\Lambda = 0.033$. We especially show the value of $r\phi'/(1 - \phi)$. The asymptotic value of this particular combination (which tends to some constant ~ 0.75 for the parameters chosen here) confirms clearly the behaviour (32). Of course, this phenomenon is present for generic values of the parameters although not always easy to set up numerically. We were able to exhibit this property for regular solutions as well.

4.2 Magnetic monopoles in EYM theory

We have integrated the system of eqs. (15) for a range of Λ by using the same techniques. Both solutions with regular origin and cosmological black holes have been considered.

All numerical solutions we found have $w^2 \leq 1$, which implies a nonzero node number of the gauge potential w . This can be proven by using the sum rule [21]

$$-\left(\frac{N\sigma\omega'}{\omega}\right)\Big|_{r_0}^{r_c} = \int_{r_0}^{r_c} dr \, \sigma \left(\frac{1 - \omega^2}{r^2} + N \frac{\omega'^2}{\omega^2} \right), \quad (69)$$

which follows directly from the YM equations (r_0 here is $r_0 = 0$ for solutions with regular origin or $r_0 = r_h$, for black holes). Suppose that $w(r)$ never vanishes and $w^2 \leq 1$ for $r_0 < r < r_c$. Then l.h.s. of the above relation vanishes, while the integrand of the r.h.s. is positive definite. Therefore the gauge potential of the

nontrivial YM configurations with $w^2 \leq 1$ must vanish at least once in the region inside the cosmological horizon.

Thus for a given value of Λ , the solutions are indexed by the node number k . Different from the AF case, however, the asymptotic value of the gauge potential is not fixed by finite energy considerations. Similar to M , w_0 appears as a result of the numerical integration.

For small values of the cosmological constant $\lambda \ll 1/R_c^2$ (R_c corresponding to the typical solutions' radius), the contribution Λr^2 to the energy density is negligible. For $r \ll R_c$, the solutions do not considerably deviate from the corresponding flat space configurations. In the region $r > R_c$, the effects of Λ become significant.

Several characteristic features of the two node particle-like solutions of the EYM- Λ system are plotted as a function of the cosmological constant in Figure 8. One can see that both the mass inside the cosmological horizon and the mass evaluated at timelike infinity are finite. The next two plots show the profiles of typical EYM- Λ solutions with regular origin (Figure 9) and cosmological black holes (Figure 10). In both cases, $w^2(\infty) \neq 1$, which implies the existence of a nonvanishing nonabelian magnetic charge defined according to (62).

One should remarks that the EYM theory presents solutions with dS asymptotics only for values of the cosmological constant up to some $\Lambda_{max} < 3/4$ [22].

As discussed in [49], the dS EYM-SU(2) theory is a consistent truncation of $D = 11$ supergravity via Kaluza-Klein dimensional reduction on a non-compact space, the positive cosmological constant being fixed by the SU(2) gauge coupling constant $\Lambda = 4e^2$. The “internal” space is a smooth hyperbolic seven-space written as a foliation of two three-spheres, on which the SU(2) Yang-Mills fields reside. However, no solutions with dS asymptotics exist for this value of the cosmological constant [26].

5 The nonabelian monopole in cosmological coordinates

Although the line element (7) takes a simple form in a static coordinate system, the expansion of the universe is not manifest. Furthermore, the static coordinates (t, r) break down at the Killing horizons. However, one can prove that the results in the previous sections remain qualitatively unchanged when using a different parameterisation of dS spacetime. A particularly interesting case is provided by the planar coordinates (for which the spatial slices are flat) — or cosmological or inflationary coordinates — with a dS line-element

$$ds^2 = e^{2HT} (dR^2 + R^2 d\Omega^2) - dT^2,$$

(to conform with the standard notation in literature, we note here $\Lambda = 3H^2$, *i.e.* $H = 1/\ell$). The relation between the physics in a static coordinate system and in a planar coordinate system in the case of Einstein-U(1) theory is discussed in [33].

Considering the general metric ansatz (7) and the matter fields form (9), (10), the solution in the cosmological form can be written as:

$$\begin{aligned} ds^2 &= -V^{-2} dT^2 + U^2 e^{2HT} (dR^2 + R^2 d\Omega^2), \\ A &= \frac{1}{2e} \left[\omega(Re^{HT}) \tau_1 d\theta + (\cos \theta \tau_3 + \omega(Re^{Ht}) \tau_2 \sin \theta) d\varphi \right], \quad \Phi = \phi(Re^{Ht}) \tau_3 \end{aligned} \quad (70)$$

where U and V are functions of $\rho = Re^{HT}$. The coordinate transformation that relates (7) to the above metric form is:

$$U(\rho) = \frac{r(\rho)}{\rho}, \quad t = T - \int \frac{Hr dr}{\sigma(r)N(r)\sqrt{\sigma(r)^2 N(r) + H^2 r^2}}, \quad (71)$$

$$V^{-2}(\rho) = \sigma^2(r(\rho)) \left(\frac{U + \rho U'}{U} \right)^2, \quad (72)$$

while $r(\rho)$ is determined implicitly by:

$$\int \frac{\sigma(r) dr}{r \sqrt{\sigma^2(r) N(r) + r^2 H^2}} = \ln \rho. \quad (73)$$

As an example, let us consider the $U(1)$ solution with:

$$\omega(r) = 0, \quad \sigma(r) = 1, \quad \phi(r) = 1, \quad N(r) = 1 - \frac{2M}{r} + \frac{\alpha^2}{r^2} - Hr^2 \quad (74)$$

Then we can easily integrate (73) and we obtain:

$$U(\rho) = 1 + \frac{M}{\rho} + \frac{M^2 - \alpha^2}{4\rho^2}, \quad V(\rho) = \frac{1 + \frac{M}{\rho} + \frac{M^2 - \alpha^2}{4\rho^2}}{1 - \frac{M^2 - \alpha^2}{4\rho^2}}. \quad (75)$$

which is readily seen to be the RNdS solution in cosmological coordinates. The extremality condition can be written in this case as $|\alpha| = M$.

One advantage of using the planar coordinates is that the expansion of the universe is now manifest: the cosmological expansion chart corresponds to $H > 0$, while the contracting chart is given by $H < 0$. However, this comes with the price that the manifest time-translation symmetry is broken while the charts that cover the horizons are highly distorted. The boundary geometry \mathcal{I}^\pm is now approached for large T and the boundary topology is R^3 (written above in spherical coordinates).

Let us notice that $\rho = Re^{HT}$ is unchanged under the transformations:

$$T \rightarrow T + a, \quad R \rightarrow e^{-Ha} R, \quad (76)$$

which means that the geometry in the cosmological ansatz is preserved. The first term in the above transformation generates the time translations in the bulk, while the second term corresponds to a scale transformation of the radial coordinate on the boundary. There exists a Killing vector associated with this symmetry and it can be written as:

$$\xi = -HR \frac{\partial}{\partial R} + \frac{\partial}{\partial T}. \quad (77)$$

Similar to the (electro-)vacuum case, the norm of this Killing vector will vanish precisely where $N(r) = 0$, that is, at the horizons $r = r_h$, r_c as determined in static coordinates.

It is straightforward to compute the boundary stress-tensor in cosmological coordinates. Choosing the boundary $\partial\mathcal{M}$ to be the three-surface given by a fixed value of T , the normal to this surface is $n_\mu = 1/V \delta_{T\mu}$ we find the following components of the extrinsic curvature:

$$K_{RR} = -\frac{HU^2 a^2}{\sigma}, \quad K_{\theta\theta} = -\frac{HU^2 a^2 r^2}{\sigma}, \quad K_{\varphi\varphi} = -\frac{HU^2 a^2 r^2 \sin^2 \theta}{\sigma} \quad (78)$$

that is, $K_{ij} = -\frac{H}{\sigma} h_{ij}$, where h_{ij} is the induced metric on $\partial\mathcal{M}$ and we denoted $a(T) = e^{HT}$.

The components of the boundary stress-tensor in cosmological coordinates are then given by:

$$\begin{aligned} 8\pi GT_{RR} &= \frac{2U^2 a^2}{\sigma^2} \left(\frac{H}{\sigma} - \frac{1}{\ell} \right) - \frac{2\ell U_{,R}}{rU} \left(1 + \frac{RU_{,R}}{2U} \right) = \frac{2M\ell}{R^3 a} + O(a^{-2}) \\ 8\pi GT_{\theta\theta} &= \frac{2U^2 a^2 R^2}{\sigma^2} \left(\frac{H}{\sigma} - \frac{1}{\ell} \right) - \ell r^2 \left[-\left(\frac{U_{,R}}{U} \right)^2 + \frac{U_{,R}}{RU} + \frac{U_{,RR}}{U} \right] = -\frac{2M\ell}{Ra} + O(a^{-2}) \\ 8\pi GT_{\varphi\varphi} &= \frac{2U^2 a^2 R^2 \sin^2 \theta}{\sigma^2} \left(\frac{H}{\sigma} - \frac{1}{\ell} \right) - \ell R^2 \sin^2 \theta \left[-\left(\frac{U_{,R}}{U} \right)^2 + \frac{U_{,R}}{rU} + \frac{U_{,RR}}{U} \right] = -\frac{2M\ell \sin^2 \theta}{Ra} + O(a^{-2}) \end{aligned}$$

where we assumed an expansion of the form $U(\rho) = 1 + \frac{M}{\rho} + O(\frac{1}{\rho^2})$. Notice that in the limit $T \rightarrow \infty$ the first terms on the right-hand side should cancel out once we take advantage of the fact that in this limit $\sigma \rightarrow 1$ (recall that $H = 1/\ell$).

The conserved quantity associated with the Killing vector ξ will be the mass and it can be evaluated at past or future infinity according to the sign of H . In our case the conserved mass is readily seen to be

$$\mathfrak{M} = -M, \quad (79)$$

which coincides, as expected with the value in a static coordinate system. In particular, a divergent \mathfrak{M} is found again for $M_H > M_S$ or $M_W > M_b$.

According to the dS/CFT conjecture, the dual field theory should live in our case on an Euclidian manifold whose metric is :

$$ds^2 = \gamma_{ij} dx^i dx^j = dR^2 + R^2 d\Omega^2 \quad (80)$$

obtained by an infinite conformal rescaling of the boundary geometry on $\partial\mathcal{M}$, corresponding to a flat $D = 3$ Euclidean metric. The stress energy tensor of the dual theory will be given by:

$$8\pi G\tau_R^R = \frac{2M\ell}{R^3}, \quad 8\pi G\tau_\theta^\theta = 8\pi G\tau_\varphi^\varphi = -\frac{M\ell}{R^3}. \quad (81)$$

It is easy to see that this stress-tensor is covariantly conserved and traceless, as expected on general grounds.

6 Conclusions

This work was partially motivated by the question of how a positive cosmological constant will affect the properties of a gravitating monopole. To the best of our knowledge, this question has not yet been addressed in the literature, except for the preliminary results in [13]. Apart from this motivation, the study of gravitating matter fields configurations in asymptotically dS space may help a better understanding of the conjectured dS/CFT correspondence.

We have found that despite the existence of a number of similarities to the $\Lambda = 0$ case the asymptotically dS solutions exhibits some new qualitative features. All solutions present a cosmological horizon, on which matter fields take non-trivial values. An interesting feature of the dS solutions appears to be the absence of monopole configurations without a Higgs potential. Also, contrary to the naive expectation that a small Λ will not affect the properties of the configurations drastically, we find that the mass of dS solutions evaluated at future/past infinity by using the quasilocal tensor of Brown and York diverges (although the mass within the cosmological horizon stays finite).

Since this result is based only on the asymptotic expansion of the field equations together with the Higgs mechanism, we expect it to remain valid for the dS versions of various possible extensions of the magnetic monopole model. Moreover, our results appear to be a generic feature of the asymptotically dS particle-like solutions of a spontaneously broken gauge theory. Similar qualitative results are found for dS sphalerons with a Higgs doublet [13].

The question of the stability of our solutions is very important. For the monopoles, we know that the solutions on the first branch are stable for $\Lambda = 0$, since these are nothing else but gravitating monopoles in asymptotically flat space. Invoking arguments from catastrophe theory [50], we thus believe that the monopoles on the first branch for $0 < \Lambda < \Lambda_{max}$ are stable, while the solutions on the second branch have one mode of instability. This statement should definitely be checked by means of a detailed analysis, e.g. with the one discussed in [51].

Supplementing the EYM system with a cosmological constant leads to the occurrence of the "bag of gold" family of solutions, where an equator naturally appears in the metric and the spacetime becomes spatially compact and homeomorphic to a three-sphere [22]. Nothing like that was observed in the present context because the Higgs function is imposed to approach its expectation value asymptotically. Relaxing this condition, however, we think that it could lead to compact solutions as well for fine tuned values of the cosmological constant like e.g. in [55]. We plan to study this possibility for EYMH in near future.

Also, it would be interesting to apply the isolated horizon formalism, recently extended to asymptotically dS spacetimes [52], to the particular system discussed here.

The divergencies described above are not too disturbing for physics inside the cosmological horizon. However, a divergent mass-energy and action appear to lead to severe problems concerning the possible holographic description [6] of a gravitating spontaneously broken gauge theory in dS spacetime. A divergent ADM mass has been found also for solutions of some theories in asymptotically AdS spacetime. However, in some cases it is still possible to obtain a finite mass by allowing the regularising counterterms to depend

not only on the boundary metric but also on the matter fields on the boundary [44]. It would be interesting to generalise this method to the dS case and to assign a finite mass (evaluated outside the cosmological horizon) to the solutions of a spontaneously broken gauge theory. We believe that this may lead to further understanding of the rich structure of a field theory in dS space as well as profound implications to the evolution of the early universe.

It is likely that a re-examination of various field theory nonperturbative effects for a dS ground state may lead to further surprises.

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Appendix A: BPS solutions with a Liouville potential

In [27] it was shown that the BPS monopoles of four-dimensional YMH theory continue to be exact solutions of this model even after the inclusion of gravitational, electromagnetic and dilatonic interactions, provided that certain non-minimal interactions are included. The solutions presented in [27] are asymptotically flat. Here we prove that a similar construction can be done for a Liouville dilaton potential, *i.e.* a positive *effective* cosmological constant.

Following the conventions in [27], we consider a four dimensional action principle on the form

$$S = \frac{1}{4\pi G} \int d^4x \sqrt{-g} \left[\frac{R}{4} - \frac{1}{2}(\nabla\sigma)^2 - e^{2b\sigma} \frac{1}{4} f_{\mu\nu} f^{\mu\nu} - \frac{e^{-2b\sigma}}{4} \Lambda \right] + S_{matter} \quad (82)$$

with σ the dilaton field⁴ b the dilaton coupling constant and $f_{\mu\nu}$ the U(1) field. Different from the situation in [27], the above action principle contains a Liouville potential $V(\sigma) = -e^{-2b\sigma}\Lambda/4$. The expression of the matter action is given by

$$S_{matter} = \int d^4x \sqrt{-g} \left\{ -\frac{1}{2} e^{\frac{(1-b^2)}{b}\sigma} \text{Tr}(F_{\mu\nu} F^{\mu\nu}) - \frac{1}{4} e^{-\frac{(1+b^2)}{b}\sigma} \text{Tr}(D_\mu \Phi D^\mu \Phi) + \frac{c\sqrt{1+b^2}}{4\sqrt{-g}} \varepsilon^{\mu\nu\alpha\beta} f_{\mu\nu} (\Phi F_{\alpha\beta}) \right\}, \quad (83)$$

with $F_{\mu\nu}$ the SU(2) field, Φ the Higgs scalar, while c is a constant defined bellow. Also, to simplify the general picture, we take here the gauge coupling constant $g = 1$.

The Einstein-Maxwell-dilaton field equations are given by

$$\begin{aligned} G_{\mu\nu} - 2T_{\mu\nu}(f) - 2T_{\mu\nu}(\sigma) &= (8\pi G)T_{\mu\nu}(mat.) \\ \nabla_\mu (e^{2b\sigma} f^{\mu\nu}) &= (4\pi G)J^\nu(mat.) \\ \partial_\mu (\sqrt{-g} g^{\mu\nu} \partial_\nu \sigma) - \frac{g}{2} \sqrt{-g} e^{2b\sigma} f^{\mu\nu} f_{\mu\nu} &= -(4\pi G) \frac{\delta S_{matter}}{\delta \sigma} \end{aligned} \quad (84)$$

where we note

$$\begin{aligned} T_{\mu\nu}(f) &= e^{2b\sigma} \left(f_{\mu\lambda} f_\nu{}^\lambda - \frac{1}{4} g_{\mu\nu} f^2 \right), \quad T_{\mu\nu}(\sigma) = \left(\partial_\mu \sigma \partial_\nu \sigma - \frac{1}{2} g_{\mu\nu} (\partial\sigma)^2 - g_{\mu\nu} V(\sigma) \right), \\ T_{\mu\nu}(mat.) &= e^{\frac{(1-b^2)}{b}\sigma} \text{Tr} \left(F_{\mu\lambda} F_\nu{}^\lambda - \frac{1}{4} g_{\mu\nu} F_{\alpha\beta} F^{\alpha\beta} \right) + e^{-\frac{(1+b^2)}{b}\sigma} \text{Tr} \left(D_\mu \Phi D_\nu \Phi - \frac{1}{2} g_{\mu\nu} D_\alpha \Phi D^\alpha \Phi \right) \\ J^\nu(mat.) &= \frac{ce^{-1}}{2} \sqrt{1+b^2} \varepsilon^{\nu\rho\sigma} \text{tr} (D_\mu \Phi G_{\rho\sigma}) . \end{aligned} \quad (85)$$

⁴The dilaton σ should not be confused with the metric function $\sigma(r)$ which enters the ansatz (7).

In [27] it has been shown that for $\Lambda = 0$ the equations of motion for $F_{\mu\nu}$ and Φ are solved by any YMH configuration that solves the *flat space* Bogomol'nyi equations,

$$G_{ij} = \mp \varepsilon^{ijk} D_k \Phi, \quad (86)$$

provided $c^2 = 1$. To this aim, a special metric ansatz has been used, in terms of only one metric function. The the Einstein, Maxwell and dilaton equations are then equivalent to a single Euclidean 3-space Poisson equation. In the absence of matter fields $S_{matter} = 0$, this choice gives the "extreme" dilaton black hole solutions of [53].

This construction can be generalised for a Liouville potential term in the action principle. However, given the presence of an effective cosmological constant, the metric ansatz would be time-dependent.

A straightforward generalisation of the metric ansatz used in [27] is

$$ds^2 = -e^{-2\phi} dt^2 + R^2(t) e^{2\phi} d\mathbf{x} \cdot d\mathbf{x}, \quad \text{where} \quad e^\phi = e^{\sigma/b}/R(t), \quad R(t) = \left(\frac{t}{t_0}\right)^{1/b^2}, \quad (87)$$

the only nonvanishing component of the $U(1)$ potential being

$$A_t = \frac{1}{\sqrt{1+b^2}} R(t)^{-b^2} e^{-(1+b^2)\phi}. \quad (88)$$

The free parameter t_0 is fixed by Λ

$$\Lambda = \frac{2}{b^2} \left(\frac{3}{b^2} - 1 \right) \frac{1}{t_0^2}. \quad (89)$$

It is convenient to define

$$e^\phi = (1 + R^{-(1+b^2)} f(\mathbf{x}))^{1/(1+b^2)}. \quad (90)$$

A direct computation shows that, for $c^2 = 1$, the YMH equations implied by the action principle (83) are automatically satisfied by any flat space solution of the Bogomol'nyi equations.

Then, the function $f(\mathbf{x})$ is uniquely determined as a solution of the equation

$$\nabla^2 f = -4\pi G(1+b^2) \text{tr} (D_i \Phi \cdot D_i \Phi), \quad (91)$$

which follows from the Einstein-Maxwell-dilaton equations. In the absence of S_{matter} , one recovers the (multi-)black hole solutions in cosmological Einstein-Maxwell-dilaton theory discussed in [54].

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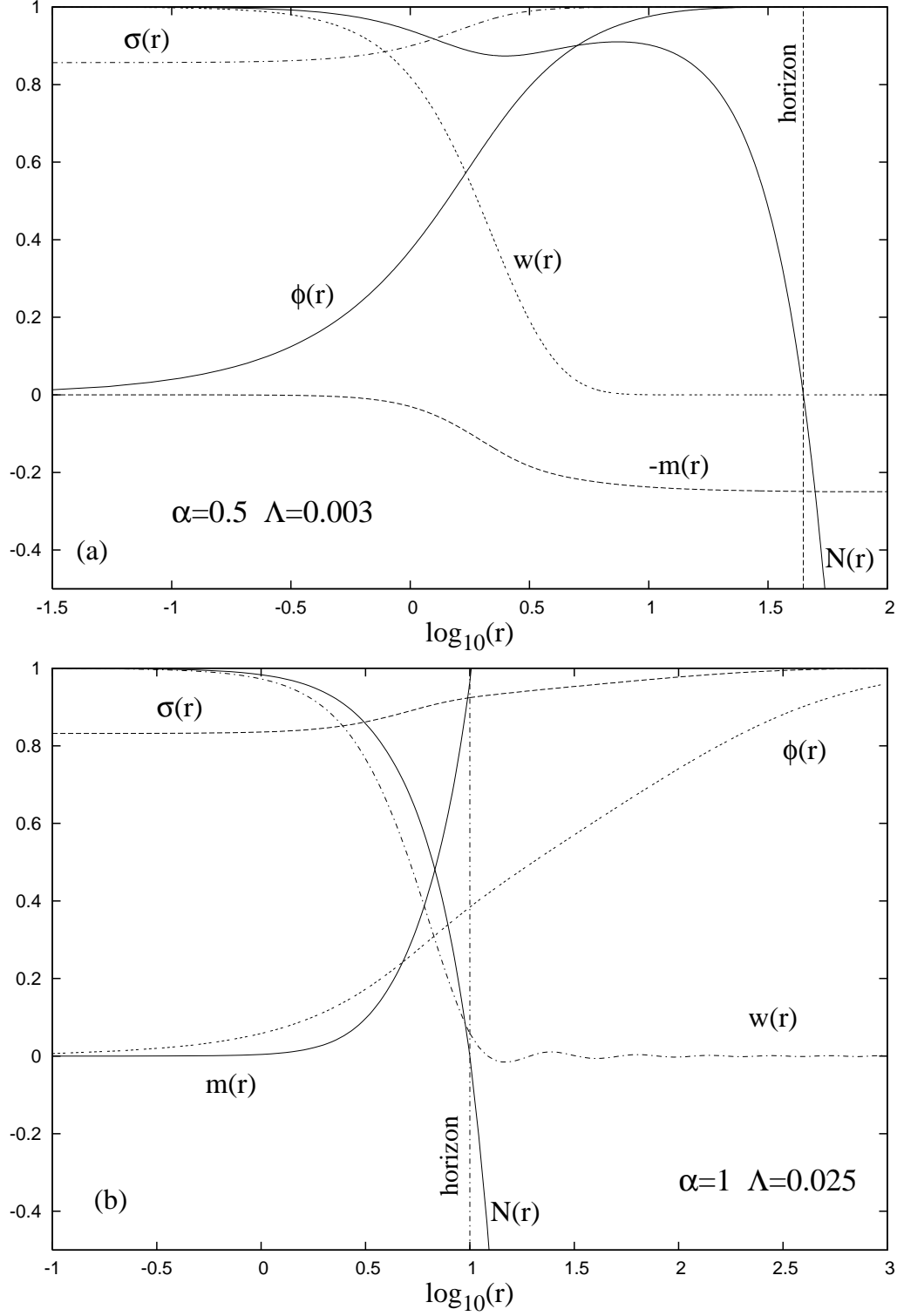


Figure 1: The profiles of two typical gravitating EYMH monopole solutions with a regular origin are plotted as a function of r for $\alpha = 1$. In (a), we show a solution on the first branch for $\alpha = 0.5$, in (b) a solution on the second branch for $\alpha = 1.0$. Here and in figures 7, 9 and 10 we have indicated also the position of the cosmological event horizon.

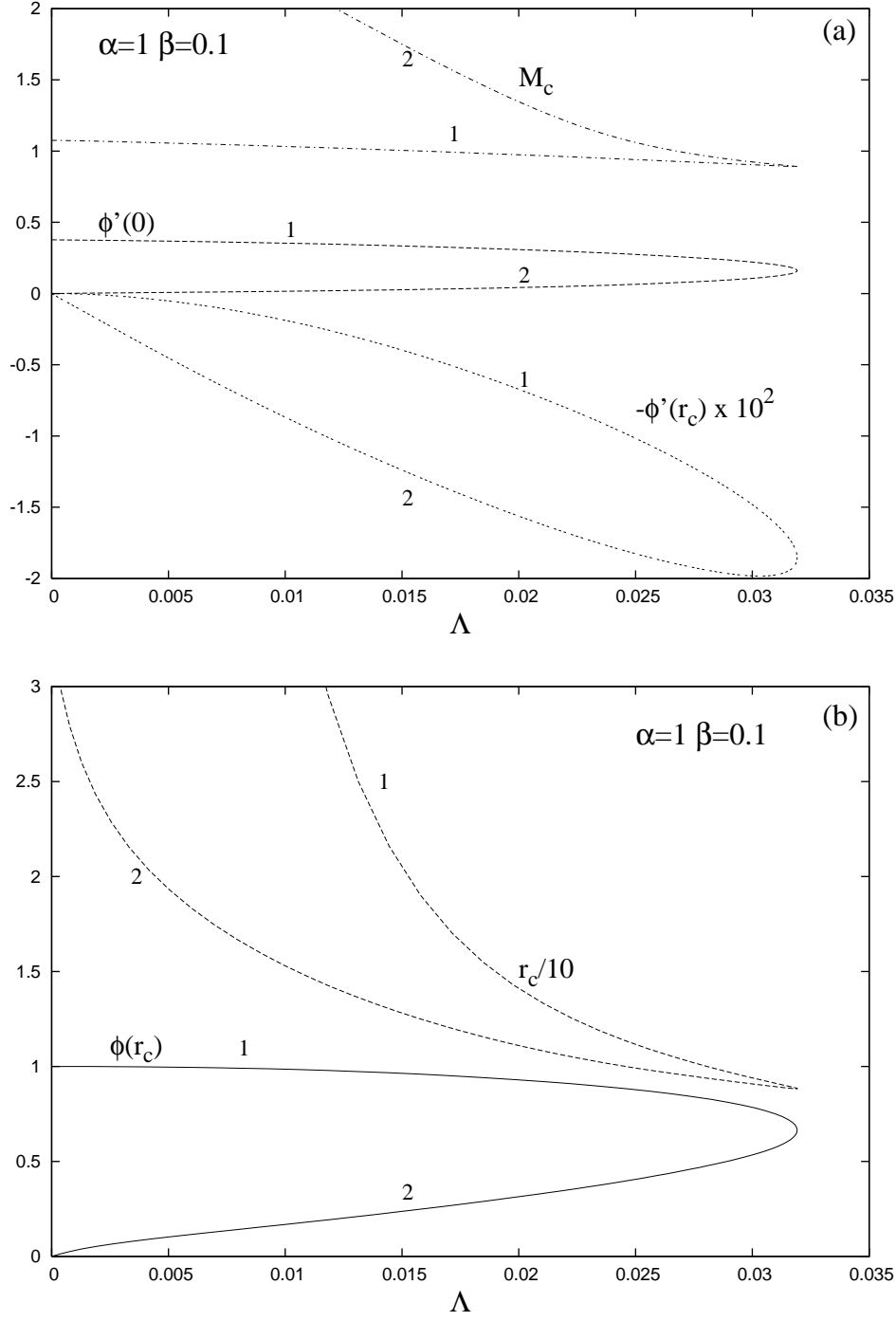


Figure 2: The dependence of solutions' properties on the value of the cosmological constant is plotted for particle-like monopole solutions. The labels 1 (resp. 2) refer to the first (resp. second) branch.

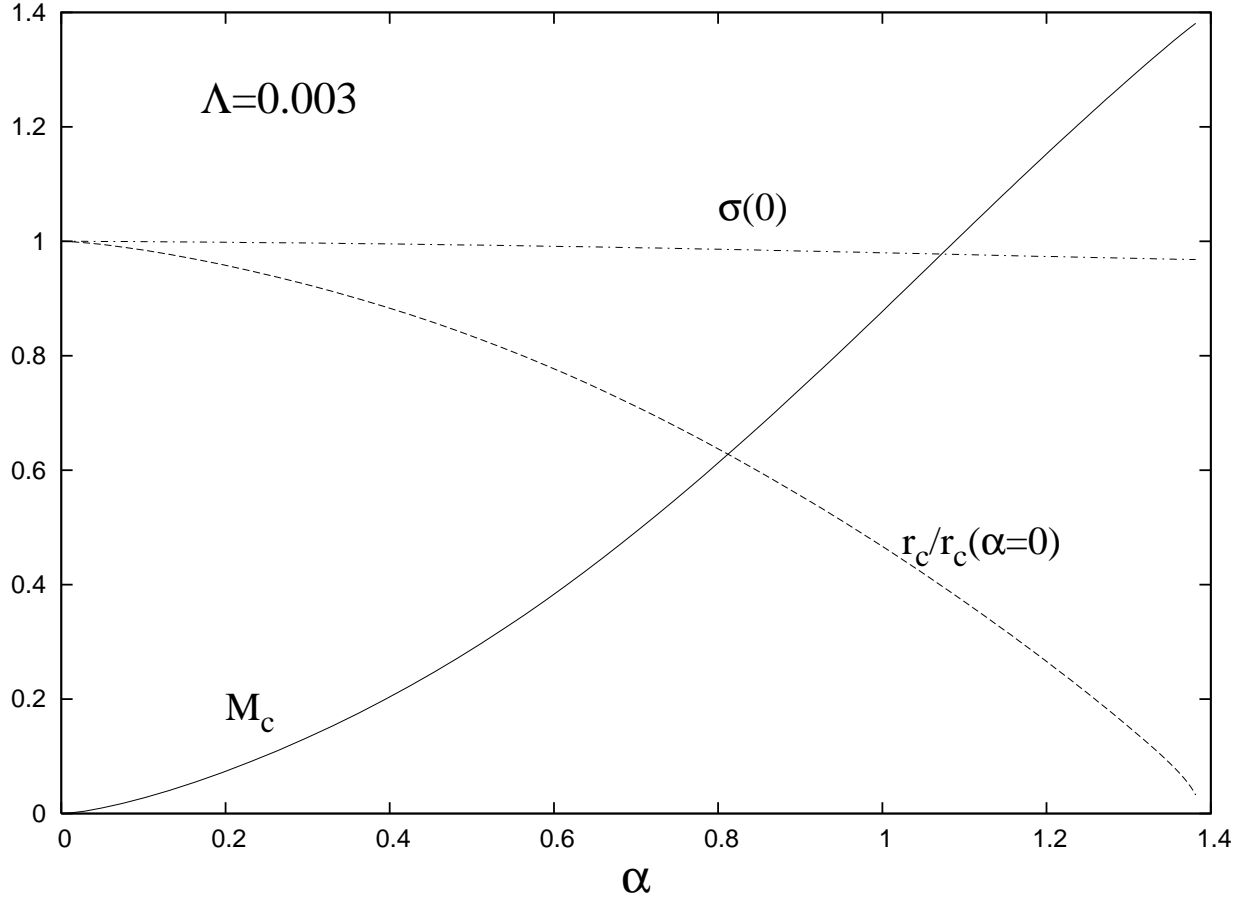


Figure 3: The mass inside the cosmological horizon M_c , the value of the metric function σ at the origin, $\sigma(0)$, and the value of the cosmological horizon r_c of the gravitating monopoles are given as functions of α for fundamental branch solutions with $\Lambda = 0.003$.

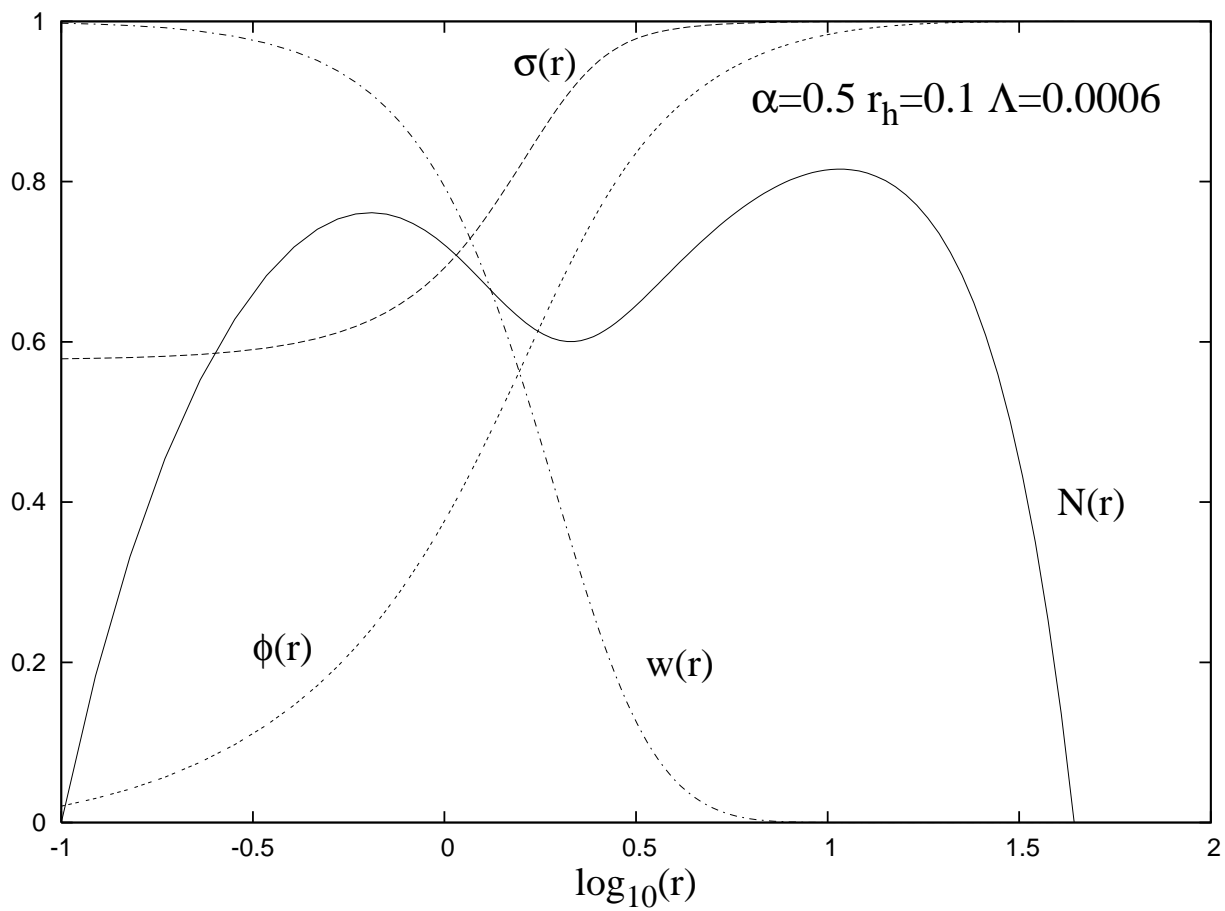


Figure 4: The profiles of the functions of the non-abelian black holes are given for $\alpha = 0.5$, $r_h = 0.1$ $\Lambda = 0.0006$.

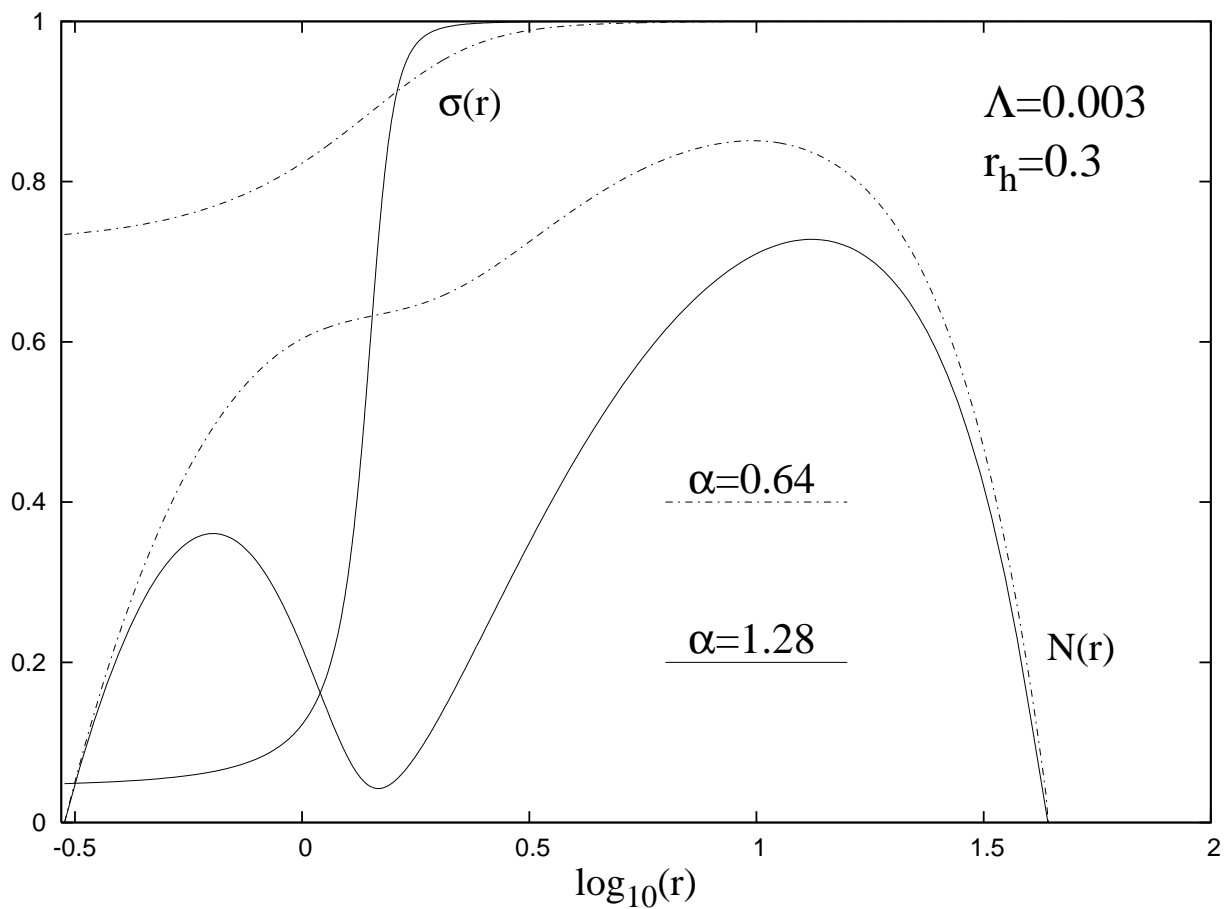


Figure 5: The metric functions N and σ of two EYMH black hole solutions with $r_h = 0.3$, $\Lambda = 0.003$ and two different values of α are shown as a function of r .

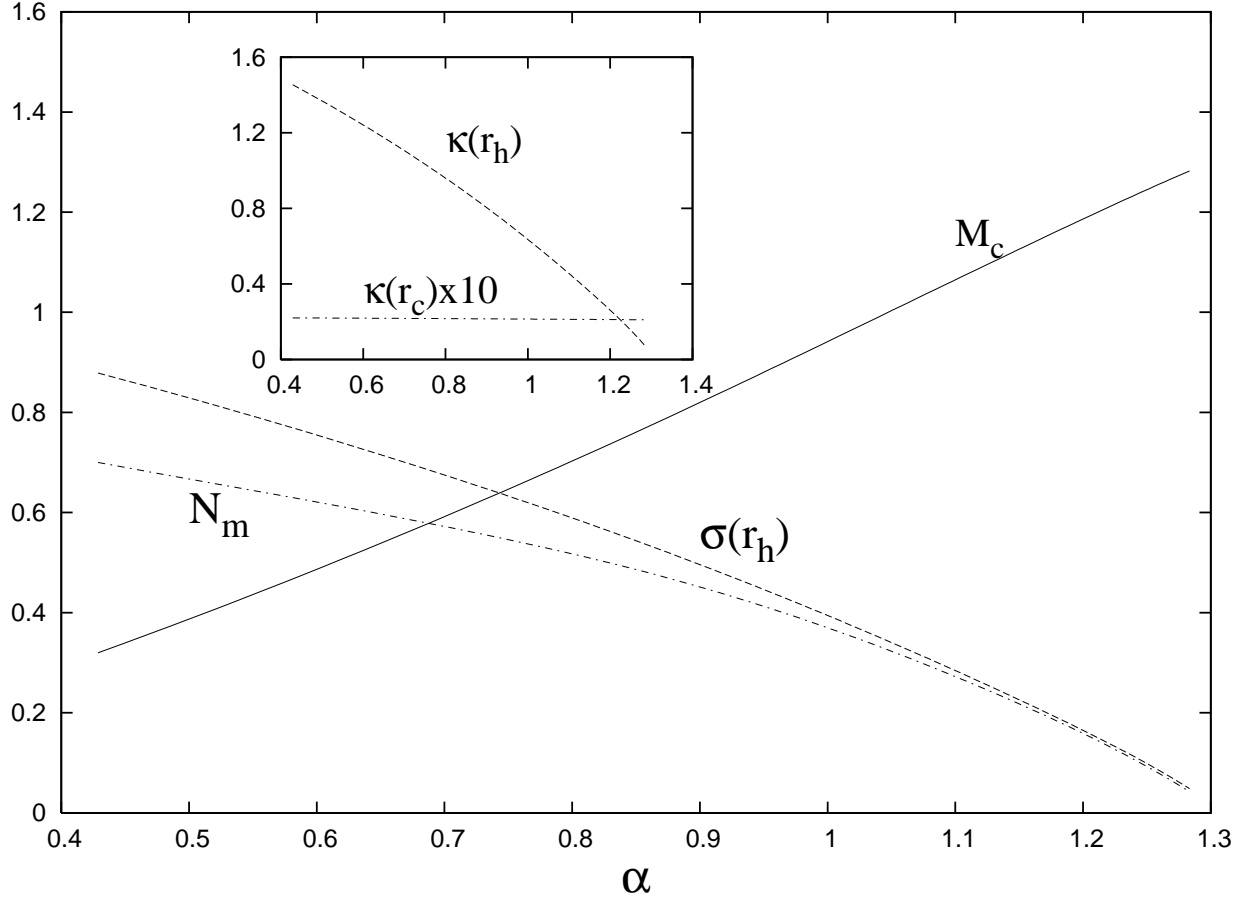


Figure 6: The surface gravity at the two horizons, the mass inside the cosmological horizon M_c and the minimal value N_m of the metric function N , are shown as functions of α for non-abelian EYMH black hole solutions with $r_h = 0.3$, $\Lambda = 0.003$.

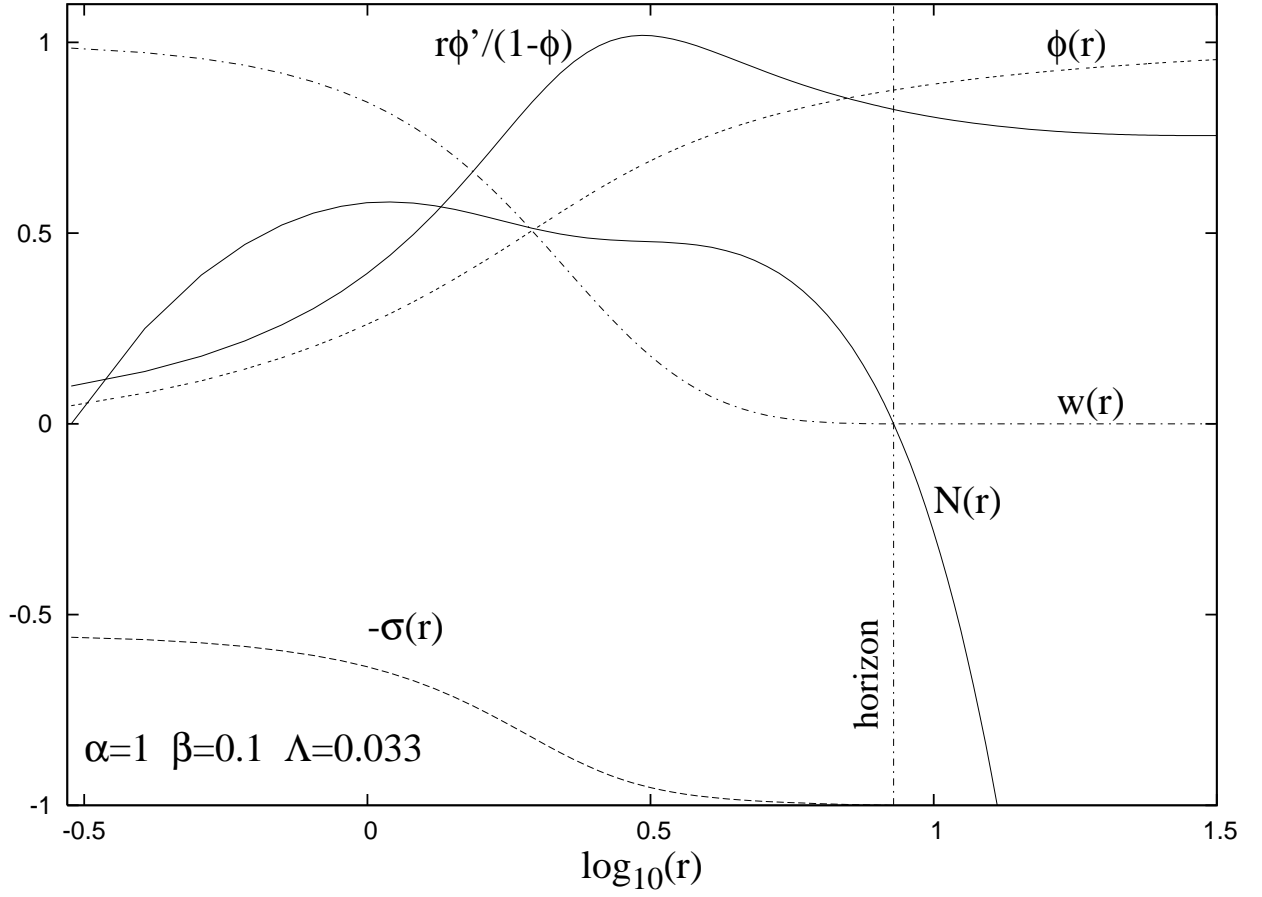


Figure 7: A typical black hole solution in EYM theory is plotted for $\alpha = 1$, $r_h = 0.3$ and $\Lambda = 0.033$. We also plot $r\phi'/(1 - \phi')$ which indicates the power with which ϕ reaches its asymptotic value.

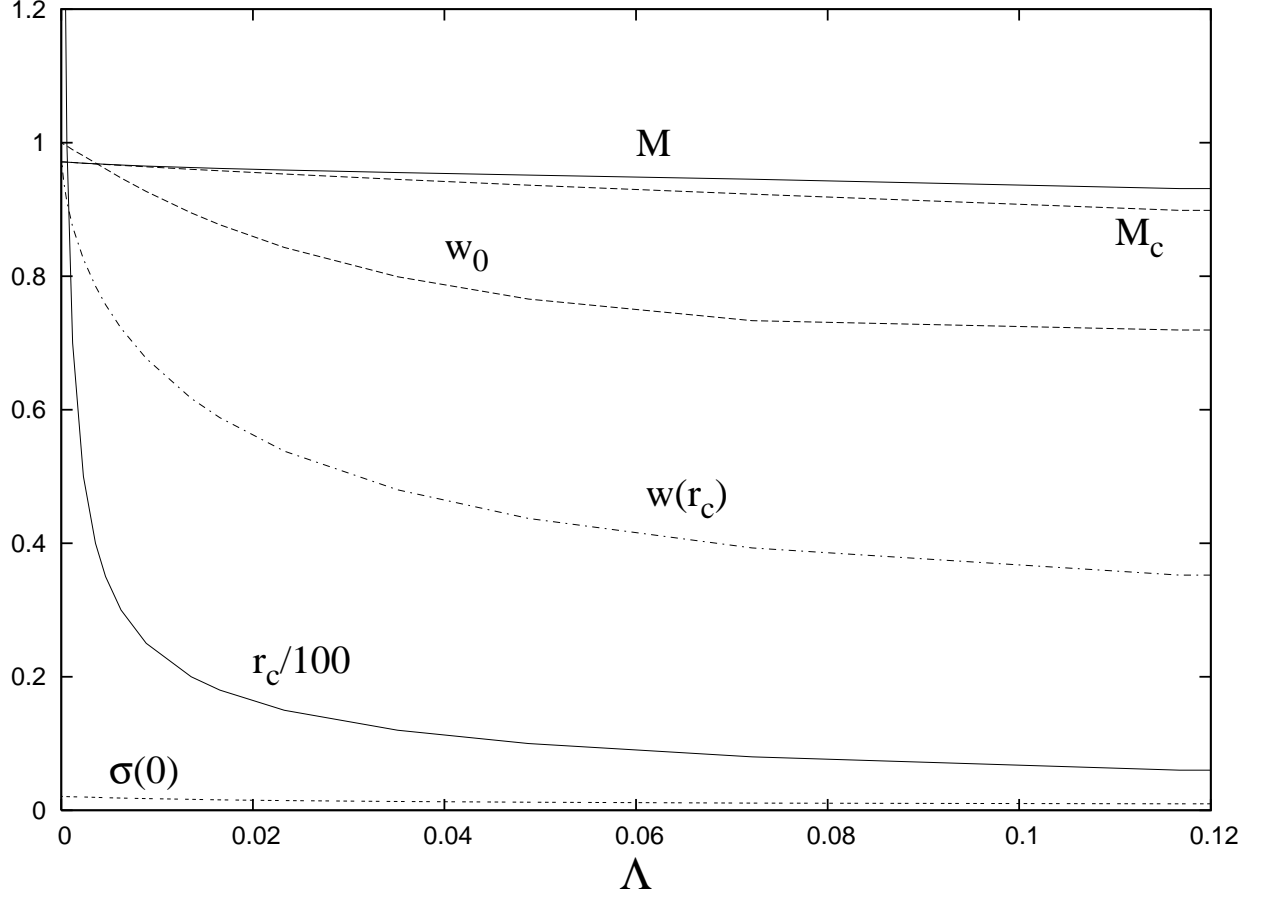


Figure 8: Several characteristic features of the two node particle-like solutions of the EYM- Λ system are plotted as a function of the cosmological constant. Here M and M_c are the values of the mass function $m(r)$ at infinity and at the cosmological horizon, r_c is the event horizon radius and $\sigma(0)$ is the value of the metric function σ at the origin. Also, w_0 and $w(r_c)$ are the values of the magnetic gauge potential at infinity and at the cosmological horizon.

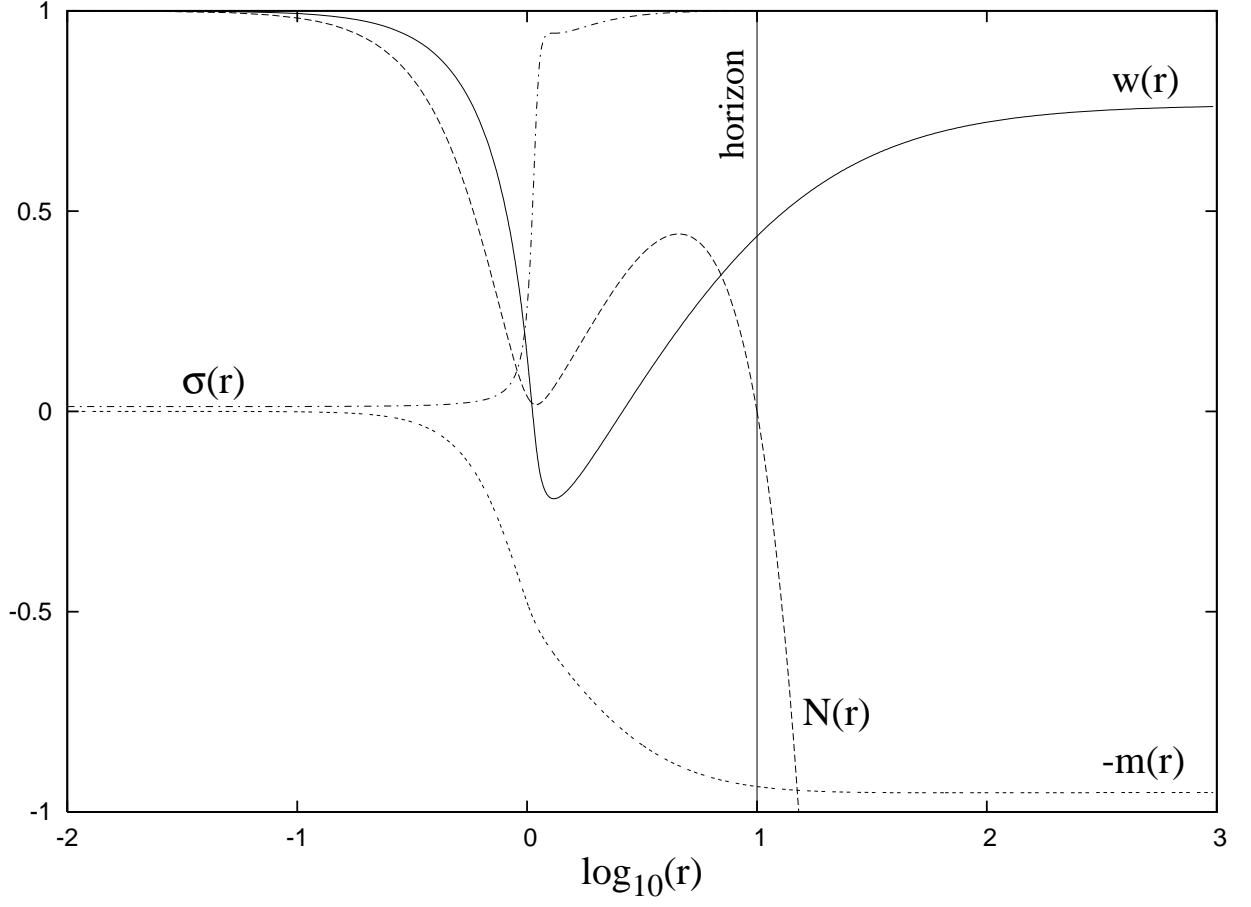


Figure 9: A typical asymptotically de Sitter monopole solution with a regular origin in EYM- Λ theory. The value at infinity of the magnetic gauge potential is $w_0 \simeq 0.761$.

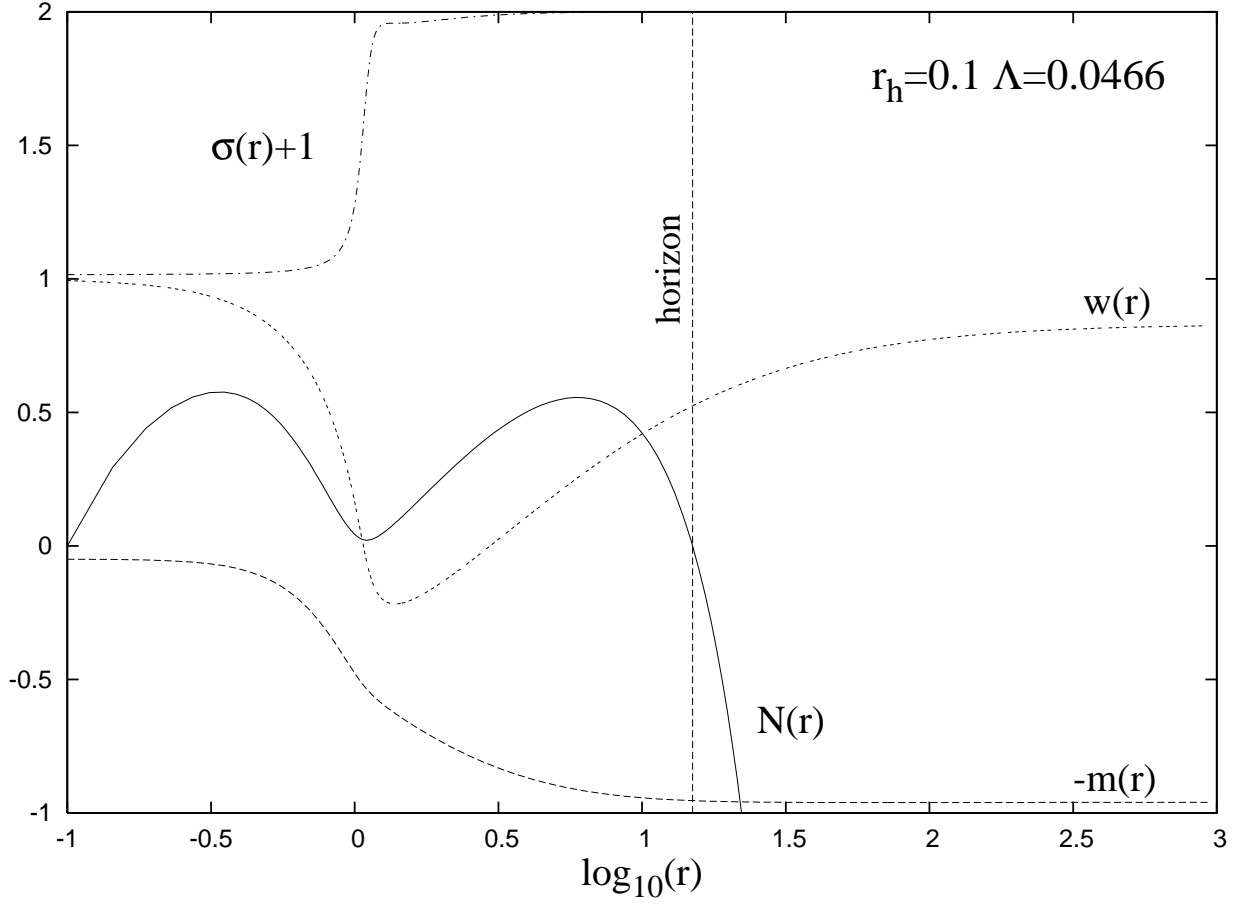


Figure 10: A typical asymptotically de Sitter black hole monopole solution in EYM- Λ theory. The cosmological horizon radius is $r_c = 15$. The value of the gauge field on the event horizon is $w_h = 0.9936$, while at infinity $w_0 = 0.8244$.