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Sums of powers vs. powers of sums

The remarkable identity

$$(1 + 2 + 3 + \ldots + n)^2 = 1^3 + 2^3 + 3^3 + \ldots + n^3$$

is not as singular as is often believed. Similar but less concise identites like

$$\Big(\sum_{k=1}^{n} k\Big)^3 = \frac{1}{4} \sum_{k=1}^{n} k^3 + \frac{3}{4} \sum_{k=1}^{n} k^5$$

date back to the 19th century. In joint work with G. Bennett we study non-integer exponents, in which case only inequalities are available. As an application we obtain the best constant in an inequality studied by R. P. Boas.