ADVANCES IN COMPUTATIONAL MULTIBODY DYNAMICS
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# APPLICATION OF EVOLUTIONARY STRATEGIES TO OPTIMAL DESIGN OF MULTIBODY SYSTEMS

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**Abstract**. *This paper is concerned with the optimization of the kinematic or dynamic behaviour of mechanical systems with the help of a multibody systems simulation tool. The authors first recall the classical form of an optimization problem, whose purpose is to find the set of design variables which minimizes a given objective function while verifying eventual constraints, and how kinematic or dynamic criteria can be transposed to this form. After having recalled the principal optimization techniques, the evolutionary strategies are presented in detail. They are inspired from the natural evolution: the genes of individuals mutate from generation to generation and the ones who survive are those that are the best fitted to their environment. The analogy with an optimization problem is quite straightforward, a set of design variables can be considered as the genes of an individual and the value of the objective function for this set of design variables represents the fitness for survival of the corresponding individual. Practically, the mutation is performed by modifying the design variables of $\mu$ parents, according to a normal distribution with zero as average and gives rise to $\lambda$ offsprings whose best $\mu$ individuals form the new parent population. The paper gives some indications for the choice of the principal parameters or options and explains how to manage the mutation in order to control the speed of convergence. The performances of the evolutionary strategies are then illustrated by two examples: the kinematic optimization of a suspension and the dynamic optimization of the comfort of a railway vehicle. Evolutionary strategies, although slower than hill-*

*climbing methods, are an interesting alternative. They indeed have several advantages: the optimization engine remains completely independent of the simulation one and can be adapted to any field of engineering, they are very robust and converge to global and not local optimal solutions.*

## 1 INTRODUCTION

Multibody simulation codes have become an inescapable tool for designing mechanical systems like, for example, manipulators or vehicles. Such tools allow engineers to virtually test the influence of several design parameters on the behaviour of the system in order to reach optimal performances while respecting technical constraints. However, if the number of design variables is important, it becomes hard to intuitively manage their evolution and a systematic optimization process is desirable. The aim of this paper is to present a systematic approach, based on evolutionary strategies, for the optimal kinematic or dynamic design of multibody systems under time-dependent constraints.

Haug[1] and his collaborators were certainly pioneers for the application of mathematical techniques to the optimization of the dynamic behaviour of multibody systems. Their approach, based on "hill-climbing" methods, especially emphasizes the calculation of the derivatives of the time-dependent functions defining the objective function or the constraints, and presents the major disadvantage that some parts must be added to the simulation code. The complexity of this task urged engineers to explore other types of methods like the ones based on stochastic principles, among which genetic algorithms and evolutionary strategies. These methods, inspired from the biological evolution have been advantageously applied in the field of structures[2–7] and were then tested for the kinematic and dynamic optimization of multibody systems[8,9]. This paper will focus only on the evolutionary strategies.

We will first recall the classical form of an optimization problem intended to find the set of design variables that minimize a so-called objective function, the design variables being otherwise subjected to inequality constraints. The principal types of methods likely to solve this kind of problem are then briefly exposed, with their advantages and drawbacks. After having presented how dynamic criteria can be transposed to the classical form, the evolutionary methods, based on successive mutation-selection processes are described in detail. Some guidelines are given for the choice of the principal options. Two examples, concerned respectively with the kinematic optimization of a suspension and the dynamic optimization of a railway vehicle, are presented for the sake of illustration.

## 2 GENERAL PROBLEM OF OPTIMIZATION

A constrained optimization problem consists in finding that set of $n_b$ design variables $\underline{\mathbf{b}}_{opt}$ leading to the minimal value of a so-called *cost-function* or *objective-function* $\psi_0(\underline{\mathbf{b}})$, expressed in terms of the design variables, while respecting $n_c$ inequality constraints on the design variables of the form $\psi_i(\underline{\mathbf{b}}) \leq 0$.

The methods able to solve such a problem can be classified in three principal categories:

- enumerative methods which systematically scan the whole range of design variables;
- deterministic or hill-climbing methods which attempt to reach the minimum through successive variations of the design variables according to the gradient of the objective function;
- stochatic methods, which try to reach the minimum by varying the design variables according to probabilistic rules.

The enumerative methods have of course the advantage of being very robust but are unpracticable for large problems as they would need a prohibitive computation time. Deterministic methods are computationally the most efficient but don't necessarily lead to the global optimum and require derivatives of the cost-function, which can't be obtained without adaptation of the software intended to evaluate the cost-function. Stochastic methods, which can also be interpreted as some kind of "organized enumerative methods", constitute a good compromise: they appear to be robust, easy to implement while requiring a reasonable computational burden.

## 3 OPTIMIZATION OF MULTIBODY SYSTEMS

When applied to multibody systems, optimization encounters peculiar difficulties as the objective function is often related to the evolution of a physical value during a given motion. For example, it is asked to minimize the maximum acceleration of the passengers during the maneuver of a transport vehicle. The objective function is said to be *time-dependent* and can be written in the following form

$$\psi_0(\underline{\mathbf{b}}) = \max_{t\in[0,\tau]} f_0(\underline{\mathbf{b}}, \underline{\mathbf{q}}, \underline{\dot{\mathbf{q}}}, \underline{\ddot{\mathbf{q}}}, t) \tag{1}$$

with $\tau$ the simulation duration, $f_0$ the physical value of interest and $\underline{\mathbf{q}}$, $\underline{\dot{\mathbf{q}}}$ and $\underline{\ddot{\mathbf{q}}}$ the vector of configuration parameters of the multibody system and its first and second time-derivatives.

The constraints themselves are also the most often time-dependent. They can represent for example a distance condition between bodies so as to avoid interferences. Such a constraint can be expressed by

$$\phi(\underline{\mathbf{b}}, \underline{\mathbf{q}}, \underline{\dot{\mathbf{q}}}, \underline{\ddot{\mathbf{q}}}, t) \leq 0 \quad t \in [0, \tau] \tag{2}$$

In order to simplify its mathematical treatment and particularly the determination of derivatives[1,11], the constraint is generally transformed into an equivalent time independent form $\psi$, according to

$$\psi = \int_0^\tau < \phi(t) > dt \tag{3}$$

where the *equivalent operator* $<>$ is defined by

$$< \phi(t) >= \begin{cases} \phi(t) & if\ \phi(t) \geq 0 \\ 0 & if\ \phi(t) < 0 \end{cases} \tag{4}$$

The design variables can also be subjected to simpler constraints, called boundary constraints expressing for example that a physical characteristic, like a stiffness or damping coefficient must remain between lower and upper boundaries $\underline{\mathbf{b}}^L$ and $\underline{\mathbf{b}}^U$

$$\begin{aligned} \underline{\mathbf{b}}^L - \underline{\mathbf{b}} &\leq 0 \\ \underline{\mathbf{b}} - \underline{\mathbf{b}}^U &\leq 0 \end{aligned} \tag{5}$$

Deterministic and stochastic methods have been systematically tested by the first author[8] on several representative systems. The use of deterministic methods, like Powell or Fletcher-Reeves, was based on the work of Haug[1] and Schittkowski[11]. They principally developed a method to compute the derivatives of the objective (1) and constraint (3) functions with respect to the design variables, through so-called adjoint variables. Although it gives interesting results, it becomes delicate for highly damped systems[8] and has the major disadvantage that the simulation software must be deeply reprogrammed in order to be able to yield all the necessary growths to the optimization procedure. It is also unpracticable for people disposing only of a commercial software. Moreover, it may converge to only a local optimum.

That is why the first author turned to stochastic methods, and particularly genetic algorithms and evolutionary strategies, although they intially inspire some septicism to the rational spirit of an engineer. These methods are indeed simpler as they only require the evaluation of the objective function and of the constraints which even needn't to be transformed to the time-independent form. Evolutionary strategies will be the subject of this paper. Developments about genetic algorithms will be found in other contributions[8,10].

## 4 EVOLUTIONARY ALGORITHMS

### 4.1 Introduction

The evolutionary strategies are iterative optimization methods which try to find an optimal solution from stochastic and small variations of the design variables. The book of Schwefel[12] constitutes a basic reference, some more recent developments being described by Fogel[13]. Evolutionary strategies are based on the principles of natural selection: the individuals of a species mutate from generation to generation by small variation of their genes, and only the best fitted to their environment will survive and be selected for further reproduction. Although other phenomenons intervene in the natural selection, mutation and selection are recognized by Darwin to be most important. The analogy with the optimization problem is quite straightforward, a set of design variables can be considered as the genes of an individual and the value of the objective function for this set of design variables represents the fitness for survival of the corresponding individual. As we are concerned with a minimization process of the objective function, the fitness for survival will be the higher as the objective function is lower. Of course, if the constraints are violated, the individual is supposed to have very low or no fitness at all for survival and will rarely or never be selected.

The basic implementation of the evolutionary strategy is composed of the following steps:

1. An initial population of $\mu$ parent individuals is selected at random and uniformly in the feasible range of each design variable. Ideally, each initial parent should verify the constraints.

2. A population of $\lambda$ offsprings is created from mutation of the parents. For each offspring, a parent is selected randomly, and each of its design variable $b_i$ is mutated by adding a Gaussian random variable with zero mean and preselected standard deviation $\sigma_i$

$$b_i(\text{offspring}) = b_i(\text{parent}) + N(0, \sigma_i) \quad i = 1, n_b \tag{6}$$

   As we will develop later, the standard deviation $\sigma_i$ may be chosen for the whole population or may be linked to the parent. By chosing a Gaussian distribution, we insure that, like in the nature, small changes occur frequently while large ones very rarely.

3. The new parents are selected as the $\mu$ individuals with the best fitness, that's to say with the lowest cost function. The new parents may be chosen either from the set of parents and offsprings (*plus* strategy $\mu + \lambda$) or only from the offsprings (*comma* strategy $\mu, \lambda$).

4. The process of mutation-selection continues until a sufficient solution is reached. The convergence criterion will be explained later.

Let us note that the plus strategy with one parent and one offspring (1+1) is referred to as the two membered evolutionary strategy.

### 4.2 Size of the population

There is so far no accurate rule to determine the size of the parent population. A good indication is to have as many or a few times as many members as the number of design variables.

On the other hand, some theoretical studies have been realized on (1,$\lambda$) strategies[12] to estimate the optimal ratio $\lambda/\mu$. It has been shown that this optimal ratio depends on the objective function and increases with its complexity. A ratio $\lambda/\mu$ equal to 5 can be considered as a good starting point.

### 4.3 Step length control

The standard deviation $\sigma_i$ plays an important role as it permits to control the speed of convergence. As it also corresponds to the mean variation of the corresponding parameter, it is often called the step length. Like in all optimization procedures, the control of the step length is the most important part of the algorithm after the recursion formula.

The theoretical study of the two membered evolutionary strategy[12] leads to the so-called success rule of Rechenberg, stated as

> *After every $n_b$ (number of design variables) iterations, check how many sucesses have occurred over the preceding 10 $n_b$ mutations. If this number is less than 2 $n_b$, multiply the step lengths by the factor 0.85; divide them by 0.85 if more than 2 $n_b$ successes occurred.*

For the initial value of the step lengths, Schwefel proposes to use the following estimation

$$\sigma_i^0 = \frac{\Delta b_i}{\sqrt{n_b}} \tag{7}$$

where $\Delta b_i$ is the expected distance from the optimum for the corresponding design variable. The accuracy on this initial value is not critical as the success law seems to rapidly adapt the step length to a suitable value, at least when it is too large.

The success law itself doesn't allow to change independently the standard deviations relative to each design variable, and consequently to adapt to the contours of the objective function. One possibility is to consider the standard deviation of each design variable as a supplementary gene, also likely to undergo some mutations, for example by multiplying it during the transition from parent to offspring by a random number of mean 1. We have preferred here another alternative based on the recombination of the parents after mutation. Let us note that in this context, each initial parent has its own initial step lengths, chosen randomly in a given range. The recombination will be explained later.

### 4.4 Comma or Plus Strategy

An other option of the evolutionary strategies concerns the choice between comma or plus strategies. The plus strategy has the advantage to assure no worsening of the objective function. On the other hand, the comma strategy is recognized to have better adaptation properties with regard to step length.

As the probablility of worsening becomes very low when the ratio $\lambda/\mu$ is large, Schwefel[12] recommends to use the comma strategy when this ratio is larger than 5.

### 4.5 Recombinations

The basic evolutionary strategies can be enriched by adding a supplementary step of recombination between the parents before mutation. Pairs of parents are randomly selected and are recombined to yield a new set of $\mu$ parents for mutation. The classical options are the following

- arithmetic recombination: the design variables or standard deviations of the new parent are the mean arithmetic value of the corresponding parameters of the two selected parents;
- discrete recombination: the design variables or standard deviations of the new parent are randomly chosen from one or the other selected parent, with equal probablity;
- geometric recombination: the design variables or standard deviations of the new parent are the geometric mean value of the corresponding parameters of the two selected parents.

Recombinations introduce the principle of sexual propagation which is expected to be very favourable for evolution as only few primitive organisms do without it. As mentioned before, it also offers the possibility to independently vary the step lengths of each design variable.

### 4.6 Convergence criteria

In the two membered strategy, the convergence criterion is based on the evolution of the best value of the objective function along generations. The optimum is assumed to be reached as far as the best value hasn't significantly changed in the last generations.

With the multimembered strategy, the criterion still becomes simpler. We can indeed consider that the optimum has not been reached as far as the best individuals of a generation differ too much with respect to their objective function values. If we denote $\psi_{0,min}$ and $\psi_{0,max}$ the minimal and maximal values of the objective functions inside a given parent generation, the iterative process will be stopped if

$$|\psi_{0,min} - \psi_{0,max}| < \epsilon \tag{8}$$

A relative error criterion can also be used, as

$$|\psi_{0,min} - \psi_{0,max}| < \epsilon_r \bar{\psi}_0 \tag{9}$$

where $\bar{\psi}_0$ is the mean value of the objective function in the considered generation.

To avoid endless processes, it is also safe to impose a maximum number of iterations after which the optimization is automatically stopped, and eventually adapted for further investigations.

### 4.7 Constraints

Ideally, the intial parent population should verify the constraints. However, it can be difficult to find a uniformly distributed initial population which respects all the constraints. This condition can then be dropped, the constraints being taken into account through a penalty of the objective function as soon as a constraint is violated. In this way, the selection generally makes the parent population licit in a few generations. Although simple, this approach wastes computation time in the first generations and introduces the risk of having a population coming only from a small number of licit initial parents.

That is why the problem of finding an initial licit population is often substituted to an optimization problem whose cost function $\psi'_0$ corresponds to the sum of the violated constraints

$$\psi'_0(\underline{\mathbf{b}}) = \sum_{j=1}^{n_c} \psi_j(\underline{\mathbf{b}}) \cdot \delta(\psi_j(\underline{\mathbf{b}})) \tag{10}$$
$$\text{with } \delta(x) = \begin{cases} -1 & \text{if } x < 0 \\ 0 & \text{otherwise} \end{cases}$$

If an illicit starting point arises during the construction of the initial population, the process is performed from that point, until all the constraints are satisfied. This initialization step is easy to implement, as it uses only available tools, and yields a well distributed population.

## 5 KINEMATIC OPTIMIZATION - SUSPENSION

The potentialities of the evolutionary strategies will be first illustrated on the example of the short-long arm front suspension represented in figure 1.

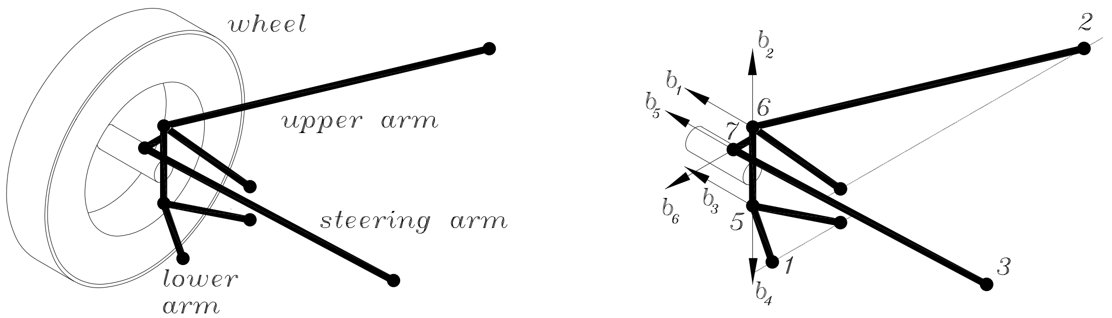


Figure 1: Short-long arm suspension

It is composed of the lower and upper control arms and the steering arm connected to the wheel carrier and the body frame. Athough the design of a suspension must take into account several criteria, we will here only try to minimize the variation of the toe angle during the vertical motion of the suspension, the steering wheel (node 3) being assumed to be fixed. There will be 6 design variables, corresponding to coordinates of particular nodes of the suspension. They are detailed in table 1 with their variation range, the nodes being shown in figure 1. There are no time-dependent constraints.

Let us note that this example, related to a kinematic optimization cannot be treated in the same manner as a dynamic optimization if the method of adjoint is used. It makes no difference in the case of stochastic methods.

| | |
|---|---|
| lateral coordinate of node 6 | $0.1207 \leq b_1 \leq 0.2007$ m |
| vertical coordinate of node 6 | $0.1830 \leq b_2 \leq 0.3030$ m |
| lateral coordinate of node 5 | $0.1167 \leq b_3 \leq 0.1967$ m |
| vertical coordinate of node 5 | $0.1167 \leq b_4 \leq 0.1967$ m |
| lateral coordinate of node 7 | $0.0900 \leq b_5 \leq 0.2100$ m |
| longitudinal coordinate of node 7 | $0.2965 \leq b_6 \leq 0.4165$ m |

Table 1: Design variables and their variation range

An evolutionary strategy of type (10,20) with recombination has been used, the maximum number of iterations being fixed to 200. The initial step lengths were chosen randomly between 0 and 1 for all parameters. The recombination was discrete for the design variables and arithmetic for the step lengths. These parameters are summarized in table 2.

| | | | |
|---|---|---|---|
| Number of parents ($\mu$) | 10 | $\sigma_{min}$ | 0 |
| Number of offsprings ($\lambda$) | 20 | $\sigma_{max}$ | 1 |
| Number of iterations | 200 | Recombination for $\underline{\mathbf{b}}$ | discrete |
| Type of strategy | ($\mu$, $\lambda$) | Recombination for $\sigma$ | arithmetic |

Table 2: Parameters of the evolutionary algorithm

The optimum could be considered to be reached at the 200th iteration. The design variables of the best individual are given in table 3. The optimization process required 4010 evaluations of the objective function and lasted around 3 hours on a Pentium 120 Mhz.

| | |
|---|---|
| $b_1 = 0.193$ m | $b_4 = 0.160$ m |
| $b_2 = 0.302$ m | $b_5 = 0.204$ m |
| $b_3 = 0.182$ m | $b_6 = 0.405$ m |

Table 3: Optimal set of design variables

The process has clearly decreased the extrema of the toe angle, whose evolution is shown in figure 2 before and after optimization. On the other hand, the shape of the curve remains the same as the chosen design variables don't really allow to modify it.

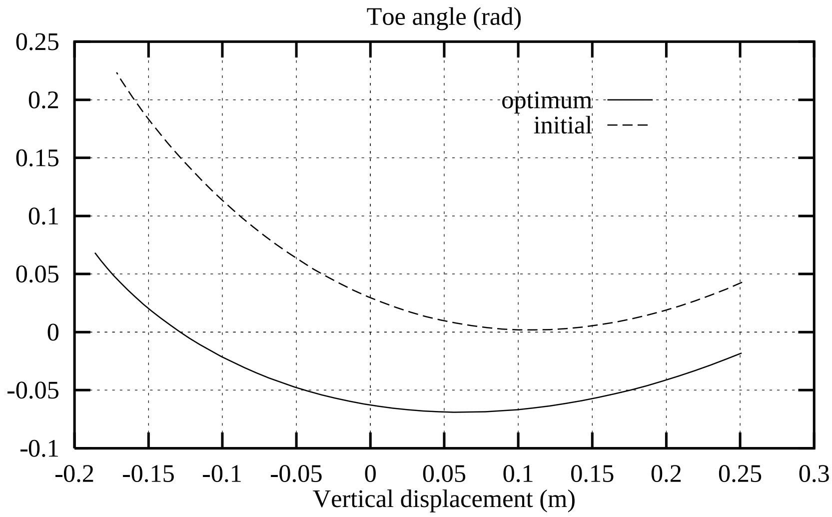


Figure 2: Evolution of the toe-angle

## 6 DYNAMIC OPTIMIZATION - RAILWAY VEHICLE

The evolutionary strategy has been applied on the example of the railway vehicle depicted in figure 3, called the *cityrunner*. This vehicle is characterized by the fact that it owns more carbodies than bogies, with the goal of having a vehicle able to ride in networks with very narrow curves, as in old european cities. As the central carbody, also called bridge car, is not directly related to the rail, its motion and consequently the comfort of the passengers are driven by the global dynamics of the vehicle, itself dependent upon global characteristics like mass distribution and suspension parameters. The problem is to find a design which satisfies or even optimizes comfort and stability of the vehicle.

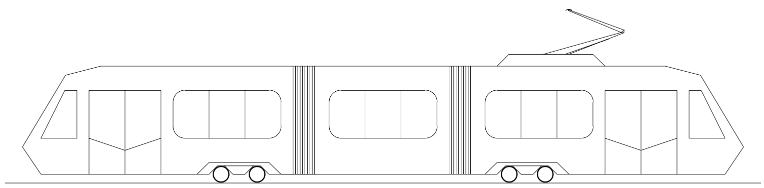

Figure 3: Railway vehicle cityrunner

The mass distribution being constrained by other considerations, the remaining parameters are the suspension ones, in particular the yaw and lateral stiffness and damping coefficients of the bogie-carbody suspensions and the damping coefficients of the articulations between the cars. It is easy to imagine that the influence of these parameters is

| | |
|---|---|
| Yaw secondary stiffness, front bogie | $5 \leq b_1 \leq 500$ KNm/rad |
| Yaw secondary stiffness, rear bogie | $5 \leq b_2 \leq 500$ KNm/rad |
| Yaw secondary damper, front bogie | $5 \leq b_3 \leq 50$ KNms/rad |
| Yaw secondary damper, rear bogie | $5 \leq b_4 \leq 50$ KNms/rad |
| Yaw damper, front car articulation | $5 \leq b_5 \leq 50$ KNms/rad |
| Yaw damper, rear car articulation | $5 \leq b_6 \leq 50$ KNms/rad |
| Lateral secondary stiffness, front bogie | $100 \leq b_7 \leq 1000$ KN/m |
| Lateral secondary stiffness, rear bogie | $100 \leq b_8 \leq 1000$ KN/m |
| Lateral secondary damper, front bogie | $5 \leq b_9 \leq 30$ KNs/m |
| Lateral secondary damper, rear bogie | $5 \leq b_{10} \leq 30$ KNs/m |

Table 4: Design variables and their variation range

| | | | |
|---|---|---|---|
| Number of parents ($\mu$) | 10 | $\sigma_{min}$ | 0 |
| Number of offsprings ($\lambda$) | 30 | $\sigma_{max}$ | 1000 |
| Number of iterations | 100 | Recombination for $\underline{\mathbf{b}}$ | discrete |
| Type of strategy | ($\mu$, $\lambda$) | Recombination for $\sigma$ | arithmetic |

Table 5: Design variables and their variation range

highly coupled so that it is very difficult to intuitively manage the process. Different optimization procedures have then been applied to this example, among which evolutionary strategies.

The objective function corresponds to the mean lateral acceleration undergone by the centers of gravity of the carbodies while crossing a 90 degrees curve of radius 14 $m$, at the speed of 4 $m/s$. The design variables correspond to the suspension parameters. They are detailed in table 4, as well as their admissible range. The process is subjected to 2 constraints, limiting the relative angle of the cars.

The parameters of the chosen evolutionary strategy are similar to those of the previous example and are summarized in table 5.

The optimum was reached at the 98th iteration and led to the values given in table 6.

| | |
|---|---|
| $b_1 = 8.648$ KNm/rad | $b_6 \leq 29.661$ KNms/rad |
| $b_2 \leq 5.431$ KNm/rad | $b_7 \leq 32.022$ KN/m |
| $b_3 \leq 29.070$ KNms/rad | $b_8 \leq 3.364$ KN/m |
| $b_4 \leq 6.350$ KNms/rad | $b_9 \leq 18.539$ KNs/m |
| $b_5 \leq 7.408$ KNms/rad | $b_{10} \leq 29.809$ KNs/m |

Table 6: Optimal solution

The optimization process required 3010 evaluations of the objective function and lasted around 10 hours on a Pentium 120 Mhz.

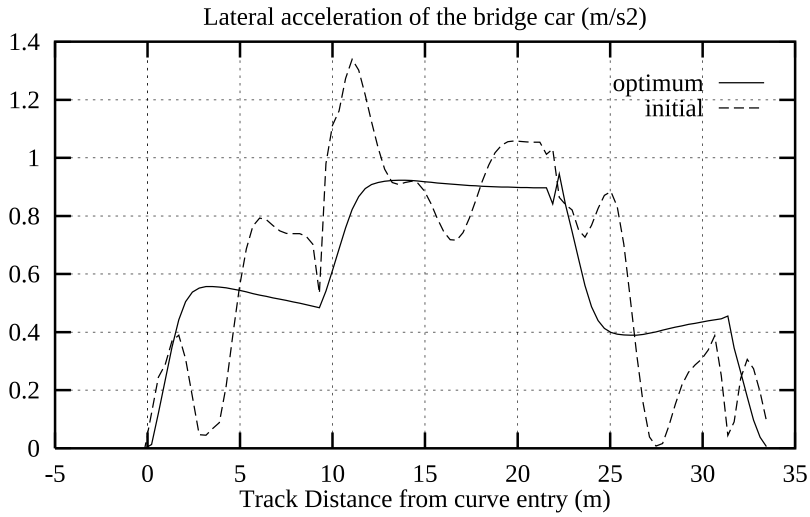


Figure 4: Evolution of the acceleration of the bridge car

The optimum objective function is worth 2.71 $m/s^2$. The evolution of the acceleration of the center of gravity of the bridge car is presented in figure 4 for the initial and optimal cases and shows how the optimization process has polished the acceleration peaks at the beginning and end of the curve.

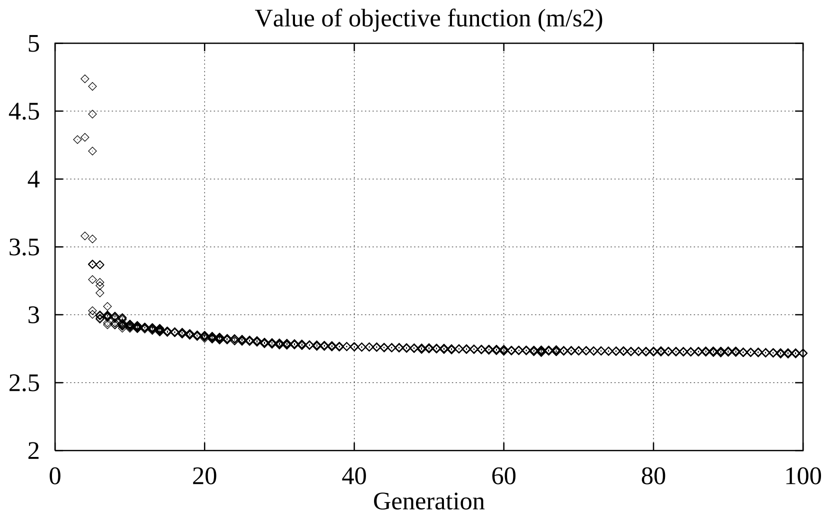


Figure 5: Evolution of the objective function of the parent population

Figure 5 shows the evolution of the objective function of the parent population with respect to the number of generations. At the beginning, the high values are due to the penalty that has been used to take the constraints into account. After less than 10 generations, all the individuals verify the constraints and the objective function slowly evolves to the optimum. The population is quite homogeneous as there are few variations of the objective function inside a given population.

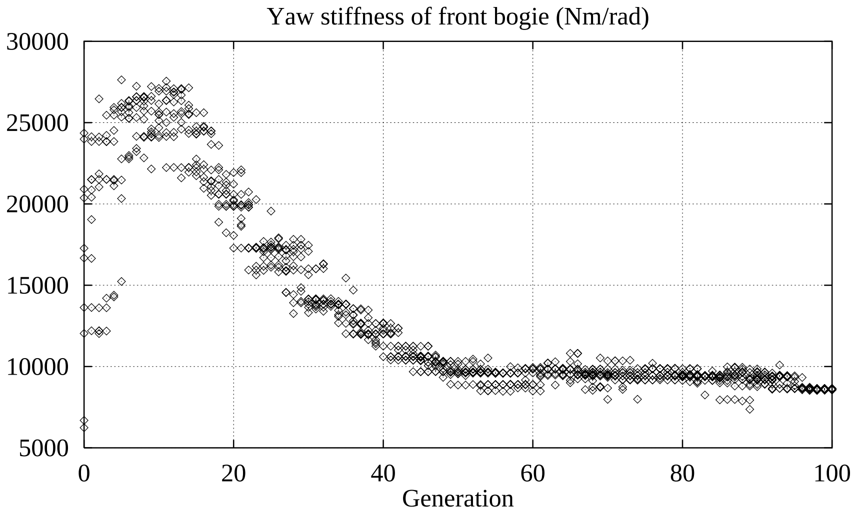


Figure 6: Evolution of the yaw stiffness of the front bogie

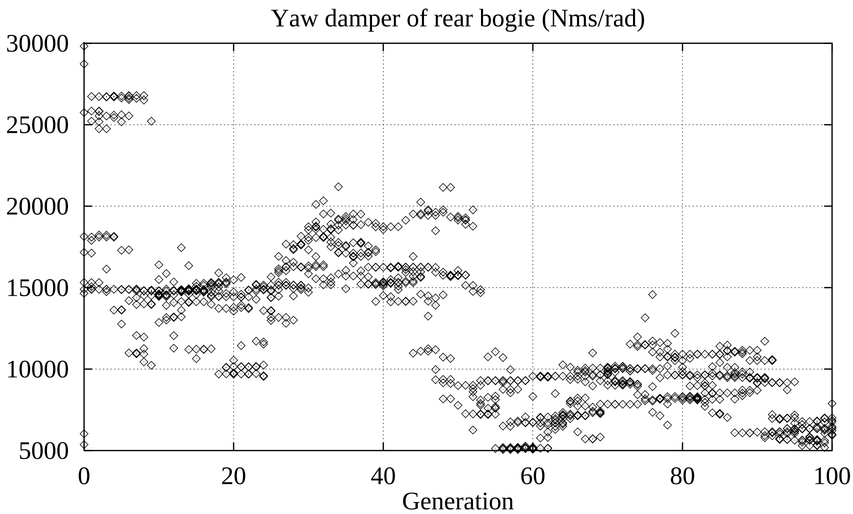


Figure 7: Evolution of the yaw damping of the rear bogie

Figures 6 and 7 show two typical evolutions of design variables, corresponding respectively to the yaw stiffness of the front bogie and the yaw damping of the rear bogie. In figure 6, the population starts from values distributed between 5000 and 30000, regularly decreases until fiftyth generation, and is then stabilized around a value of 10000, certainly forced by one of the constraints. The variation range also rapidly becomes limited. In figure 7, another behaviour arises: while oscillating around the value of 15000 until fortyth generation, the population progressively switches to a mean value between 5000 and 10000. The variation is larger, showing that the design variable has less influence on the behaviour of the system.

## 7 CONCLUSION

This paper presented the application of evolutionary strategies to the kinematic and dynamic optimization of multibody systems. The method has been described and two examples have shown its capabilities.

The initial scepticism of the authors with regard to these strange stochastic methods rapidly became a real enthusiasm. Although classical hill-climbing methods are globally more efficient, evolutionary strategies appear to be much more robust, are much easier to use and can use the simulation software as is. Consequently, the optimization engine can also be used for any engineering problem. Genetic algorithms, which are another kind of stochastic method, were also tested by the autors and lead to comparable results. Stochastic methods can then be considered as a better alternative for the transfer of technology.

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