Skyrmion and Skyrme-Black holes in de Sitter spacetime

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Abstract

Numerical arguments are presented for the existence of regular and black hole solutions of the Einstein-Skyrme equations with a positive cosmological constant. These classical configurations approach asymptotically the de Sitter spacetime. The main properties of the solutions and the differences with respect to the asymptotically flat ones are discussed. It particular our results suggest that, for a positive cosmological constant, the mass evaluated as timelike infinity in infinite. Special emphasis is set to De Sitter black holes Skyrmions which display two horizons.

1 Introduction

It is almost two decades than the first examples of hairy black holes are known [1]. This early construction of gravitating objects presenting both, an event horizon and a non trivial structure of the matter fields, was first achieved in a context where the matter fields are described in terms of a non-linear sigma-model, for instance the Skyrme model [2]. Initially, the Skyrme model was proposed more than forty years ago as an effective model for chiral symmetry breakdown in quark models; the main fields are the *pion* particles and soliton-like solutions of the equations -the Skyrmions- are interpreted as the *nucleons*; see e.g. [3] for a complete and recent review of the topic.

Although the initial purpose of the Skyrme model was far from being coupled to gravity, the classical equations resulting from its coupling to gravity constitute a rich system of equations where both (stable) gravitating solitons and hairy black holes solutions exist. These were studied in great details in [4, 5], a recent review of these solutions and motivations can be found in [6]. When space-time is imposed to be asymptotically flat, the gravitating skyrmions exist in two branches indexed by an effective coupling constant $\alpha^2 \equiv 4\pi GF_{\pi}$ where G denotes Newton's constant and F_{π} , the pion decay constant, is the coupling constant of the standard Skyrme model. The two branches merge at a maximal value, say α_{max} . The solution with the lowest energy smoothly approaches the flat Skyrmion in the $\alpha \to 0$ -limit and is known to be stable on the basis of topological arguments.

Recently, the Einstein-Skyrme model was reconsidered by supplementing the equations with a negative cosmological constant [7] and strong numerical evidence was given that asymptotically anti-DeSitter hairy black hole Skyrmion exist as well. More precisely, the authors of Ref. [7] have shown that gravitating Skyrmion solution exist with a metric approaching asymptotically the Anti-deSitter space for values of the cosmological constant $|\Lambda|$ lower than a maximal value, say $|\Lambda| = |\Lambda_{max}|$. Similarly to the case $\Lambda = 0$, two branches of solutions exist. When α is fixed and Λ varies, the two branches terminate at the maximal value. Another interesting issue of these calculations is that not only the solutions corresponding to the branch of lowest energy are stable. It seems that stable solutions are available on the two branches.

Although there are many reasons to study AdS black holes and solutions with such an asymptotics (see e.g. the AdS/CFT correspondence [8, 9] and/or the brane world cosmology arguments [10, 11]), De Sitter

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(dS) space-time enjoyed recenty a huge interest in theoretical physics for a variety of reasons. First at all, the observational evidence accumulated in the last years (see, *e.g.*, ref. [12]), seems to favour the idea that the physical universe has an accelerated expansion. The most common explanation is that the expansion is driven by a small positive vacuum energy (*i.e.* a cosmological constant $\Lambda > 0$), implying spacetime to be asymptotically dS. Furthermore, dS spacetime plays a central role in the theory of inflation (the very rapid accelerated expansion in the early universe), which is supposed to solve the cosmological flatness and horizon puzzles. Several results in the literature suggest that the conjectured dS/CFT correspondence has a number of similarities with the Anti-de Sitter/CFT correspondence, although many details and interpretations remain to be clarified (see [13] for a recent review and a large set of refences on this problems). In view of these developments, an examination of the classical solutions of gravitating fields in asymptotically dS spacetimes seems appropriate.

Several solutions of this type were considered in the framework of the Einstein-Yang-Mills equations [14, 15, 16]. The solutions of the Einstein-Maxwell theory with $\Lambda > 0$ have been discussed in a dS/CFT context in [17], multi-black hole configurations being considered as well. In a recent paper [18] the Einstein-Yang-Mills-Higgs equations was considered in the context of a positive cosmological constant. One main result of this analysis is that DeSitter monopole and sphaleron exist for small enough value of the cosmological constant. Again, they exist in two branches for $\Lambda \leq \Lambda_{max}$ but, computing the mass at timelike infinity according to the formalism developped in [20, 21, 22] we find that they do not have a finite mass as long as $\Lambda > 0$.

In this paper we consider the Einstein-Skyrme equations for a positive cosmological constant. We explore static, spherically symmetric configurations of the metric and matter fields. As a consequence of the positive cosmological constant Λ , these solutions approach a DeSitter space-time asymptotically and present a cosmological horizon. Because of the spherical symmetry, this horizon occurs on a sphere, at an finite value of the radial variable r. We succeeded in solving the equations numerically in both cases : regular and black hole solutions.

The paper is organized as follows: in Sect.2 we discuss the lagrangian, the ansatz, the relevant boundary conditions and establish the equations. In Sect.3 we discuss the numerical solutions for both cases (i) solutions regular at the origin and (ii) solutions presenting an event horizon at some finite value $r = r_h$. In both case the solutions present a cosmological horizon at $r = r_c$. The behaviour of the fields in the interior of the event horizon is briefly discussed as well.

2 The Einstein-Skyrme Lagrangian

2.1 Action principle

The action for a gravitating SU(2) Skyrme model is

$$S = \int_{\mathcal{M}} d^4 x \, \sqrt{-g} \left(\frac{1}{16\pi G} (R - 2\Lambda) + \mathcal{L}_M \right) \tag{1}$$

with Newton's constant G and cosmological constant Λ . The matter part of the Lagrangian density chosen as the Skyrme model :

$$\mathcal{L}_{M} = \frac{F_{\pi}^{2}}{16} g^{\mu\nu} \operatorname{tr}(L_{\mu}L_{\nu}) + \frac{1}{32e^{2}} g^{\mu\nu} g^{\rho\sigma} \operatorname{tr}([L_{\mu}, L_{\rho}][L_{\nu}, L_{\sigma}])$$
(2)

The basic matter field, denoted U(x), takes value in SU(2) while the combination $L_{\mu} \equiv U^{\dagger} \partial_{\mu} U$ has values in the Lie algebra su(2). Here F_{π} , *e* represent the two coupling constants of the theory. In the context of hadron physics, F_{π} is the pion decay constant and *e* is the Skyrme constant which ensures the stability of the Skyrmion. Throughout the paper, we assume the pion fields to be massless.

2.2 Spherically symmetric ansatz

In flat space, the fields equations associated with the Skyrme model admit an extremely rich pattern of solutions (see e.g. [3] for a recent review and references therein). However, here we will restrict to the spherically symmetric solutions. The spherically symmetric Skyrmion solution is constructed by imposing the static, hedgehog ansatz for the chiral field :

$$U(x) = U(\vec{r}) = \cos f(r) + i\hat{x} \cdot \vec{\tau} \sin f(r)$$
(3)

where $\hat{x} \equiv \vec{r}/r$ and $\vec{\tau}$ denotes the Pauli matrices.

For the metric, we use the standard spherically symmetric line element

$$ds^{2} = \frac{dr^{2}}{N(r)} + r^{2}(d\theta^{2} + \sin^{2}\theta d\varphi^{2}) - \sigma^{2}(r)N(r)dt^{2}$$
(4)

where we conveniently parametrize the metric function N(r) according to

$$N(r) = 1 - \frac{2m(r)}{r} - \frac{\Lambda r^2}{3}$$
(5)

The classical energy due to the matter fields can then be expressed in term of the following reduced functional

$$E_{S} = 4\pi \frac{F_{\pi}}{e} \int (\frac{1}{8} N u f'^{2} + \frac{v}{4x^{2}}) \sigma \, dx \quad , \quad x \equiv F_{\pi} er \tag{6}$$

with

$$u \equiv x^2 + 8\sin^2 f$$
 , $v \equiv \sin^2 f(x^2 + 2\sin^2 f)$ (7)

Here a rescaled radial coordinates x is introduced. From now on, the primes denote derivatives with respect to x. Accordingly, it is convenient to define $\mu(x) = eF_{\pi}m(r)$ and $\tilde{\Lambda} = \Lambda/e^2F_{\pi}^2$.

2.3 Field equations

The variational equations associated with the above functional are called the Einstein-Skyrme equations. Within the spherically symmetric ansatz, they reduce to the following system of three non-linear differential equations

$$\mu' = \frac{\alpha^2}{8} (Nuf'^2 + \frac{2v}{x^2}) \tag{8}$$

$$\sigma' = \sigma \frac{\alpha^2}{4x} u f'^2 \tag{9}$$

$$(N\sigma u f')' = \sigma (4Nf'^2 + 1 + \frac{4\sin^2 f}{x^2})\sin 2f$$
(10)

(11)

2.4 Boundary conditions

We want the generic line element (4) to describe a nonsingular, asymptotically de Sitter spacetime outside a cosmological horizon located at $x = x_c > 0$. In addition we require both possibilities of either a regular solution on the line $[0, \infty]$ or an event horizon at $x = x_h \leq x_c$. Here $N(x_h) = 0$ and $N(x_c) = 0$ are only coordinate singularities where all curvature invariants are finite. Nonsingular extensions across these null surfaces can be found. The regularity assumption implies that all curvature invariants at $x = x_c$ are finite.

The regularity conditions at x = 0 are

$$\mu(0) = 0 \quad , \quad f(0) = \pi \tag{12}$$

Examining closely the asymptotic values of the equations for $\Lambda > 0$, it turns out that the fields can take one of the two following asymptotic forms

$$f = q + \frac{1}{x^2} \frac{3}{2\Lambda} \sin 2q + O(1/x^3) \quad , \quad m(r) = \frac{\alpha^2}{4} (\sin^2 q)x + M + O(1/x) \quad , \quad \sigma(x) = 1 - \frac{\alpha^2}{4} \frac{c^2}{x^4} + O(1/x^5) \quad (13)$$

here q denotes an arbitrary constant; or

$$f = \frac{F}{x^3} + O(1/x^5) \quad , \quad m = M + \frac{\alpha^2 \Lambda}{8} \frac{F^2}{x^3} + O(1/x^5) \quad , \quad \sigma = 1 - \frac{3\alpha^2}{8} \frac{F^2}{x^6} + O(1/x^7) \tag{14}$$

where F is a constant. The form (14) leads to a finite mass M for the solution; for $\Lambda < 0$, it is precisely the form obeyed by the solutions constructed in [7].

At the event or cosmological horizon, (i.e. a value corresponding to a zero of the metric function N(r)) the regularity of the equation of the chiral field leads to the following condition

$$(uf'(x(\Lambda x^2 + \alpha^2 \frac{v}{x^2}) - x) + 2\sin(2f)(x^2 + 4\sin^2 f)|_{x=x_h \text{ or } x=x_c} = 0$$
(15)

The numerical integration is first performed on $[0, x_c]$ (or on $[x_h, x_c]$ in the case of black holes) with the condition (15) imposed as a boundary condition at $x = x_c$ (or at $x = x_h$ and $x = x_c$ in case of black hole). Then the solution is continued by integrating on $[x_c, \infty]$, again, imposing the condition (15) at $x = x_c$. The asymptotic behaviour (13) or (14) will be determined by means of this second integration, together with the corresponding value of q or F.

Both the event and the cosmological horizon have their own surface gravity κ given by

$$\kappa_{h,c}^2 = -\frac{1}{4}g^{tt}g^{rr}(\partial_r g_{tt})^2\Big|_{r=r_h,r_c}$$

the associated Hawking temperature being $T_H = |\kappa|/(2\pi)$.

2.5 Known solutions

For several limits on the different parameters, the solutions of the above equations are well known. The Schwarzschild-de Sitter solution corresponds to

$$f(r) = k\pi, \quad \sigma(r) = 1, \quad N(r) = 1 - \frac{2M}{r} - \frac{\Lambda r^2}{3},$$
 (16)

and describes a black hole inside a cosmological horizon as long as N(r) has two positive zeros, *i.e.* $M < 1/3\sqrt{\Lambda}$.

In the flat limit $\alpha = \Lambda = 0$, the Skyrmion solution is recovered. For $\Lambda = 0$, $\alpha \neq 0$ the gravitating Skyrmion (resp. Skyrme-black holes) are obtained.

3 Numerical results

The system of equations depends on three parameters Λ , α and x_h . The case $\Lambda > 0$ leads to the occurence of a cosmological horizon at $x = x_c$ with $N(x_c) = 0$. To integrate the equations, we used the differential equation solver COLSYS which involves a Newton-Raphson method [23]. The equations are first solved on the interval $[0, x_c]$ for regular solutions and on the interval $[x_h, x_c]$ for black holes where x_h corresponds to the event horizon. From this, the value of Λ can be determined numerically. From the data of the numerical profiles obtained on this finite interval, we are able to extend the solution outside the cosmological horizon i.e. for $x \in [x_c, \infty]$. In the case where an event horizon is present, we further integrated inside the event horizon, i.e. for $x \in [\epsilon, x_h]$; here ϵ denotes a small cut off. Since the origin constitutes an essential singularity of the metric $(N(0) = -\infty)$, the integration cannot be performed at the origin. We will discuss this case in a special section. The different solutions are constructed numerically with a absolute error lower than 10^{-6} .

3.1 Skyrme solutions in a fixed dS background

If we set $\alpha = 0$, we have trivially $\sigma = 1$ and N corresponds to the dS solution in the vacuum. The matter equation then leads to the Skyrme equation in the background of a deSitter space-time. One solution of this type is represented in Fig. 1. for $x_c = 44.0$, corresponding to $\Lambda \approx 0.0031$. Many features of these solutions are recovered in the presence of gravity and will be discussed at length in the next sections.

3.2 Gravitating DeSitter Skyrmion

We now set $\alpha > 0$. If regular conditions are imposed at the origin, the system can be integrated first on $x \in [0, x_c]$, fixing x_c by hand imposing (15) at $x = x_c$. The corresponding value of Λ is then determined numerically. While α increases, we observe that the function N(x) develops a minimum at $x = x_m$ with $x_h < x_m < x_c$, as illustrated on Fig. 1 for $\alpha = 0.3$.

In the case $\Lambda \ll 1$, the function μ seems to attains a constant asymptotic value inside the sphere $x = x_c$, accordingly the parameter M of Eq.(13) can be determined directly. However, the cosmological horizon decreases while Λ increases, as a consequence the occurrence of x_c outside the core of the Skyrmion does not persist for large values of Λ (typically for $\Lambda > 0.01$) and then the integration of the equation for $r \in [x_c, \infty]$ turns out to be necessary to refine the evaluation of the parameter M. This can be achieved by using the data of numerical the solution at $x = x_c$.

However, this is not the end of the story. Indeed, the integration on $[x_c, \infty]$, reveals that the solutions do not fall asymptotically on the configuration (14) but rather on (13) where the value q depends non trivially on Λ . The occurrence of the cosmological constant therefore prevents the chiral field to reach the zero value f = 0 asymptotically. Our numerical analysis strongly suggests that no solutions decaying according to (14) exist. An analytical argument would however be necessary to state this result.

The parameter q typically depends on Λ and is determined numericaly (typically, $q \approx 0.05$ for the values corresponding to Fig. 2). The numerical evaluation clearly suggests that $\lim_{\Lambda \to 0} q/\Lambda$ is finite. Accordingly the standard Skyrmion decay i.e. $f(r) \sim C/r^2$, is recovered in the gravitating but asymptotically flat limit (note: we do not include any mass term for the chiral field in this paper). This result deeply contrasts with the case $\Lambda < 0$ (see [7]) which has no cosmological horizon and where the asymptotic form (14) is obtained by a direct integration on $[0, \infty]$. In fact we were able to construct numerically the anti-de Sitter counterpart of our solution (i.e. for $\Lambda < 0$ and decay of type (13)). These solutions do not have a finite mass and were not emphasized in [7].

The physical consequences of this result are important. Indeed, because of (13) the function m(r) acquires a small linear dependance and does not stay constant asymptotically, preventing the mass to stay finite. [We do not illustrate this on a graphic but we refer to Fig. 2 where an (identical) phenomenon is present in the asymptotic behaviour of a black hole.] After an appropriate redefinition of the radial variable x and by using a standard argument, it can be shown that this property of the mass leads asymptotically to a locally deSitter space-time with an angular deficit given by $4\pi(1 - \alpha^2(\sin^2 q)/2)$.

It is worth to point out that the feature of non finiteness of the energy of soliton in asymptotically deSitter space-time was already observed in [18]; this was in the context of spontaneously broken SU(2)-gauge fields theories, respectively with the magnetic monopole (case of a Higgs triplet) and for sphaleron (case of a Higgs doublet). Global monopoles are also studied in space-times involving a cosmological constant [19], in this case also the mass function increases linearly and leads to an angular deficit. This property persists in the presence of a cosmological constant. In the present case, however, the mass evaluated at the cosmological horizon is finite (see [18]) are references therein.

The novel feature present in the case of the Skyrme field is that the cosmological constant drives the radial function f(r) away from its standard asymptotic value f(r) = 0. As a consequence the chiral field $U(\vec{r})$ does not approach $U = 1_2$ asymptotically.

In flat space, the Skyrme solitons are largely characterized by their baryon number. This charge is defined as the integral of the zero-component B^0 of the topological current B^{μ} :

$$B \equiv \int \sqrt{-g} B^0 d^3 x \quad , \quad B^\mu = -\frac{1}{24\pi^2} \varepsilon^{\mu\nu\rho\sigma} \frac{1}{\sqrt{-g}} \operatorname{Tr}(L_\nu L_\rho L_\sigma) \tag{17}$$

over a time-fixed section of space-time. It has an integer value and is interpreted as the baryon number of the solution.

When the Skyrmion is considered in a asymptotically flat space-time [5] the charge B is still an integer. In the present context however, B stops to be an integer for several reasons. In principle, we have to limit the integral defining B to the domain $0 \le r \le r_c$ which corresponds to the limit of the observable universe inside the cosmological horizon. The value of B then deviates from an integer because in general we have $f(r_c) \ne 0$. However, even if we take advantage of the continuation of the solution for $r \in [r_c, \infty]$ to extend the space maximally, we find after some algebra (see [24, 7])

$$B = \frac{1}{2\pi} (2f - \sin 2f)_{r=0} - \frac{1}{2\pi} (2f - \sin 2f)_{r=\infty} = 1 - \frac{1}{2\pi} (2q - \sin(2q))$$
(18)

which is obviously not an integer.

Different mechanisms leading to non conservation of the baryon number were constructed and examined e.g. in [24]. Our analysis just reveals that supplementing the Skyrme model with a positive cosmological constant leads to the same feature.

3.3 Black hole solutions

When the conditions of an event horizon are imposed at $r = r_h$ and for $\Lambda > 0$, the function N possesses two zeros. The system is solved first on the interval $[x_h, x_c]$ (we assume $r_h \ll r_c$ throughout all simulations) Again, the corresponding value of Λ is determined numerically. On Fig. 2 we present the profiles of the solutions for $\alpha = 0.3$, $r_h = 0.1$ and $x_c = 10$, corresponding to $\Lambda \approx 0.04$. On this figure, we clearly see that the chiral function f(r) does not approach zero asymptotically and that the mass function μ starts increasing after it stays on a plateau in the region of the cosmological horizon. We insist that this is in full aggrement with (13).

While increasing the value of the cosmological constant the numerical analysis shows that the solution (in fact black holes and regular at the origin) exist up to some maximal value of Λ , say up to $\Lambda = \Lambda_{max}$. No solution seems to exist for $\Lambda > \Lambda_{max}$ but a second family of solution exist for $\Lambda < \Lambda_{max}$. For a given value of Λ the mass inside associated with the solutions on the second branch is greater than the corresponding mass for the first branch.

Some physically relevant quantities characterizing the solutions are presented in Fig. 3 in functions of the cosmological constant parameter. Here we plot namely the mass, the value of the cosmological horizon and the values of the temperature at two horizons. The temperatures at the cosmological horizon corresponding to the two branches are the same, contrasting with the temperature at the event horizon which comes out to be larger for the solutions on the second branch.

Our numerical results further indicate that, for fixed values of α , Λ , the value of the parameter q is larger on the second branch than on the first (or main) one. Not that the construction of the second branch becomes rather difficult when reaching small values of Λ . That's why the second branch on Fig. 3 seems incomplete but we believe that the second branch extend backward to $\Lambda = 0$ and we plan to solve this numerical difficulties in near future.

3.4 Inside the event horizon

The question of integrating an hairy black hole solution inside the event horizon was adressed in [25] for Einstein-Yang-Mills (EYM) black holes with and without a Higgs field. The authors pointed out serious numerical difficulties that are met when the integration inward the horizon is performed by using the numerical data available from the integration in $x \in [x_h, \infty]$. They called the different phenomenon attached to the interior solution "mass inflation inside hairy black holes". When we attempt to integrate the Einsten-Skyrme equation for $x < x_h$ by using the data available from the integration on $x \in [x_h, x_c]$ we are immediately faced numerical difficulties which, likely, have the same origin than in [25]. Similarly to the case of EYM and EYMH equations, we notice the occurrence of regions inside the event horizon where the derivative of the function f varies suddently. This is illustrated on Fig. 4. In addition, the chiral function f(x) seems to deviate from the standard value $f(0) = \pi$ and to approach a different value in the limit $x \to 0$. In the case of EYM, it was observed, similarly, that the gauge function w(r) does not approach w = 1 when $r \to 0$. A more detailed analysis of this part of the solution is under investigation.

4 Conclusions

This work was partially motivated by the question on how a positive cosmological constant will affect the properties of a gravitating skyrmion. To the best of our knowledge, this question has not yet been addressed in the literature. The unexpected result of our analysis is, in our opinion, the fact that the presence of a positive cosmological constant prevents the skyrmion to have an integer topological number. The physical consequence of this result is that the solutions do not have a finite mass evaluated at timelike infinity. This suggests that, in the background of a varying cosmological constant, e.g. during inflation, the baryon number of the system could be violated. The analysis of the solutions reported here is minimal and will be extended in near future.

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Figure 1: The profile for the DeSitter-Skyrme soliton for $\alpha=0.0$ and $\alpha=0.3$



Figure 2: The profile for the DeSitter-Skyrme black hole for $\alpha = 0.3, x_h = 0.1$



Figure 3: The value of the cosmological horizon x_c , the value of the metric function σ at the event horizon, the mass of the solution of the DeSitter Black holes and the temperatures at the two horizons are given as functions of Λ for $\alpha = 0.3$, $x_h = 0.1$.



Figure 4: The profiles, inside the horizon, of the functions N, f, f' of the DeSitter-Skyrme black holes are given for $\alpha = 0.3, x_h = 0.1$