A robust voltage control algorithm incorporating model uncertainty impacts

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Abstract: This paper addresses the voltage control problem of the medium-voltage distribution systems under uncertainty of the network model. A Robust Voltage Control Algorithm (RVCA) is developed in order to manage the voltage constraints considering uncertainties associated with the parameters of load, line and transformer models. The RVCA determines a corrective solution that remains immunized against any realization of uncertainty associated with the parameters of network model. To this end, prior to formulating the voltage control problem, Monte Carlo (MC) simulations are used to characterize uncertain parameters of the network component models and Load Flow (LF) calculations are carried out to evaluate their impacts. The voltage constraints management under the uncertain environment is then formulated as a Robust Optimization (RO) problem. The latter is constructed based on the results obtained through the MC simulations and LF calculations. Once the RO is solved, in order to check the robustness of the solution, system voltages are evaluated using the LF calculations considering the new set-points of control variables and uncertainty of network parameters. The simulation results reveal that neglecting model uncertainty in the voltage control problem can lead to infeasible solutions while the proposed RVCA, at an extra cost determines a corrective solution which remains protected against the studied uncertainties.

1. Introduction

The massive integration of Distributed Generation (DG) units in the electric distribution systems has created serious voltage violation issues. In order to maintain the system voltages within the permitted limits, different Voltage Control Algorithms (VCAs) have been proposed in the literature. Despite their differences, the VCAs have been developed relying on similar assumptions that the exact power factors of loads are known [1-11], load powers are independent of the voltage [1], [3-11], lines can be modelled with the series impedances [2, 5, 6, 9, 10], which remain unchanged over the time [1-10], and the substation transformer is equivalent to a pure reactance [2, 5, 9]. The models based on which the VCAs in [1-11] were developed do not represent the real characteristics of the network components because in practice, load power factors are not available accurately, power consumptions of loads depend on the voltage, shunt admittances of lines must be taken into consideration, line resistances vary in function of the conductor temperatures, and internal resistance of transformer has important impact on the node voltages [12]. Consequently, corrective decisions of the Voltage Control Algorithm (VCA) obtained by relying on the simplified network model may be insufficient to solve a specific voltage violation problem of the real case.

Given that an accurate and up-to-date model of the network is not quantifiable and the simplified deterministic network model leads to erroneous analyses, the alternative solution is to consider that parameters of the network model are not deterministic but are rather uncertain varying within the predefined bounds. To the best of our knowledge, the first work that considers the model uncertainty in the VCA is [11]. In the latter paper, it is assumed that the line resistances vary due to the thermal dependency effect. The Robust Optimization (RO) has been adopted to account for the uncertainty of the line resistances. In addition, on the basis of a posteriori analysis, impacts of the model uncertainty on the VCA relying on the simplified deterministic network model have been investigated in [12].

In this paper, a Robust Voltage Control Algorithm (RVCA) is developed that manages the voltage constraints considering uncertainties associated with the parameters of load, line and transformer models. The proposed RVCA determines a corrective solution that remains immunized against any realization of uncertainty associated with the parameters of network model. To this end, the RVCA modifies DG active and reactive powers as well as the transformer tap position. The RVCA is developed on the basis of an optimization procedure relying on the linear approximations of the relations between nodal voltages and control variables obtained through the voltage sensitivity analysis.

Compared to [11], beside the thermal dependency of line resistances, we consider complementary sources of model uncertainty, which are arisen from voltage dependency of loads, power factor of loads, shunt admittances of lines and internal resistance of transformer. Our proposed RVCA firstly manages the voltage constraints subject to uncertainty arisen from each of the abovementioned factors (individually). Then, we consider that uncertainties of load, line and transformer parameters are present simultaneously and the RVCA determines a corrective solution that remains protected against any realization of uncertainty associated with the network model. Moreover, in our work, the robustness of the RVCA solution is verified with the numerical simulations. Finally, the proposed RVCA is adapted for voltage management of 3-phase unbalanced systems.

Compared to [12], in the current work, the model uncertainty is considered inside the VCA when taking the corrective decisions of the control variables by adopting a RO formulation. The proposed framework of [12] determines the upper and lower bounds of voltage variations due to uncertainty of the network model. In order to be robust against the uncertainty effects, in [12], it is suggested to keep the VCA simple (relying on the simplified deterministic network model) and modify its targeted bounds based on the maximum deviations that the node voltages can have due to the model uncertainty. In contrast with this idea, the proposed RVCA of the current work leads to solutions which are less conservative since the solutions are immunized against the considered working point while in the approach according to [12], the targeted bounds of the VCA are modified based on the maximum deviations that the node voltages can have due to model uncertainty.

In view of the above discussion, the main contribution of this work is to propose a robust VCA that incorporates wide sources of model uncertainty and to validate its robustness through the numerical simulations. The comparative studies of the RVCA and simple VCA (relying on the simplified deterministic network model) reveal that the solution of the latter will be insufficient to solve the voltage control problem of the real case.

The rest of this paper is organized as follows. In section 2, the RO is introduced. In section 3, the proposed methodology to construct the RVCA is described. Afterwards, the studied sources of uncertainty are presented in section 4 that is followed by introducing the investigated test system in section 5. Then, numerical simulations are carried out in section 6 in order to evaluate performance of the proposed RVCA. The final conclusion is reported in section 7.

2. Robust optimization

In many optimization applications, the problem data are assumed to be known with certainty. In practice, however, the realistic data are very often subject to uncertainty due to their random nature, measurement errors, or other reasons. Since the solution of the optimization problem exhibits high sensitivity to data perturbations, ignoring the data uncertainty could lead to solutions which are infeasible in practice [13]. Robust optimization presents methodology for dealing with the optimization problem subject to data uncertainty. Under this approach, we are willing to accept a suboptimal solution for the nominal values of data in order to ensure that this solution remains feasible when data change within the predefined ranges. In contrast to the stochastic optimization, RO formulates the uncertainty assuming that an uncertain value varies within a predefined interval rather than proposing a probability distribution function for it. Therefore, in the RO, uncertainty modelling is not stochastic, but rather deterministic and set-based. Consequently, no assumption on the distribution of the uncertainty has to be made which is an attractive aspect of RO, especially, in the case of lack of full information about the nature of the uncertainty [14].

In the electric power systems, data uncertainty can be arisen from the electricity price change, load or DG power variation, measurement noise, state estimation error, unobservability of network state, and partial knowledge of network model. In the literature, RO techniques have been applied to problems such as volt-var control [15], voltage constraints management [11], optimal power flow [16-17], economic dispatch [18], generation planning [19-20], and microgrid planning [21].

2.1. Robust optimization counterpart

Consider the generic linear optimization problem given in below:

$$\textbf{Min:}\ \mathbf{C^T x} \tag{1}$$

$$\mathbf{Ax} \le \mathbf{b} \tag{2}$$

$$\mathbf{l_b} \le \mathbf{x} \le \mathbf{u_b} \tag{3}$$

where $\mathbf{x}$ is vector of decision variables, $\mathbf{C^T}$ is the transpose vector of coefficients of objective function, $\mathbf{A}$ is matrix of coefficients of structural constraints (2), and $\mathbf{b}$ is vector of Right-Hand Side (RHS) of the structural constraints (2). The upper and lower bounds on the control variables are defined by $\mathbf{u_b}$ and $\mathbf{l_b}$, respectively. It is assumed that data uncertainty affects Left-Hand Side (LHS) and RHS of structural constraints (2) [13]. Consider a particular row i of the matrix $\mathbf{A}$ (i.e., the LHS of (2)) and let J_i be set of column indices in row i that are subject to uncertainty. Each entry a_{ij} of $\mathbf{A}$ ($j \in J_i$) is modelled as a symmetric and bounded random variable $\tilde{a}_{ij}$ that takes values from the range $[a_{ij} - \hat{a}_{ij}, a_{ij} + \hat{a}_{ij}]$ where a_{ij} is the nominal value of $\tilde{a}_{ij}$ and $\hat{a}_{ij}$ denotes its maximum positive perturbation. The uncertain data $\tilde{a}_{ij}$ is given by

$$\tilde{a}_{ij} = a_{ij} + \xi_{ij}\hat{a}_{ij} \tag{4}$$

where ξ_{ij} is a random variable which is subject to uncertainty and perturbs in the range [-1,1]. Regarding the RHS uncertainty of (2), we have

$$\tilde{b}_i = b_i + \xi_{i0}\hat{b}_i \tag{5}$$

where $\tilde{b}_i$, b_i and $\hat{b}_i$ are the uncertain RHS of the *ith* structural constraint, its nominal value, and its maximum perturbation respectively. Also, ξ_{i0} is the random variable associated with uncertainty of RHS of the *ith* structural constraint. In order to derive the robust counterpart of the presented generic linear optimization problem (given in (1) to (3)), the structural constraint (2) needs to be modified as below given that the data uncertainty affects its RHS and LHS.

$$\sum_{j \notin J_i} a_{ij}x_j + \sum_{j \in J_i} \tilde{a}_{ij}x_j \le \tilde{b}_i \quad \forall i \tag{6}$$

The above constraint can be reformulated as

$$\sum_j a_{ij}x_j + \left[-\xi_{i0}\hat{b}_i + \sum_{j \in J_i} \xi_{ij}\hat{a}_{ij}x_j\right] \le b_i \quad \forall i \tag{7}$$

In the robust optimization with a predefined uncertainty set E, the robust solution is the one that remains feasible for any ξ in the given uncertainty set. The corresponding structural constraint of the RO problem under uncertainty set E is given by [13].

$$\sum_j a_{ij}x_j + \max_{\xi \in E}\left[-\xi_{i0}\hat{b}_i + \sum_{j \in J_i} \xi_{ij}\hat{a}_{ij}x_j\right] \le b_i \quad \forall i \tag{8}$$

In the literature, different uncertainty sets have been introduced and discussed. The most prominent ones are the box [22], ellipsoidal [23] and polyhedral [24] uncertainty sets. The RVCA of this work adopts the model under box uncertainty set. This choice is motivated by the fact that the ellipsoidal robust formulation converts the initial linear optimization problem into a non-linear second-order one and results in increasing complexity and calculation burden of the RVCA [24]. In addition, the polyhedral uncertainty set leads to a bi-level nested optimization problem. Although the latter has a linear dual model, due to introduction of dual variables, the size of the RO counterpart increases in the polyhedral model. Consequently, given that under the box uncertainty, the RVCA has the same number of variables and constraints as the initial problem (with nominal values) and the RVCA formulation remains linear similarly to the initial one, the RO under the box uncertainty set is adopted in this work.

In the box uncertainty set, it is assumed that ξ_i can vary independently between 0 and Ψ_i where Ψ_i represents perturbation bounds of the uncertain coefficients in the *ith* row of (2). The interaction of perturbations creates a box-shaped space, which represents the box uncertainty set. The RO counterpart of the structural constraint (8) under box uncertainty set is equal to [13]

$$\sum_j a_{ij}x_j + \Psi_i\left[\sum_{j \in J_i} \hat{a}_{ij}|x_j| + \hat{b}_i\right] \le b_i \quad \forall i \tag{9}$$

For the bounded uncertainty $\xi_i \in [-1,1]$, when Ψ_i is set to 1, the entire uncertain space is covered by the box. This is a special case of the box uncertainty set, which is known as the interval uncertainty set, and will result in the most conservative solution. On the contrary, Ψ_i equal to zero leads to the nominal optimization problem (i.e., (1) to (3)). Consequently, the level of the conservatism of the solutions can be controlled by adjusting Ψ between 0 and 1.

3. Robust voltage control algorithm

The proposed RVCA of this work modifies active and reactive powers of DGs as well as the transformer tap position in order to manage the voltage constraints. It aims at finding the corrective actions of the abovementioned control variables such that the obtained solution remains immunized against the uncertainties associated with the network component models. The RVCA is developed on the basis of an optimization procedure relying on the linear approximations of the relations between nodal voltages and control variables. The linearization of the voltage control problem is carried out using the concept of the voltage sensitivity analysis. Thanks to information provided by the voltage sensitivity analysis, impacts of control variables on node voltages are known. Therefore, there is no need to consider the equality constraints relating to balance of the nodal active and reactive powers (which are non-linear and non-convex) in the optimization formulation. Consequently, the simplified linearized optimization formulation of the voltage control problem can be solved in a faster and more efficient way.

Figure 1 presents the proposed RVCA of this work, which consists of the pre-processing stage, RO formulation and post-processing stage. The pre-processing stage determines perturbation bounds of the RHS and LHS of the RO structural constraints due to the model uncertainty effect using Monte Carlo (MC) simulations and Load Flow (LF) calculations. The RO formulation is constructed then on the basis of information provided by the pre-processing stage. The solution of the RO defines needed changes of control variables in order to solve the voltage control problem subject to model uncertainty. The obtained solution of the RO is finally validated in the post-processing stage and robustness of the solution is evaluated. The three parts of the proposed methodology are discussed further in the following sections.

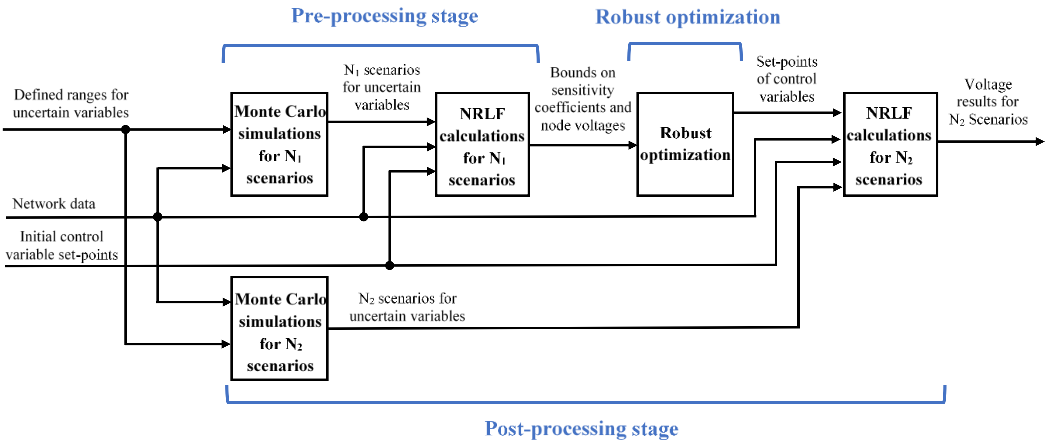


Fig. 1. *The proposed approach to develop the robust voltage control algorithm*

3.1. Pre-processing stage

In the first step of the pre-processing stage, it is aimed to characterize the uncertain models of the studied components. To this end, MC simulations are utilized to create N_1 scenarios within the predefined bounds for the uncertain variables of the network component models. The scenario creation technique in the MC simulations follows the same procedure as presented in [12] and [25]. As a result of the model uncertainty, in each generated scenario, the elements of matrix **A** (i.e., the voltage sensitivity matrix) as well as the RHS of the structural constraints (representing node voltages) will be perturbed with respect to their initial values. Given that perturbation bounds of the RHS and LHS of the structural constraints are not known, in the second step of the pre-processing stage, LF calculations are performed for each of N_1 scenarios created by the MC simulations. The Newton-Raphson Load Flow (NRLF) approach is used in this regard. Once the perturbation bounds of the RHS and LHS of the structural constraints are obtained, the RO formulation of the voltage control problem under uncertainty of the network model can be constructed as presented in the next section.

3.2. Robust optimization

The RVCA aims at minimizing the weighted sum of the control variables subject to uncertain voltage constraints and restrictions of control variables. It leads to the following RO problem [11].

$$\textbf{Min}: OF = \sum_{x=1}^{N_G}\left(C_Q|\Delta Q_{DGx}| + C_P\Delta P_{DGx}\right) + C_{TR}|\Delta Tap_{TR}| \quad (10)$$

$$\sum_{x=1}^{N_G}\left(\frac{\widetilde{\partial V_u}}{\partial Q_{DGx}}\Delta Q_{DGx} + \frac{\widetilde{\partial V_u}}{\partial P_{DGx}}\Delta P_{DGx}\right) + \frac{\widetilde{\partial V_u}}{\partial V_{Tap}}\Delta Tap_{TR} \le \widetilde{\Delta V_u}^{req} \quad \forall u, u \in U \quad (11)$$

$$\sum_{x=1}^{N_G}\left(\frac{\widetilde{\partial V_l}}{\partial Q_{DGx}}\Delta Q_{DGx} + \frac{\widetilde{\partial V_l}}{\partial P_{DGx}}\Delta P_{DGx}\right) + \frac{\widetilde{\partial V_l}}{\partial V_{Tap}}\Delta Tap_{TR} \ge \widetilde{\Delta V_l}^{req} \quad \forall l, l \in L \quad (12)$$

$$0 \le \Delta P_{DGx} \le |P_{DGx}| \quad \forall x, x \in G \quad (13)$$

$$\Delta Q_{DGx}^{min} \le \Delta Q_{DGx} \le \Delta Q_{DGx}^{max} \quad \forall x, x \in G \quad (14)$$

$$\Delta Tap_{TR}^{min} \le \Delta Tap_{TR} \le \Delta Tap_{TR}^{max} \quad (15)$$

where OF presents the objective function of the RO, ΔP_{DGx} and ΔQ_{DGx} are the active and reactive power changes of the DG number x ($x \in G$), G is set of DG numbers, and N_G is the total number of DGs that contribute in the voltage control problem. Also, C_P and C_Q are the weighting coefficients for the active and reactive power changes of DGs. ΔTap_{TR} and C_{TR} denote the transformer tap changes, and its corresponding weighting coefficient, respectively. $\frac{\widetilde{\partial V_u}}{\partial Q_{DGx}}$, $\frac{\widetilde{\partial V_u}}{\partial P_{DGx}}$ and $\frac{\widetilde{\partial V_u}}{\partial V_{Tap}}$ stand for uncertain sensitivity coefficients of voltage at bus u with respect to reactive power of DGx, active power of DGx, and transformer tap position, respectively, where u is index for the buses with the voltage rise and set U includes all the buses with the voltage rise violations. Similarly, l is index for the buses with the voltage drop and set L contains all buses with the voltage drop issue. As it can be noticed, inequality constraints (11) and (12) take into account the required values of voltage modifications at the buses with the voltage rise and drop violations in order to bring back those voltages within the permitted voltage range. The RHS of the structural constraint (11) (or (12)) denoted by $\widetilde{\Delta V_u}^{req}$ (or $\widetilde{\Delta V_l}^{req}$) gives the uncertain needed voltage modification at bus u (or l) in order to return its voltage rise (or drop) within the permitted voltage range. The constraints (13) to (15) consider the upper and lower bounds of the control variables. Under the box uncertainty with $\Psi_i = 1$ (i.e., the interval uncertainty set), the deterministic equivalent of the uncertain constraints (11) and (12) can be obtained according to (9) as follows:

$$\sum_{x=1}^{N_G}\left[\left(\frac{\partial V_u}{\partial Q_{DGx}} + \frac{\widehat{\partial V_u}}{\partial Q_{DGx}}\right)\Delta Q_{DGx} + \left(\frac{\partial V_u}{\partial P_{DGx}} + \frac{\widehat{\partial V_u}}{\partial P_{DGx}}\right)\Delta P_{DGx}\right] + \left[\left(\frac{\partial V_u}{\partial V_{Tap}} + \frac{\widehat{\partial V_u}}{\partial V_{Tap}}\right)\Delta Tap_{TR}\right] + \widehat{\Delta V_u}^{req} \le \Delta V_u^{req} \quad \forall u, u \in U \quad (16)$$

$$\sum_{x=1}^{N_G}\left[\left(\frac{\partial V_l}{\partial Q_{DGx}} + \frac{\widehat{\partial V_l}}{\partial Q_{DGx}}\right)\Delta Q_{DGx} + \left(\frac{\partial V_l}{\partial P_{DGx}} + \frac{\widehat{\partial V_l}}{\partial P_{DGx}}\right)\Delta P_{DGx}\right] + \left[\left(\frac{\partial V_l}{\partial V_{Tap}} + \frac{\widehat{\partial V_l}}{\partial V_{Tap}}\right)\Delta Tap_{TR}\right] + \widehat{\Delta V_l}^{req} \ge \Delta V_l^{req} \quad \forall l, l \in L \quad (17)$$

where $\frac{\widehat{\partial V_u}}{\partial Q_{DGx}}$, $\frac{\widehat{\partial V_u}}{\partial P_{DGx}}$ and $\frac{\widehat{\partial V_u}}{\partial V_{Tap}}$ are perturbations of voltage sensitivity coefficients of bus u with respect to the control variables. Also, $\widehat{\Delta V_u}^{req}$is the perturbation of needed voltage modification at bus u due to model uncertainty impact. The required voltage modifications (i.e., the RHSs of (16) and (17)) at buses with the voltage rise and drop violations are calculated with respect to the 1.03 pu and 0.97 pu, which are considered as the upper and lower permitted voltage limits as below.

$$\Delta V_u^{req} = 1.03 - V_u \quad \forall u, u \in U \quad (18)$$

$$\Delta V_l^{req} = 0.97 - V_l \quad \forall l, l \in L \quad (19)$$

where V_u and V_l are the initial voltages of bus u and l, respectively.

Given that the predefined bounds of the variation for the uncertain variables of the network model are not necessarily symmetrical (see section 4), the model uncertainty does not create always symmetrical variation around the nominal value of each entry of the voltage sensitivity matrix. In this case, perturbations of sensitivity coefficients in (16) or (17) must be selected such that the maximum protection against the worst uncertainty scenario is guaranteed (as the interval uncertainty set is adopted). In this regard, the perturbation that reduces the absolute value of each entry of the sensitivity matrix at most is selected because in this way, the biggest value of control variable changes will be demanded. Consequently, the highest protection against the model uncertainty is provided. In addition, the perturbation that creates the biggest voltage violation at the *lth* or *uth* bus of the system (among N_1 scenarios) will be chosen since it gives the worst voltage violation scenario at bus *l* or *u*.

The voltage sensitivity coefficients with respect to nodal active and reactive powers in each of N_1 created scenarios are obtained through the inverse of the Jacobian matrix, which is in our disposal in the NRLF study. The nodal voltage sensitivities with respect to transformer tap movement are calculated using the perturb-and-observe technique. To this end, the voltage variation in the observed point is calculated using the NRLF when the transformer tap position (i.e., the perturbation point) is moved by one step.

3.3. Post-processing stage

Once the abovementioned linear RO problem is solved, the new set-points of control variables (i.e., active and reactive power changes of DGs as well as the transformer tap movement) are determined. As stated, the obtained solution of the RVCA must remain immunized against all possible realizations of uncertainties associated with the network component models. In order to verify the latter, further analyses are carried out on the new set-points of control variables. In this regard, MC simulations are used to create N_2 scenarios for uncertain parameters of the network component models. Then, LF calculations are done on each of the N_2 scenarios considering the set-points of control variables obtained by the RO and the rest of the network data. Finally, node voltages in N_2 scenarios will be in our disposition, which will present the robustness of the RVCA solution in N_2 realizations of uncertainties associated with the network component models.

In the proposed approach shown in figure 1, in the pre-processing stage (prior to composing the RO problem), when choosing needed number of scenarios (i.e., N_1) for characterizing uncertainties and defining their impacts, the requirement regarding the execution time of the RVCA must be taken into account. Such a limit does not exist when N_2 scenarios are created to validate the RVCA results since the corrective decisions have been already made. Consequently, N_2 can be much bigger than N_1. In this way, the RVCA results will be tested for extra scenarios that are not necessarily included among N_1 generated scenarios in the first stage of the MC simulations. It is worth noting that the defined variation ranges for uncertain parameters of network component models are identical when creating scenarios in the pre-processing and post-processing stages.

4. Studied sources of uncertainty

In this paper, it is considered that exact values of load powers, line resistances and admittances as well as transformer internal resistance are not known at the specific studied time. Therefore, the above parameters are taken into account as uncertain variables changing within the predefined bounds. The uncertainty in parameters of network model is arisen from the voltage dependency of loads, power factor of loads, thermal dependency of lines, shunt admittances of lines and internal resistance of substation transformer. The abovementioned sources of uncertainty are described in below and the variation bounds of the uncertain variables are presented.

The power consumption of an electric load is known at the nominal voltage (i.e., 1 pu). Depending on the nature of the load, its real consumption however can be different when the voltage is not equal to the nominal value due to the voltage dependency effect. In the Medium-Voltage (MV) distribution systems, nodal load powers are aggregated from the low-voltage side and consist of various load types. Since load powers are changing continuously and we do not have information about the aggregate voltage-power dependency of loads at the studied time, we cannot obtain the exact values of load powers. Voltage dependency of load powers is taken into account here with the exponential load model [12]. It is supposed that the exponent for active power (denoted by α) can change within the interval of [0, 2.6] and the one regarding the reactive power (shown by β) varies between 0 and 4 [26]. The NRLF formulation is modified according to the approach presented in [12] in order to consider the voltage dependency of loads.

Due to insufficient measurements in the electric distribution systems, load power factors are not available accurately. Consequently, an uncertainty is added to the load model relating to the power factor. Supposing that the load active power is known, the power factor uncertainty changes the reactive power consumption of load. In this work, it is assumed that the load power factor (denoted by *PF*) can vary between 0.9 (lagging) and 1.

The resistance of line depends not only on the conductor size and type, but also on the temperature at which the conductor is operating [12]. The conductor temperature, by itself, is in function of the line loading and the ambient temperature. In practice, the relation between the line loading variation and conductor temperature change is unknown. Also, the real ambient temperature of the conductor is not available. Therefore, the line resistances cannot be obtained with certainty due to the thermal dependency effect. In the network studied in this paper, total powers of DGs are almost 3 times bigger than sum of the load powers [12]. Therefore, the temperature variations of cable conductors as a function of the cable loadings in the voltage rise case are expected to be bigger than ones of the voltage drop state. According to experiments which have been performed on a 15 kV underground cable in [27], we suppose that the line resistances can increase up to 11% and decrease to 4% of their nominal values due to temperature variations of the cable conductors in the voltage rise case as explained in [12]. In the voltage drop case, temperature variations create resistance changes (denoted by ΔR) equal to ±5.8% of their nominal values [12].

In the electric distribution systems, lines are usually considered with their series impedances while their shunt admittances are neglected. Shunt admittances however can have important impacts on the node voltages in case of long cable lines [12]. Shunt admittances of the lines can be calculated theoretically if we know the cable characteristics and its exact installation configuration. Such data are not always available in practice. Considering line capacity equal to 0.25 μF/km [8] and having length of the lines in the studied network, the upper bound of the predefined range for the admittance variation in all the lines can be determined. In the case that shunt admittances of lines are ignored, the lower bound of the admittance variation is obtained, which is equal to 0.

The power transformer is generally modelled with a series impedance. Given that the internal resistance of the transformer (denoted by R_T) is very small compared to its reactance, R_T is mostly considered to be negligible. The internal resistance of transformer however can have considerable effects on the node voltages as shown in [12]. The nominal value of transformer resistance can be obtained by doing specific electrical tests on the transformer. However, the transformer resistance can vary in an unknown manner due to temperature variations arisen from the transformer loading changes and ambient temperature variations. The typical reactance to resistance ratio of the power transformers is within the range of 20 to 40 [28]. Assuming that the reactance of the transformer is known, the variation bounds of R_T can be determined. It is considered here that the resistance of the transformer can vary based on its loading conditions. In this regard, an extension of ±10% with respect to the aforementioned range is adopted. Therefore, it is supposed that the resistance of the transformer can take values from the interval starting at 2.25% (1/40×0.9) and ending at 5.5% (1/20×1.1) of the transformer reactance [12].

5. Investigated test system

Performance of the proposed RVCA is tested on the 77-bus, 11 kV radial distribution system shown in figure 2 [2], [12], [29]. It is the so-called ‘‘HVUG’’ test case of the United Kingdom Generic Distribution System (UKGDS). In the investigated network, bus number 1 is considered as the slack node while all other buses are of PQ (load) type. The substation transformer located between nodes 1 and 2 is modelled with a pure reactance equal to 12.5% pu in the transformer base power (80 MVA). The studied network feeds 75 loads which have total active and reactive powers equal to 24.27 MW and 4.85 Mvar, respectively. It also hosts 22 DGs, which are identical with the rated powers equal to 3.5 MW. The capability curves of DGs are obtained from the points given in [9]. In the studied network, loads are of power constant type, lines are modelled with the series impedances, and DG active power is considered as a negative load.

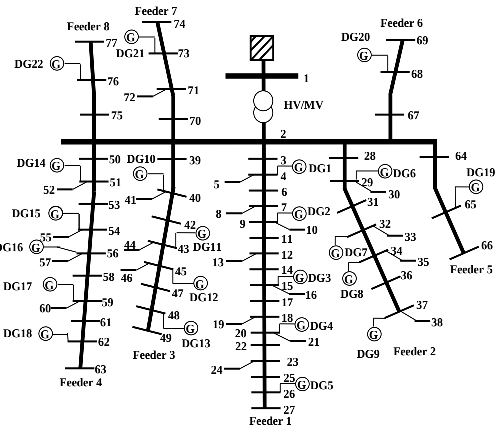


Fig. 2. *77-bus radial distribution system*

6. Simulation results

The proposed RVCA including MC simulation, NRLF calculation, and the presented RO formulation is implemented in the MATLAB environment. Performance of the RVCA is tested on the UKGDS shown in figure 2 considering two different working points corresponding to the voltage drop and rise states. In the voltage drop condition, it is assumed that load demands are maximum (equal to their nominal values) while active powers of DGs are zero. In the voltage rise state, it is supposed that the load demands are low (equal to 10% of their nominal values) while active powers of DGs are at 90% of their rated values. The initial reactive powers of DGs in both cases are set to zero.

In order to consider the constraint regarding the calculation time of the RVCA, in the pre-processing stage (prior to forming the RO problem), 500 scenarios (N_1=500) are created by the MC simulations. However, to validate the RO results, number of scenarios is increased to 2000 (N_2=2000). In the voltage control procedure, it is supposed that the transformer tap changer action has the smallest weighting coefficient compared to other control variables which is equal to 1 (C_{TR}=1) while the reactive power changes of DGs are weighted by a coefficient which is 50% bigger than the tap changer one (C_Q=1.5). Also, active power curtailment of DGs is assigned to a coefficient which is 100% bigger than the tap changer one (C_P=2).

Table 1 presents the demanded contributions of DGs and necessary transformer tap movements in order to manage voltage violations in the voltage drop and rise cases when the model uncertainty is neglected. In table 1 and hereafter, NA is used to indicate that a specific control action is not applied. In addition, DGs with the power changes are only mentioned in the table and for the rest of DGs (which are not listed), power changes are equal to zero. The initial system voltages (with voltage violations) as well as the ones obtained after the voltage regulation using the simple VCA (which considers the nominal network model) are depicted in figure 3.

Table 1 VCA results considering the simplified deterministic models of network components

	Voltage drop	Voltage rise
ΔQ_{DGx} (Mvar)	DG5=-1.266	DG5=1.363
ΔP_{DGx} (MW)	NA	NA
ΔTap_{TR}	2	-4
OF	3.897	6.044

From figure 3, it can be noticed that if there is no uncertainty in the parameters of network model, the simple VCA can bring back the initial voltage violations inside the permitted voltage range. However, for any realization of uncertainty (i.e., inevitable in reality), the node voltages will be different from those depicted in figure 3. In what follows, the RVCA is utilized to manage the voltage constraints of the same working points (corresponding to the voltage rise and drop cases) under uncertainty of the network component models. The RVCA results are compared with the ones obtained through the simple VCA (which does not consider the model uncertainty and relies on the simplified deterministic network model). The proposed RVCA firstly manages the voltage constraints under uncertainty arisen from each of the sources mentioned in section 4 (individually). Then, all studied sources of uncertainty are considered to be present simultaneously and the RVCA solves the voltage control problem under uncertainties of load, line and transformer models. In the end, the proposed RVCA is adapted for voltage management of 3-phase unbalanced systems.

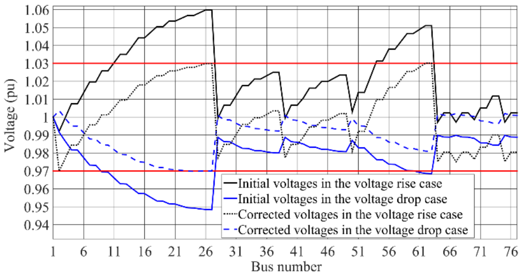


Fig. 3. *Initial nodal voltages as well as the corrected ones obtained by the simple VCA relying on the simplified deterministic models of network components*

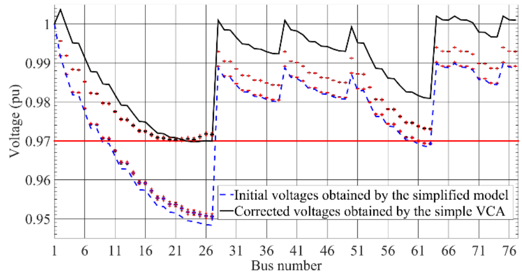
6.1. On the uncertainty linked with the voltage dependency of loads

In the first studied case of this work, the uncertainty due to the voltage dependency of loads is taken into consideration. The RVCA manages the node voltages in the voltage drop and rise conditions under uncertainty of the load-voltage dependency. Table 2 presents the control variable changes demanded by the RVCA. Moreover, figure 4 shows the boxplots of initial and corrected voltages subject to the studied uncertainty of this section as well as the initial and corrected voltages obtained by relying on the simplified deterministic component models. The boxplots of initial voltages show the possible perturbations of node voltages due to the model uncertainty, which correspond to the uncertainty in RHS of the structural constraints. The boxplots of corrected voltages give the voltage results obtained in N_2 scenarios considering the solution of the RO problem. Hereafter, boxplots of the initial voltages are shown in blue while ones related to the corrected voltages are illustrated in black.

In the voltage drop condition, the voltage control problem considering the uncertainty linked with the voltage dependency of loads has been solved with a smaller value of objective function compared to the one obtained by the simple VCA using the simplified (power constant) load model (see tables 1 and 2). This is due to the fact that in the voltage drop condition, node voltages are smaller than 1 pu; therefore, the load-voltage dependency reduces the load powers. Consequently, the load-voltage uncertainty decreases the severity of the voltage control problem. In other words, perturbations caused by the studied uncertainty release (smooth) the structural constraints of the RO problem such that less control effort is needed to solve the voltage control problem in the voltage drop condition. Similar interpretation can be also done on the basis of the voltage results shown in figure 4(a) where it is seen that boxplots of initial voltages are placed above the initial voltages obtained by the simplified load model.

Table 2 Robust VCA results considering uncertainty associated with voltage dependency of loads

	Voltage drop	Voltage rise
ΔQ_{DGx} (Mvar)	DG5=-2.577	DG5=1.364
ΔP_{DGx} (MW)	NA	NA
ΔTap_{TR}	NA	-4
OF	3.845	6.046



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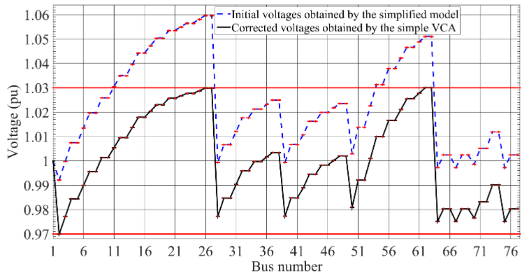

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Fig. 4. *The voltages obtained using simplified network component models as well as boxplots of initial and corrected voltages considering the uncertainty associated with the voltage dependency of loads*
(a) in the voltage drop case, (b) in the voltage rise case

Furthermore, it can be concluded that the uncertainty impact due to the load-voltage dependency in the voltage rise condition is negligible since the RVCA and simple VCA have led to almost similar results (see tables 1 and 2). The latter point can be verified further considering the figure 4(b) where it is seen that boxplots of initial and corrected voltages have very narrow bounds.

6.2. On the uncertainty linked with the power factor of loads

Performance of the RVCA under uncertainty of load power factors is investigated here on the studied voltage rise and drop conditions. Similar to the previous case, it can be expected that the power factor uncertainty impact appears mostly on the voltage drop condition since the load powers are maximal in this case. Table 3 and figure 5 present the RVCA results under uncertainty of the load power factors.

The average power factor of loads in the studied UKGDS is equal to 0.98 [12]. Considering the defined range for the power factor variation from 0.9 to 1, the load reactive powers will (mostly) increase because of the power factor

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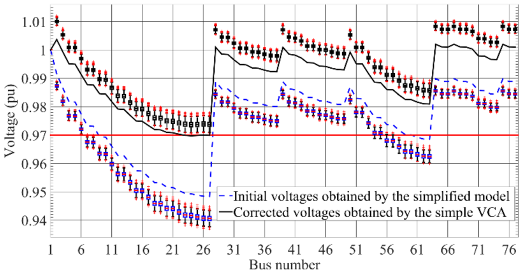
uncertainties. Consequently, in figure 5(a), it is observed that boxplots of the initial node voltages are noticeably lower than the initial voltages obtained by neglecting load power factor uncertainties. The difference between the former and latter can reach almost 0.01 pu. Therefore, a bigger value of control variable changes (with respect to the simple VCA results) is needed to manage the voltage control problem under uncertainty of load power factors in the voltage drop case as it can be seen in table 3.

Table 3 Robust VCA results considering the uncertainty associated with power factor of loads

	Voltage drop	Voltage rise
ΔQ_{DGx} (Mvar)	DG5=-1.327	DG5=1.317
ΔP_{DGx} (MW)	NA	NA
ΔTap_{TR}	4	-4
OF	5.991	5.976



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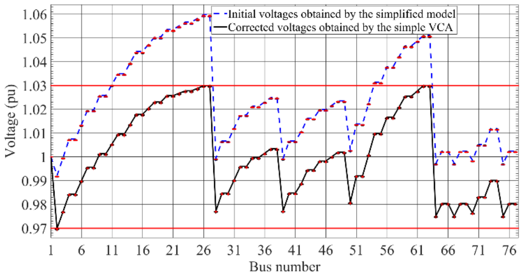

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Fig. 5. *The voltages obtained using simplified network component models as well as boxplots of initial and corrected voltages considering the uncertainty associated with the power factor of loads*
(a) *in the voltage drop case,* ***(b)*** *in the voltage rise case*

In figure 5(b) regarding the voltage rise condition, it is seen that as a result of the load power factor uncertainty, boxplots of the initial voltages are placed under the voltages obtained by the simplified load model. It is due to the fact that power factor uncertainties have increased the load reactive powers with respect to their initial values. Consequently, in the voltage rise condition, the voltage control problem under load power factor uncertainties has been solved with a smaller value of objective function in comparison with the one of the simple VCA (see tables 1 and 3).

From figures 5(a) and 5(b), it is noticed that the boxplots of the corrected voltages in the voltage drop and rise conditions do not violate the permitted voltage range in all N_2 created scenarios. Therefore, it is verified that the RVCA solution remains immunized against all realizations of the studied uncertainty. It is worth noting that the solution of the simple VCA is optimal and feasible for the nominal value of the uncertain variable. If the latter takes any other value than its nominal one, the solution of the simple VCA would be either infeasible or non-optimal. In the current studied case, the solution of the simple VCA is infeasible in the voltage drop case and non-optimal in the voltage rise case (for any value of the uncertain variable other than the nominal one). In contrast, the solution of the RVCA remains feasible for all realizations of uncertainty (within the predefined range) and is optimal with respect to the worst uncertainty scenario.

6.3. On the uncertainty linked with the thermal dependency of lines

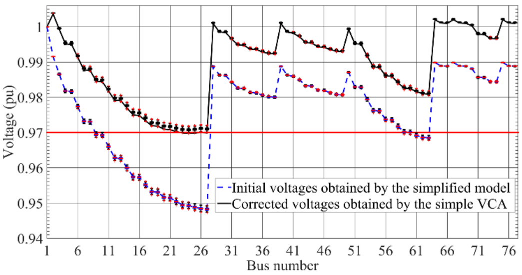
In this section, thermal dependency of lines is taken into consideration as the source of uncertainty. The RVCA is utilized to manage the voltage control problem of the studied working points under uncertainty of the line resistances due to the thermal dependency effect. Table 4 and figure 6 present the RVCA results corresponding to the voltage rise and drop conditions.

In the voltage drop case, as stated in section 4, it is considered that the line resistances can vary within the range of ±5.8% of their nominal values due to the thermal dependency effect. This creates voltage variations around the initial voltages (obtained by neglecting thermal dependency of branch resistances) as seen in figure 6(a). The RVCA solution must remain immunized against the worst possible realization of the uncertainty. Therefore, the perturbations that create the worst uncertainty scenarios are selected to compose the structural constraints of the RO problem. The worst uncertainty scenario in the voltage drop condition corresponds to the case in which the initial voltages are equal to their minimum in boxplots shown in figure 6(a) and the absolute values of voltage sensitivity indexes are reduced at most by the thermal dependency impact. Figure 6(a) confirms that when the RVCA solution is applied to N_2 simulated scenarios (that take the model uncertainty impact into account), the boxplots of the corrected voltages do not violate the permitted voltage range. In order to be protected against the uncertainty associated with the thermal dependency of line resistances, the reactive power changes of DG5 has changed from -1.266 Mvar (i.e., the simple VCA results given in table 1) to -1.403 Mvar as it can be seen in table 4.

Table 4 Robust VCA results considering the uncertainty associated with thermal dependency of lines

	Voltage drop	Voltage rise
ΔQ_{DGx} (Mvar)	DG5=-1.403	DG5=1.876 DG18=0.544
ΔP_{DGx} (MW)	NA	NA
ΔTap_{TR}	2	-4
OF	4.103	7.631

In the voltage rise condition, thermal dependency effect creates bigger voltage variations compared to the ones in the voltage drop situation as it can be noticed from boxplots of initial voltages shown in figures 6(a) and 6(b). It is explained by the fact that the defined range for the resistance variation (due to the thermal dependency effect) is wider in the voltage rise condition. In order to have a protected solution against the uncertainty effect in the voltage rise condition, the reactive powers of DG18 and DG5 are increased by 0.544 and 0.513 Mvar, respectively, with respect to the results of the simple VCA (given in table 1). Figure 6(b) verifies that the solution of the RVCA is immunized against realization of N_2 scenarios in the voltage rise condition.



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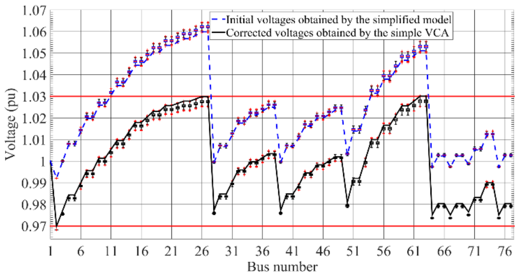

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Fig. 6. *The voltages obtained using simplified network component models as well as boxplots of initial and corrected voltages considering the uncertainty associated with thermal dependency of lines*
(a) *in the voltage drop case,* ***(b)*** *in the voltage rise case*

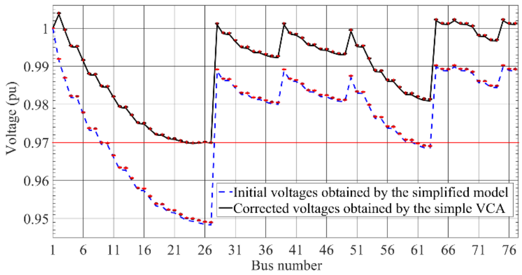
6.4. On the uncertainty linked with the shunt admittances of the lines

The RVCA performance is tested here when shunt admittances of the lines are considered as the source of uncertainty. Table 5 and figure 7 present the RVCA results under uncertainty of shunt admittances of the lines.

In table 5 regarding the RVCA results in the voltage drop condition, it is seen that as a result of incorporation of shunt admittances, objective function of the RVCA is reduced with respect to the one of the simple VCA (given in table 1) from 3.897 to 3.81. It is due to the fact that shunt admittances of lines increase the initial node voltages with respect to the ones obtained by the simple line model as it can be seen in figure 7(a). Therefore, the severity of the voltage control problem is decreased when shunt admittances are taken into account. Consequently, a smaller value of control variable changes is needed for managing the voltage violations in the voltage drop condition.

Table 5 Robust VCA results considering the uncertainty associated with shunt admittances of lines

	Voltage drop	Voltage rise
ΔQ_{DGx} (Mvar)	DG5=-1.207	DG5=1.485 DG18=0.062
ΔP_{DGx} (MW)	NA	NA
ΔTap_{TR}	2	-4
OF	3.81	6.281



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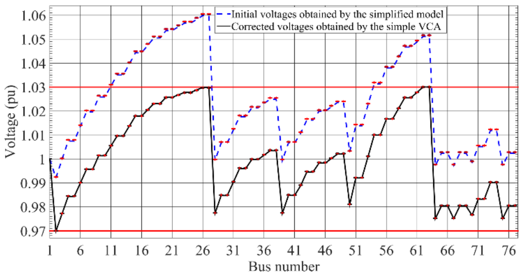

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Fig. 7. *The voltages obtained using simplified network component models as well as boxplots of initial and corrected voltages considering the uncertainty associated with shunt admittances of lines*
(a) *in the voltage drop case,* ***(b)*** *in the voltage rise case*

Unlike the voltage drop condition, in the voltage rise situation, in order to have a protected solution against the uncertainty of shunt admittances, the objective function of the RVCA increases with respect to that of the simple VCA as it can be seen in tables 1 and 5. From figure 7(b), it is noticed that boxplots of initial voltages in N_1 scenarios are found to be in above of the initial voltages obtained by the simple line model meaning that the studied uncertainty has raised the nodal voltages. As a consequence, the RO solution must have a bigger value to provide the needed protection against the worst uncertainty case. Figure 7(b) confirms that the solution of the RVCA remains protected under N_2 realizations of shunt admittance values.

6.5. On the uncertainty linked with the internal resistance of transformer

The internal resistance of the substation transformer is considered to be an uncertain variable in this section. The RVCA is employed to manage the voltage constraints in the voltage rise and drop conditions when the internal resistance

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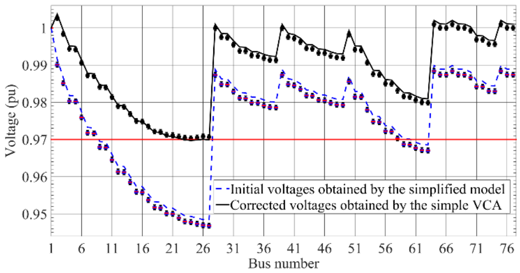
of the transformer has a random but bounded value given in section 4. Table 6 and figure 8 present the RVCA results under uncertainty of the transformer resistance.

Table 6 Robust VCA results considering the uncertainty associated with internal resistance of transformer

	Voltage drop	Voltage rise
ΔQ_{DGx} (Mvar)	DG5=-1.534	DG5=1.911 DG18=0.644
ΔP_{DGx} (MW)	NA	NA
ΔTap_{TR}	2	-4
OF	4.298	7.864

From table 6, it can be noticed that in both voltage drop and rise cases, the RVCA solution has a bigger objective function value compared to its counterpart obtained by the simple VCA (given table 1). This means that in practice, the solution obtained by the simple VCA will not be sufficient to solve the voltage control problem of the considered points due to the uncertainty that exists in the value of the transformer internal resistance. Figure 8 demonstrates that the solution of the RVCA remains immunized against N_2 realizations of the transformer resistance in both voltage rise and drop conditions since boxplots of the corrected voltages do not exceed the permitted voltage range.



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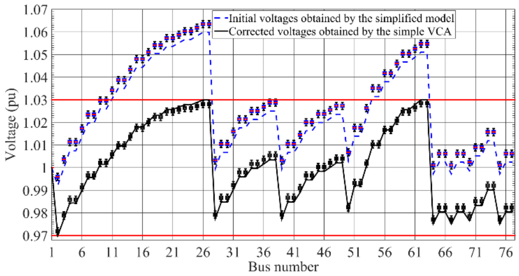

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Fig. 8. *The voltages obtained using simplified network component models as well as boxplots of initial and corrected voltages considering the uncertainty associated with the internal resistance of transformer*
(a) *in the voltage drop case,* ***(b)*** *in the voltage rise case*

In order to be protected against the uncertainty of the transformer resistance, the objective function of the RVCA has increased by 0.401 in the voltage drop and 1.82 in the voltage rise conditions (with respect to the objective function of the simple VCA given in table 1). Therefore, it can be concluded that the studied uncertainty has a more important effect on the voltage rise case though the defined range for variation of the transformer resistance is identical in both voltage rise and drop cases. It is explained by the fact that in the studied UKGDS, total powers of DGs are almost three times bigger than sum of the load powers. Therefore, in the voltage rise case where DG powers are maximal, internal resistance of transformer can create bigger impacts on the node voltages compared to the voltage drop case where the load powers are at their maximum values.

6.6. On the uncertainty linked with the load, line and transformer models

In the last case study, the uncertainties are considered to be arisen simultaneously from the load, line, and transformer models. The RVCA performance under uncertainties of the network component models is evaluated on the same working points as before corresponding to the voltage rise and drop conditions. Prior to forming the RO formulation, in order to characterize the uncertainties and to evaluate their impacts, it is needed to create scenarios for the considered uncertain variables, which are α, β and *PF* for the loads, ΔR and $b/2$ for the lines and R_T for the substation transformer. Considering the voltage dependency of load, the load power factor will be in function of α and β since by changing the voltage dependency exponents, active and reactive powers of load, as well as the load power factor will be changed. Therefore, in the current case, once the scenarios for α, β and *PF* are created, the load powers are calculated considering the uncertainties linked with the power factor and voltage dependency of loads according to the formulation presented in [12].

Given that there are more uncertain variables in the current case, number of 500 scenarios (N_1=500) that is used in the previous cases would not be sufficient to capture all the important possible realizations of the mentioned uncertainties. On the other hand, due to the constraint regarding the execution time of the RVCA, it is not possible to increase N_1. In order to deal with this issue, in the pre-processing stage of the RVCA, the uncertain variables that have bigger impacts on the voltage control problem are only taken into consideration. In the voltage rise condition, it was shown that the transformer resistance and thermal dependency of line resistances have led to the biggest changes of the RVCA objective function (with respect to the one of the simple VCA). In the voltage drop condition, the uncertainties associated with the load models and transformer resistance have resulted in the biggest variations of the RVCA objective function. Therefore, the RO is constructed considering the selected sources of the uncertainty as mentioned in above in each of the voltage rise or drop case. Once the RO problem of the RVCA is solved, then, in order to validate the results, the nodal voltages are evaluated considering the solution of the RO when all uncertain variables of the network component models are taken into account simultaneously. In the post-processing stage, we increase the total number of scenarios to 5000 (N_2=5000) since the RVCA decision is already made. It should be noted that the total number of scenarios created in the pre-processing stage remains unchanged equal to 500. Table 7 presents the RVCA results under uncertainties of network component models, and figure 9 shows the boxplots of node voltages.

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Table 7 Robust VCA results considering the uncertainties associated with models of load, line and transformer

	Voltage drop	Voltage rise
ΔQ_{DGx} (Mvar)	DG5=-2.666	DG4=0.0702 DG5=2.31 DG18=1.194
ΔP_{DGx} (MW)	NA	NA
ΔTap_{TR}	2	-4
OF	5.993	9.361

As it can be noticed from table 7, the objective function of the RVCA has raised in both voltage rise and drop conditions (compared to that of the simple VCA given in table 1) due to presence of the model uncertainty. This means that the solution of the simple VCA can be insufficient to solve the voltage control problem of the real case. The voltage results shown in figure 9 reveal that the considered simplification of the uncertainty sources in the pre-processing stage of the RVCA does not create voltage violation when all uncertainties are included in the result validation (post-processing) stage since the boxplots of the corrected voltages are within the permitted voltage limits. Therefore, it can be concluded that within the considered variation range of the uncertain variables in this work, uncertainties linked with the thermal dependency of lines and internal resistance of transformer have the most important effects on the voltage control problem of the studied system in the voltage rise condition. For the voltage management in the voltage drop case, load and transformer models are recognized as the most important sources of the model uncertainty.

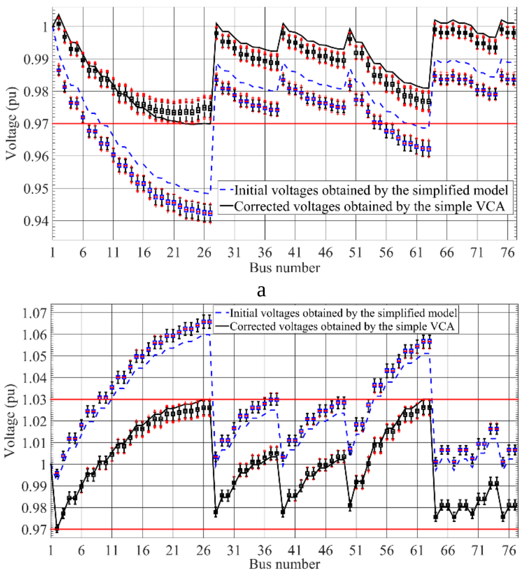

Fig. 9. *The voltages obtained using simplified network component models as well as boxplots of initial and corrected voltages considering the uncertainties associated with the load, line, and transformer models*
(a) *in the voltage drop case,* ***(b)*** *in the voltage rise case*

6.7. On the robustness level of the solutions

The result obtained by the RVCA considering the interval uncertainty set (Ψ=1) appears to be conservative since it is protected against the worst realization of uncertainty. In order to moderate the conservatism of the interval uncertainty set, the border of the uncertainty space can be reduced. In this regard, we investigate performance of the RVCA when the border of the box uncertainty space (i.e. Ψ) is reduced to 0.5 and to 0. It should be noted that the latter case corresponds to the deterministic formulation of the VCA and its relevant results have been reported in table 1. The RVCA results in the voltage drop and rise conditions when Ψ=0.5 are given in table 8.

Comparing the results given in table 7 (related to Ψ =1) with the ones of table 8 (related to Ψ =0.5) and table 1 (where Ψ =0), it is confirmed that the objective function of the RVCA with the reduced uncertainty space (Ψ <1) is smaller than that of the RVCA with the interval uncertainty set (Ψ =1) in both voltage rise and drop conditions. Therefore, the conservatism of the interval uncertainty set is moderated by decreasing the size of the box uncertainty space. However, in such a case, the solution of the RVCA will not be any more protected against all possible realizations of uncertainty. In other words, when changing Ψ from 1 to zero, on the one hand, the conservatism of the interval uncertainty set is reduced, but on the other hand, the voltage violations may appear in the system nodes.

Table 8 Robust VCA results considering the uncertainties associated with models of load, line and transformer with Ψ =0.5

	Voltage drop	Voltage rise
ΔQ_{DGx} (Mvar)	DG5=-2.601	DG5=1.843 DG18=0.592
ΔP_{DGx} (MW)	NA	NA
ΔTap_{TR}	1	-4
OF	4.902	7.654

6.8. On the adaptation of the proposed RVCA for the voltage management of 3-phase unbalanced systems

In most of studies carried out on the MV distribution systems, the phase imbalance factor has been neglected and the single-phase equivalent model of the 3-phase system has been taken into account [1-3], [5-12]. Although in the MV level, 3-phase power lines are symmetrical and injected powers from DG units (i.e. mostly wind turbines) are balanced within the three phases, the node powers aggregated and transferred from the low-voltage level are not balanced due to single-phase consumptions and generations in the low-voltage level. As a result, the MV distribution systems are not perfectly balanced in reality. In order to quantify the magnitude of the phase imbalance phenomenon within the studied system, we can compute the Voltage Unbalance Factor (VUF). According to the IEEE definition [30], the voltage unbalance is expressed as the ratio of the negative sequence voltage component V_n to the positive sequence voltage component V_p. The negative sequence voltages mainly result from unbalanced loads or single-phase

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components that infer asymmetric currents flowing in the network.

In this section, it is attempted to modify the proposed RVCA so that it can consider the phase imbalance factor. The limitation that we have in this regard is that voltage control methods in the MV level (e.g.: adjustment of transformer tap position and management of DG active and reactive powers) act equally on the three phases of the system. This means that we cannot separately correct the voltage of each phase in an optimal manner. Consequently, in order to ensure that voltage violations of three phases will be removed completely, we need to find the phase with the worst voltage violation, and then to apply the corrective solution corresponding to that phase to all three phases. Within this framework, the proposed RVCA shown in figure 1 is modified as follows in order to be able to manage the voltage violations of the unbalanced MV systems.

Firstly, in the pre-processing stage, the NRLF study method is replaced with a new algorithm that is capable of analysing the 3-phase unbalanced systems. The load flow method developed in [31] is used in this regard. It enables us to find the phase with the worst voltage violation while being able to compute the VUF at each node of the system. In the second stage, the RO principles as explained in section 3.2 are applied to the phase with the worst voltage violation. Solution of the RO formulation defines the new set-points of control variables which are obtained according to the phase with the worst voltage violation. This solution is then applied to the (3-phase) voltage control devices of the under-study network. Finally, in order to verify that the RO solution manages correctly the voltage violations of all phases, we use again the load flow method for the unbalanced systems in the post-processing stage.

In order to validate the efficiency of the modified RVCA in voltage management of a 3-phase unbalanced system, its performance under uncertainty of line resistances is tested when the 77-bus UKGDS feeds unbalanced 3-phase loads. The working point related to the voltage drop condition is considered here. It is assumed that the load power at each phase and each node can randomly take values ranging from 70% to 100% of its nominal value. It should be noted that in the voltage rise case, the load powers are small; consequently, impacts of imbalance factor are proved to be negligible in the latter case.

In the first step, before application of the robust optimization, VUF is computed for each node of the studied MV network, and for each of the N_1 scenarios in the pre-processing stage. Obtaining an average VUF of 0.05%, it is concluded that the voltage imbalance is not an important issue in the MV level. According to the standard EN50160 (i.e. European standard defining the requirements in distribution systems), the VUF shall be within the range of 0–2% over 95% of a weekly period. In the studied MV network, the VUF is always considerably inferior to this 2% limitation.

Regarding the RVCA results, by doing the load flow calculations in the pre-processing stage, it is found that the biggest voltage drop occurs in phase B of bus 27. Therefore, the RO formulation is built according to voltage violations of phase B. Solution of the RO defines that the reactive power of DG5 should be changed by -1.14 Mvar and transformer tap position should be moved to one higher position. Figure 10 presents the corrected voltages in the 3 phases of bus 27 obtained in the post-processing stage. As it can be seen, the corrective solution of the RVCA determined according to phase B manages the voltage violations of the three phases in all the N_2 scenarios of the post-processing stage. This confirms the robustness of the modified voltage control algorithm in presence of the voltage imbalance phenomenon.

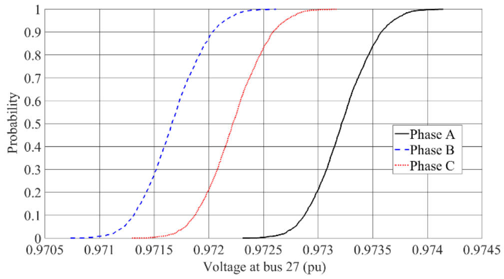

Fig. 10. *The CDFs of three phase voltages at bus 27 obtained by the RVCA in the voltage drop condition.*

Moreover, it is worth observing that in figure 10, the corrected voltage of phase B is closer to the lower permitted voltage limit (i.e. 0.97) since the RO solution is obtained with respect to voltage violations of this phase. Finally, by computing the VUF after the RVCA action, we observe that the defined control solution improves slightly (by around 3%) the quality of the voltage balance in the considered MV network (the mean VUF is moved from 0.05% to 0.0485% when the RVCA process has been executed).

7. Conclusion

This paper addresses the voltage control problem of the MV distribution systems under uncertainty of the network parameters. A RVCA is developed in order to manage the voltage constraints considering uncertainties associated with the parameters of load, line and transformer models. On the basis of the simulation results, it is found that although the model uncertainty in most of the studied cases has led to an increase of the objective function of the RVCA with respect to that of the simple VCA, in certain cases, the severity of the voltage control problem is reduced when considering the model uncertainty. For instance, the uncertainty linked with the voltage dependency of loads smooths the structural constraints of the RVCA such that the voltage control problem considering that source of uncertainty is solved with a smaller value of objective function compared to the case of neglecting the load-voltage uncertainty in the simple VCA. However, it is observed that when cumulative effects of the studied uncertainties are taken into account, in both voltage rise and drop conditions, the objective function of the RVCA is raised with respect to the simple VCA one. This indicates that the solution of the simple VCA can be insufficient to solve the voltage control problem of the real cases. Moreover, it has been shown that border of the uncertainty space can be adjusted in order to control the conservatism of the RO solution. In the end, the proposed RVCA has been modified for the voltage management of the 3-phase unbalanced systems. As the future work, application of the chance constrained optimization can be studied. It enables us to define the desired probability of satisfying the operational constraints in order to eventually control the conservatism of the solution.

8. References

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