Average-Energy Games

Patricia Bouyer[1] Nicolas Markey[2] Mickael Randour[3]
Kim G. Larsen[4] Simon Laursen[4]

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[4]Aalborg University

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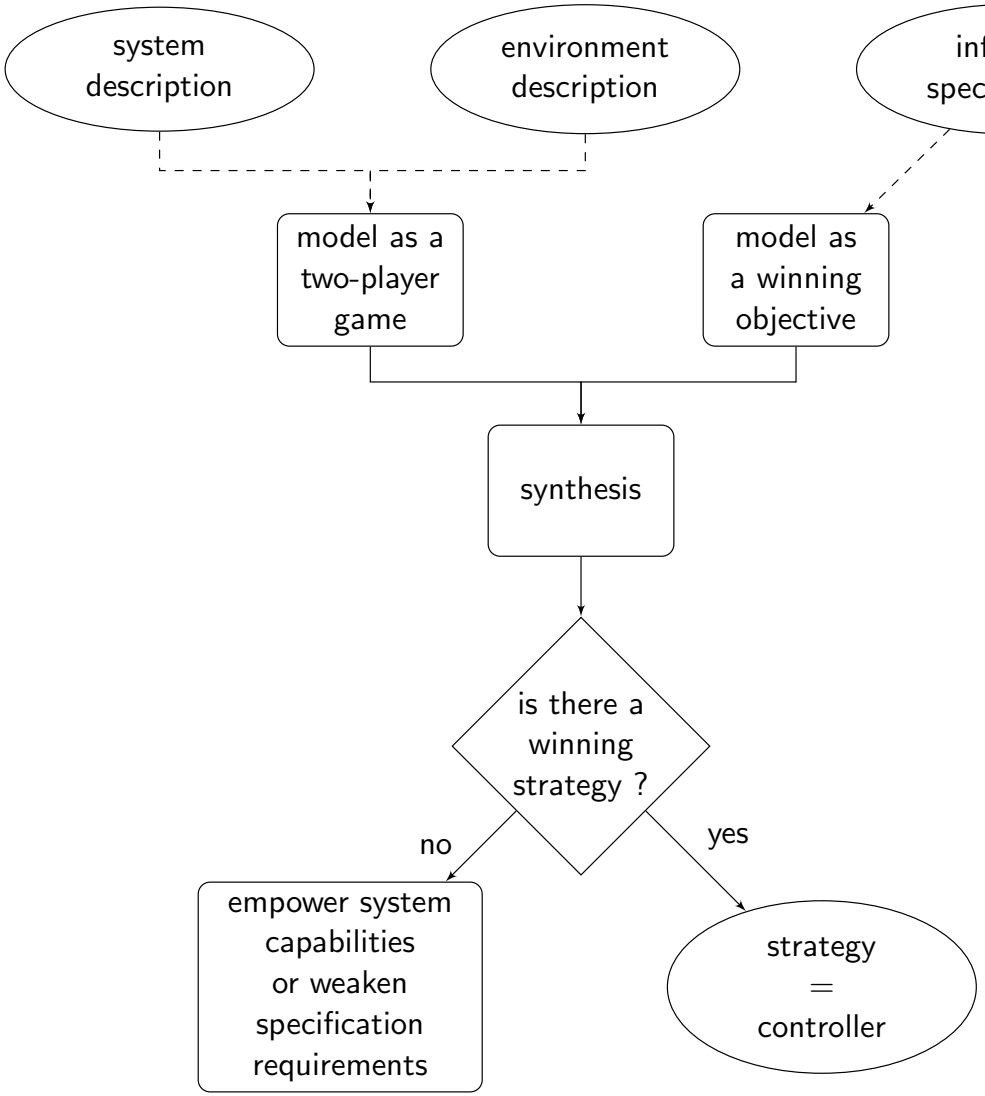
[2] Computer Science Department, Université libre de Bruxelles (ULB), Belgium
[3] Aalborg University, Denmark

Abstract. Two-player quantitative zero-sum games provide a natural framework to synthesiz
trollers with performance guarantees for reactive systems within an uncontrollable environment
sical settings include mean-payoff games, where the objective is to optimize the long-run avera
per action, and energy games, where the system has to avoid running out of energy.
We study *average-energy* games, where the goal is to optimize the long-run average of the accu
energy. We show that this objective arises naturally in several applications, and that it yields int
connections with previous concepts in the literature. We prove that deciding the winner in su
coNP and at least as hard as solving mean-payoff games, and we establish that m
We also consider the case where the system has to minimize the avera
within predefined bounds at all times: this corre
results for one-player and two-pla

General context: strategy synthesis in quantitative games



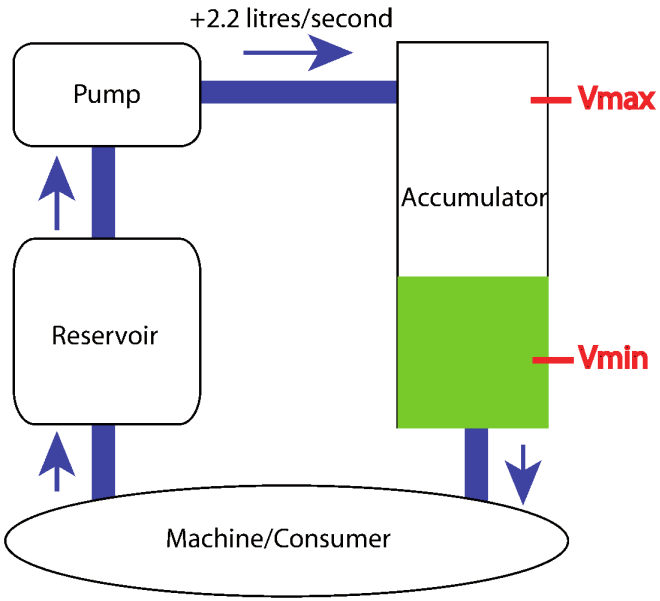
1. How complex is it to decide if a winning strategy exists?
2. How complex such a strategy needs to be? **Simpler is better**.
3. Can we synthesize one efficiently?

$\Rightarrow$ Depends on the *winning objective*.

Motivating example

HYDAC oil pump industrial case study [CJL+09] (Quasimodo research project).

Goals:



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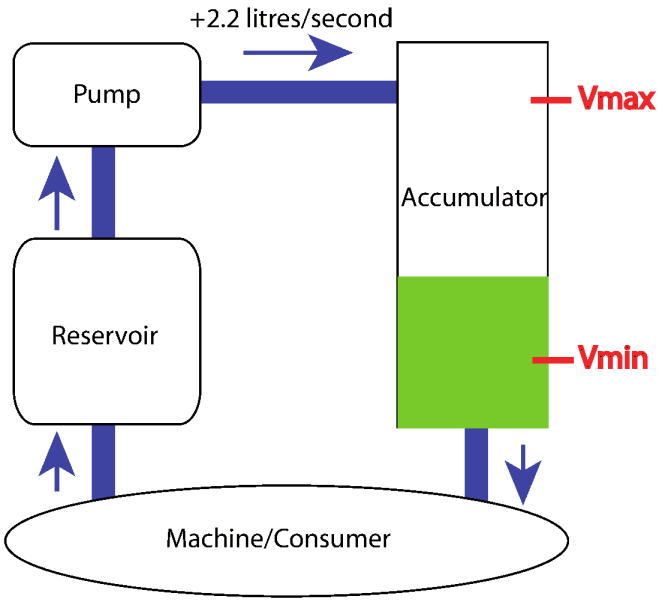

Goals:

1. Keep the oil level in the safe zone.
   ↪ **Energy objective with lower and upper bounds:** EG_{LU}

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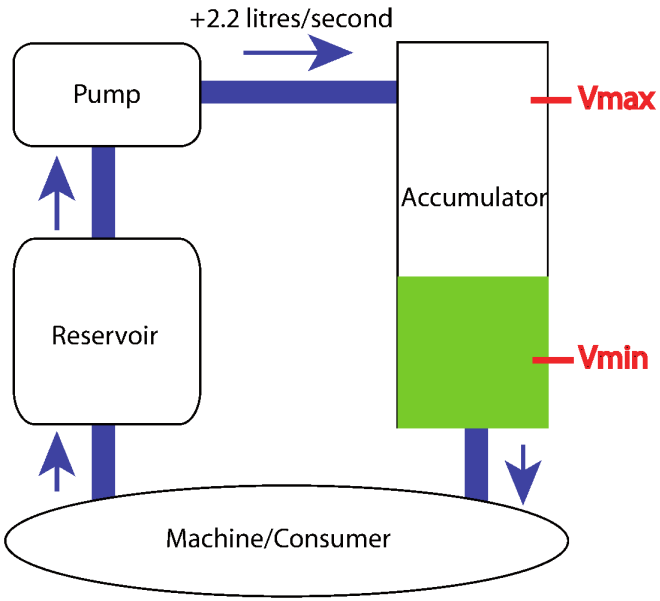

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   ↪ **Average-energy objective:** AE

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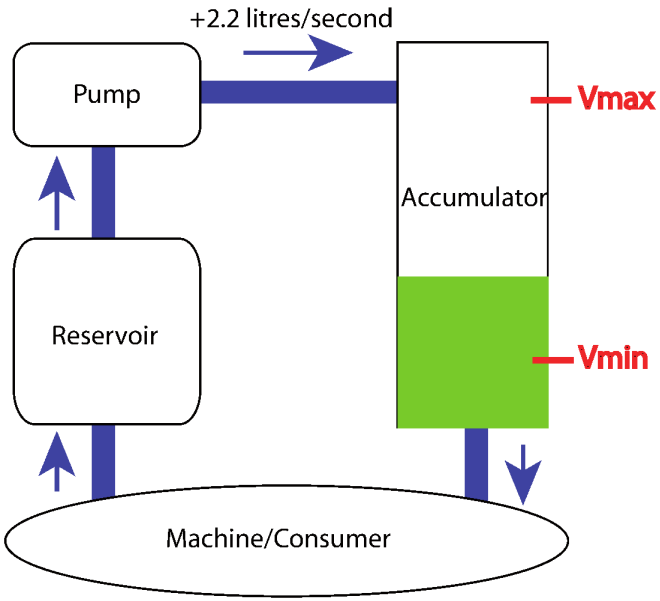

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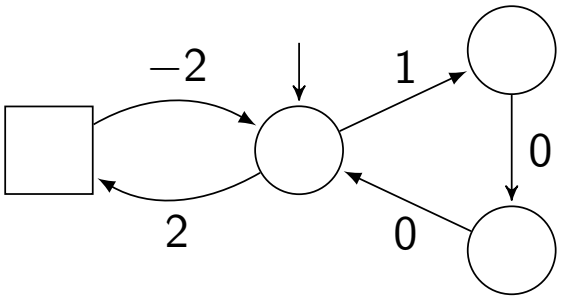
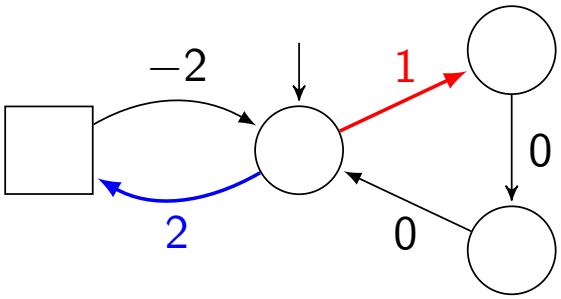
⇒ **Conjunction:** AE_{LU}

Average-energy: illustration



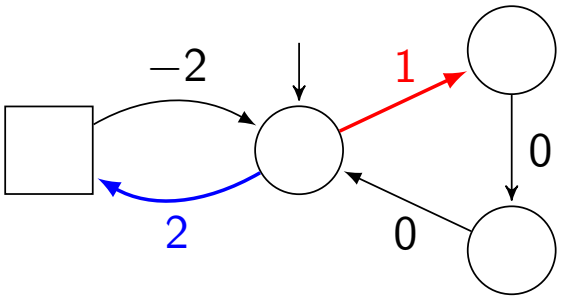
- Two-player turn-based games with integer weights.

Average-energy: illustration



- Two-player turn-based games with integer weights.
- Focus on two *memoryless* strategies.

Average-energy: illustration



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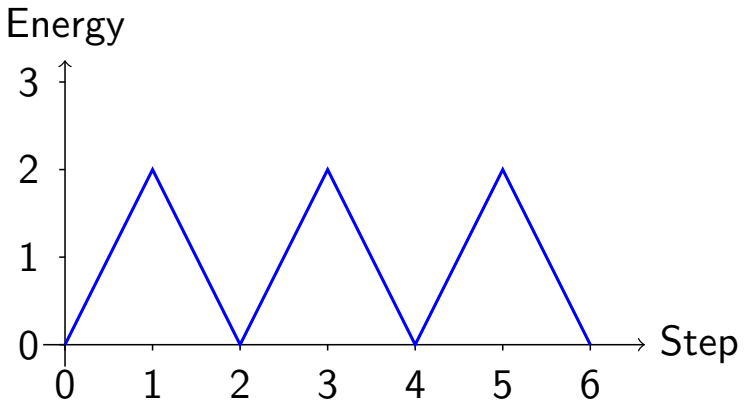
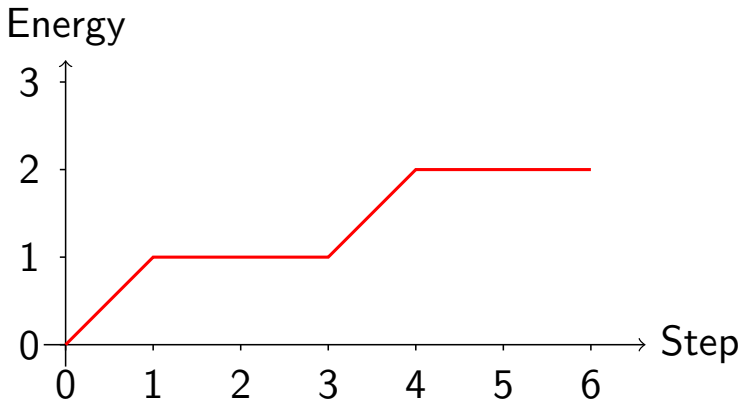
$\Longrightarrow$ We look at the **energy level** (*EL*) along a play.





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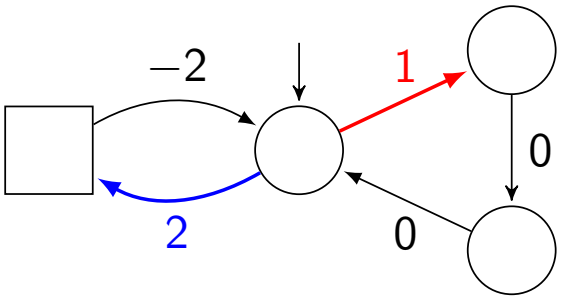

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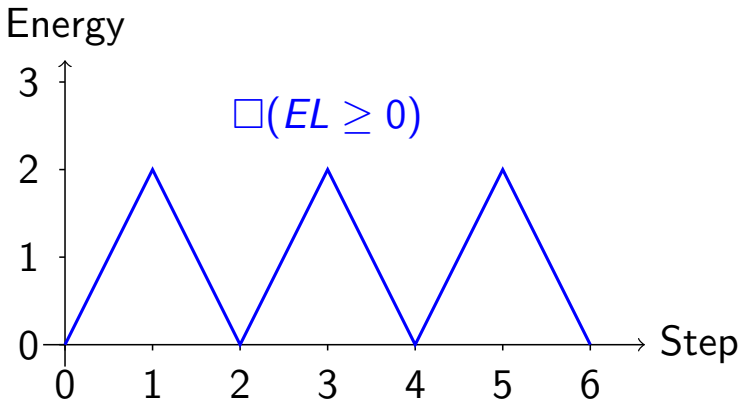
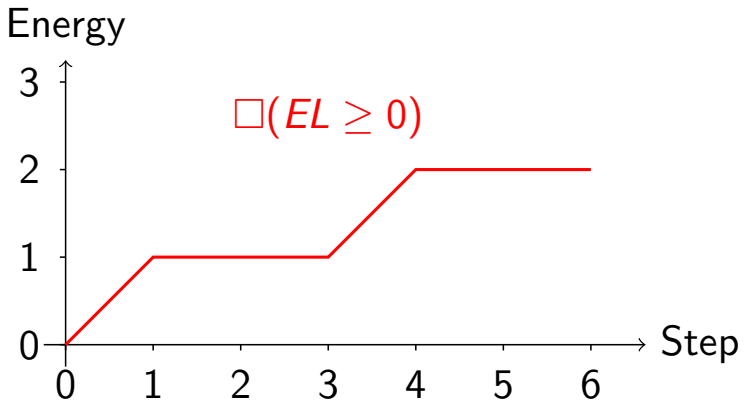
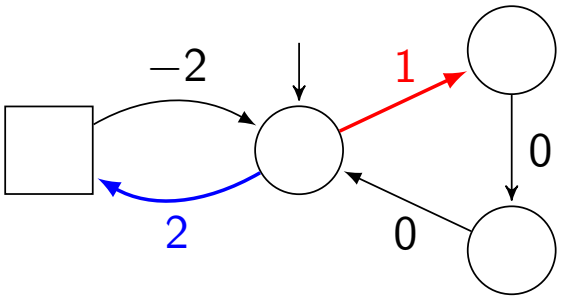




Energy objective (EG_L/EG_{LU}): e.g., always maintain $EL \geq 0$.

Average-energy: illustration



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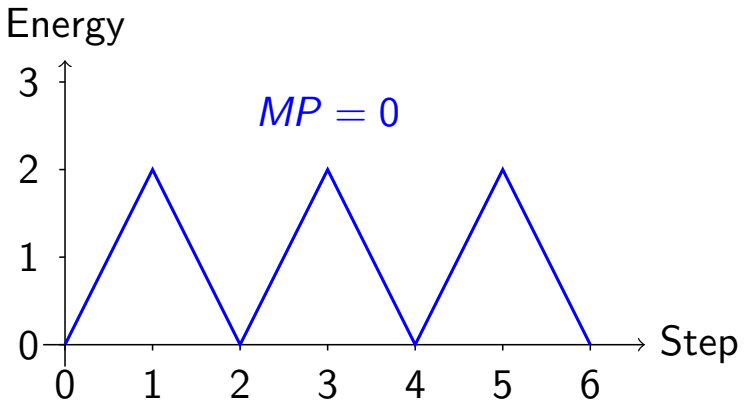
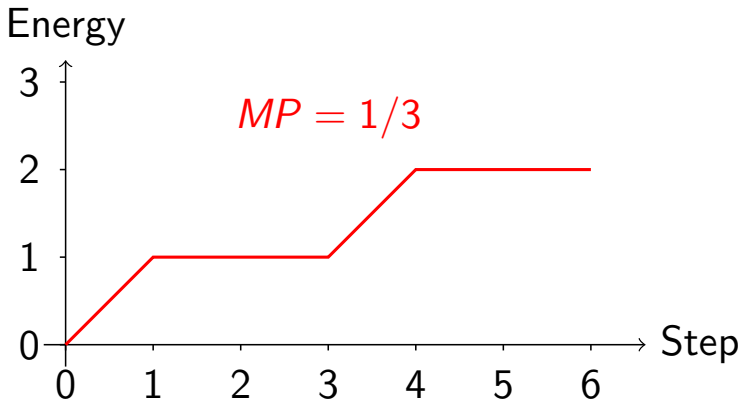
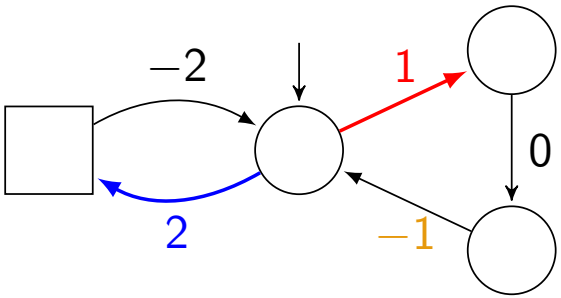




Mean-payoff (*MP*): long-run average payoff per transition.

Average-energy: illustration



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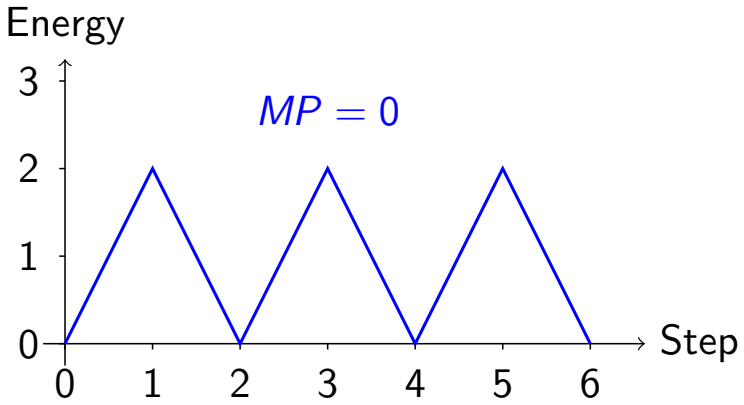
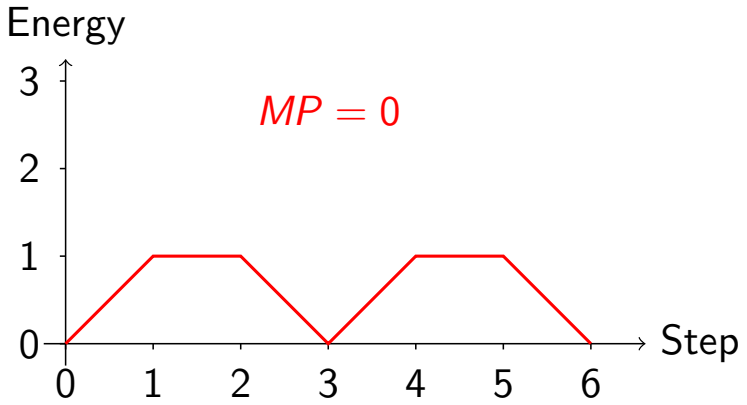
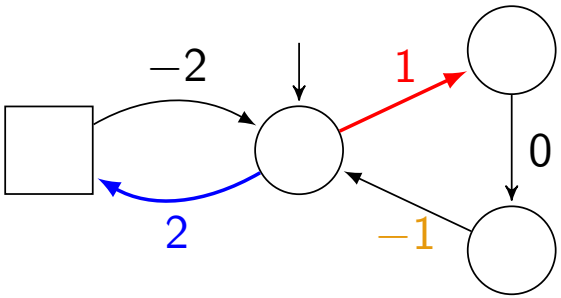




Mean-payoff (*MP*): long-run average payoff per transition.

$\Longrightarrow$ **Let's change the weights of our game.**

Average-energy: illustration



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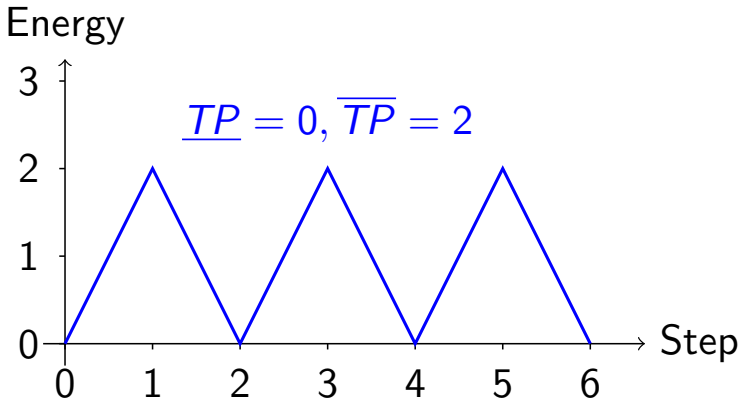
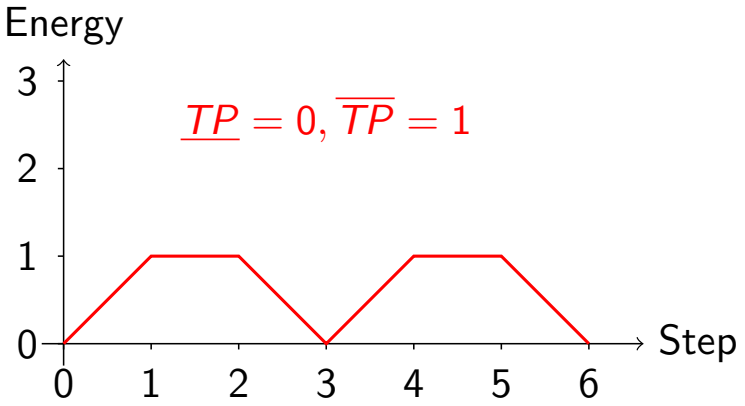
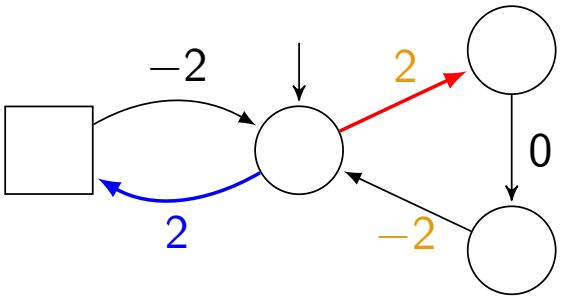




Total-payoff (*TP*) *refines MP* in the case $MP = 0$ by looking at high/low points of the sequence.

Average-energy: illustration



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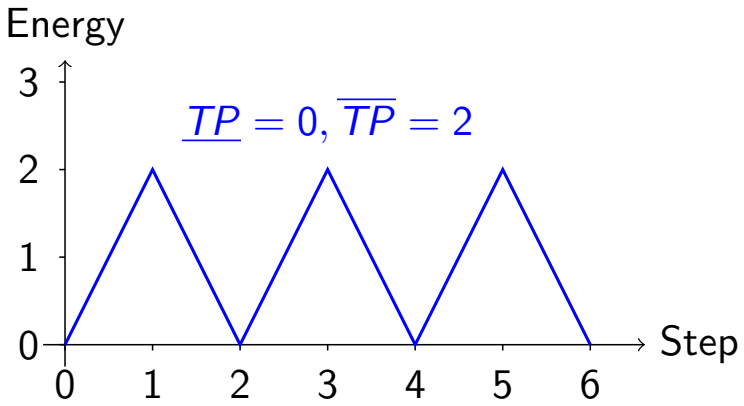

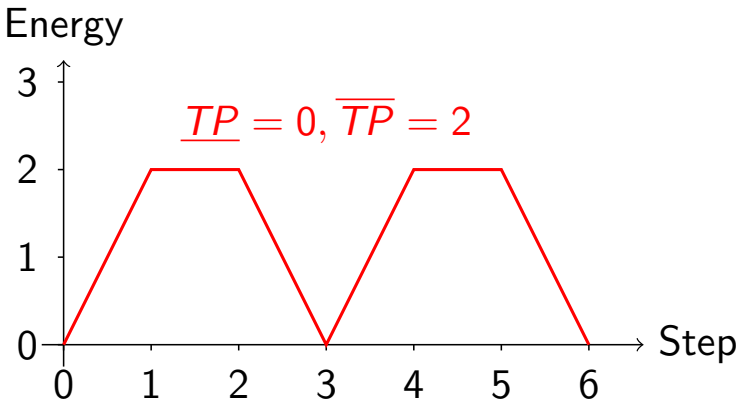

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$\Longrightarrow$ **Let's change the weights again.**

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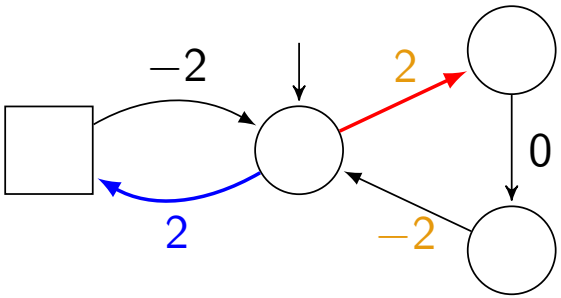

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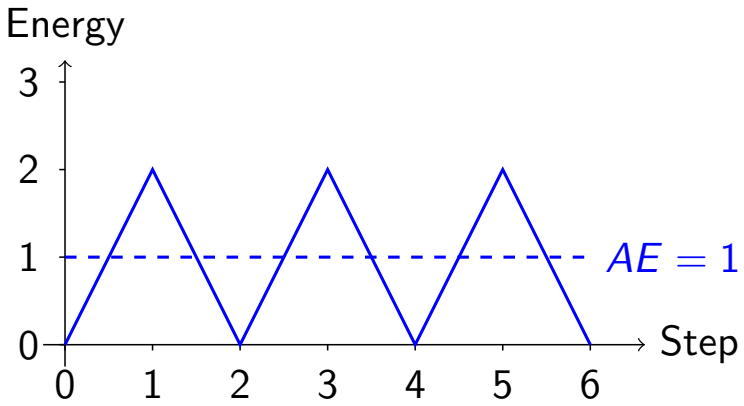
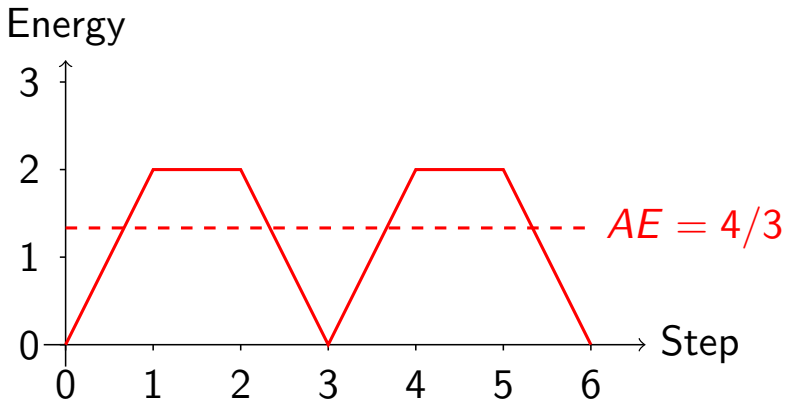
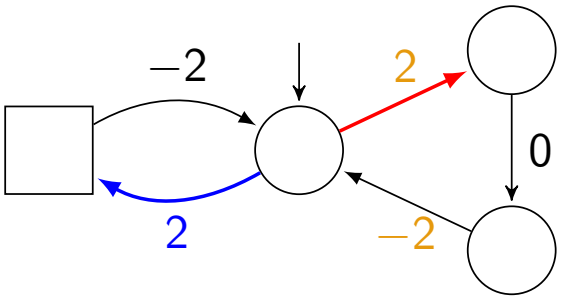




Average-energy (*AE*) *further refines TP*: average *EL* along a play.

Average-energy: illustration



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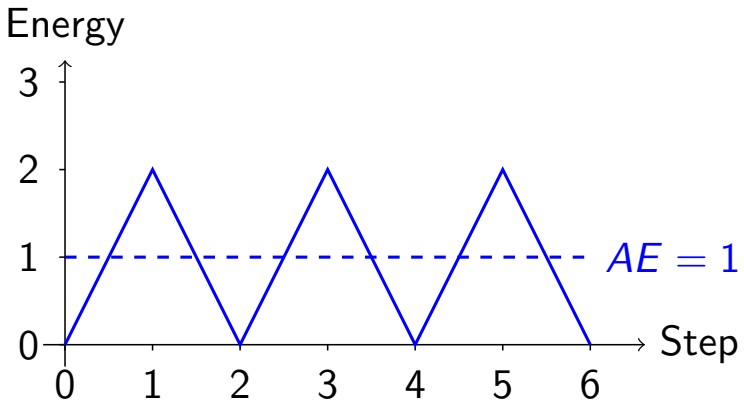

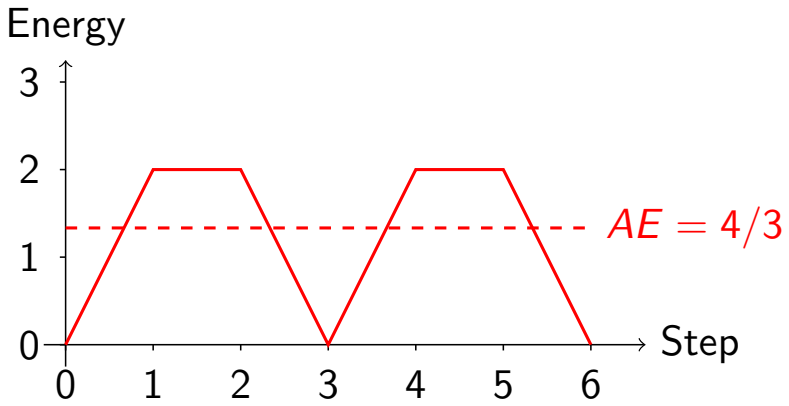

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$\Longrightarrow$ **Natural concept (cf. case study).**

Average-energy: overview

Objective	**1-player**	**2-player**	**memory**
MP	P [Kar78]	NP ∩ coNP [ZP96]	memoryless [EM79]
TP	P [FV97]	NP ∩ coNP [GS09]	memoryless [GZ04]
EG_L	P [BFL+08]	NP ∩ coNP [CdAHS03, BFL+08]	memoryless [CdAHS03]
EG_{LU}	PSPACE-c. [FJ13]	EXPTIME-c. [BFL+08]	pseudo-polynomial
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- For all but EG_{LU}, *memoryless* strategies suffice.

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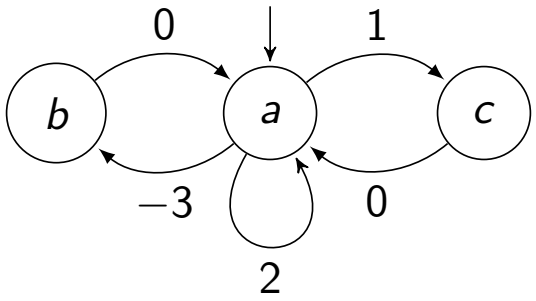
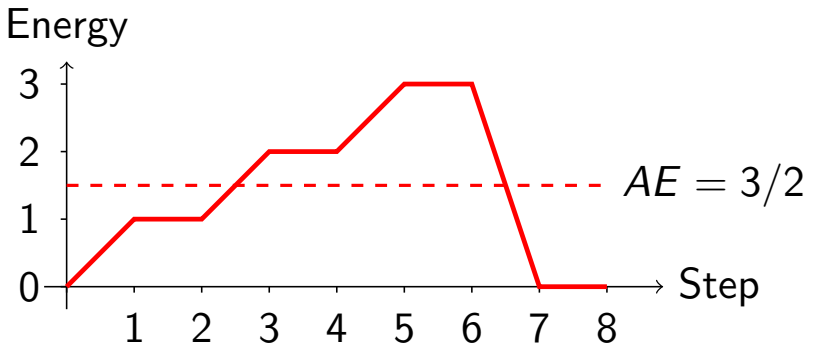
- *MP*-hardness.

With energy constraints, memory is needed!

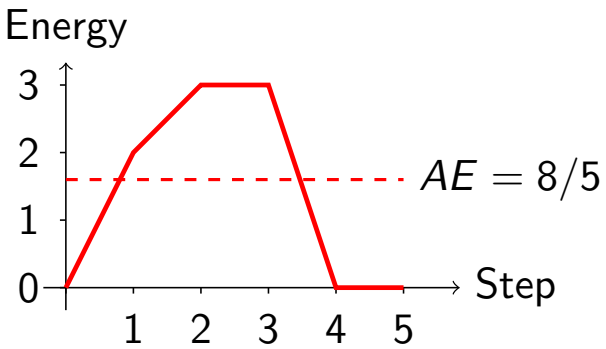
$AE_{LU} \rightsquigarrow$ minimize AE while keeping $EL \in [0, 3]$ (init. $EL = 0$).



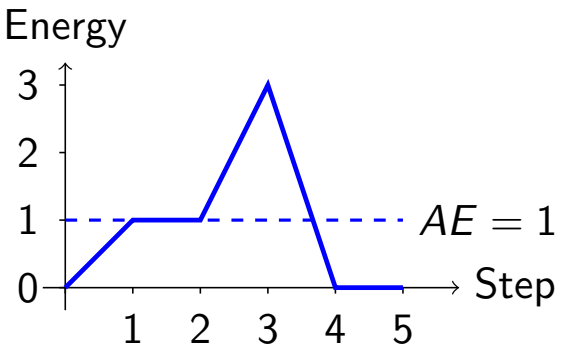
(a) One-player AE_{LU} game.



(b) Play $\pi_1 = (acacacab)^\omega$.



(c) Play $\pi_2 = (aacab)^\omega$.



(d) Play $\pi_3 = (acaab)^\omega$.

Minimal AE with π_3: alternating between the $+1$, $+2$ and -3 cycles.

With energy constraints, memory is needed!

$AE_{LU} \rightsquigarrow$ minimize AE while keeping $EL \in [0, 3]$ (init. $EL = 0$).

Non-trivial behavior in general!

$\hookrightarrow$ Need to choose carefully which cycles to play.

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Non-trivial behavior in general!
$\hookrightarrow$ **Need to choose carefully which cycles to play.**

The AE_{LU} problem is EXPTIME-complete.

$\hookrightarrow$ Reduction from AE_{LU} to AE on pseudo-polynomial game
($\Rightarrow$ $AE_{LU} \in$ NEXPTIME $\cap$ coNEXPTIME).

$\hookrightarrow$ Reduction from this AE game to MP game $+$
pseudo-poly. algorithm.

With energy constraints: results overview

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AE	P	NP ∩ coNP	memoryless
AE_{LU} (poly. U)	P	NP ∩ coNP	polynomial
AE_{LU} (arbitrary)	EXPTIME-e./PSPACE-h.	EXPTIME-c.	pseudo-polynomial
AE_L	EXPTIME-e./NP-h.	*open*/EXPTIME-h.	*open* ($\geq$ pseudo-poly.)

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AE_{LU} (poly. U)	P	NP ∩ coNP	polynomial
AE_{LU} (arbitrary)	EXPTIME-e./PSPACE-h.	EXPTIME-c.	pseudo-polynomial
AE_L	EXPTIME-e./NP-h.	*open*/EXPTIME-h.	*open* ($\geq$ pseudo-poly.)

$\Longrightarrow$ *Good news: we are closing in on the open problem and believe it to be* EXPTIME*-complete.*

Wrap-up

"New" quantitative objective.[1]

- ▷ Practical motivations (e.g., HYDAC).
- ▷ "Refines" *TP* (and *MP*).
- ▷ Same complexity class as EG_L, *MP* and *TP* games.
- ▷ AE_{LU} well-understood.
- ▷ Closing in on AE_L.

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Thank you! Any question?

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