Hairy Black Holes & Boson Stars : From shift-symmetry to spontaneous scalarization

Ludovic Ducobu

Theoretical and mathematical physics

University of Mons

UMONS

Faculté des Sciences

complexys
UMONS RESEARCH INSTITUTE FOR COMPLEX SYSTEMS

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Plan

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- Allow to explain many phenomenons :
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  2. Existence and shape of gravitational waves : GW150914 (2016)
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. . . there are some unexplained phenomena within General Relativity (GR) :

- Origin and value of the cosmological constant
- Low intensity of gravitational interaction
- Existence of singularities within space-time
- Origin and composition of dark matter and dark energy
- Accelerated expansion of the universe

Not all of them are related to quantum correction problems !

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In GR, all the degrees of freedom are encoded in the metric $g_{\mu\nu}$.
But, formally, the equivalence principle does not rule out the possible existence of other kind of fields in the model.

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- Simplest covariant object
- Important element of many models :
  - Cosmology
  - Standard model of particle physics
  - Kaluza-Klein reduction
  - Effective theory
  - ...
- Also experimentally motivated since the Brout-Englert-Higgs boson's discovery (CERN 2012)

Introduction : Why not considering the simplest case ?

Why not just using $\mathscr{L}_{EKG} = \kappa \left(R - 2\Lambda \right) - \nabla_\mu \phi \nabla^\mu \phi - V(\phi)$?

Introduction : Why not considering the simplest case ?

No Hair Theorem (*Schematically*)

Consider an asymptotically flat black hole spacetime

Hypothesis 1 : (Symmetries of spacetime)

Hypothesis 2 : (Coupling condition)

Hypothesis 3 : (Symmetries of the scalar field)

Hypothesis 4 : ("Energetic" condition)

Then, *the scalar field must be trivial :* $\phi\left(x^{\mu}\right)=c^{te}, \forall x^{\mu}$.

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Then, *the scalar field must be trivial :* $\phi\left(x^{\mu}\right) = c^{te}, \forall x^{\mu}$.

Note : Generically, the proof makes **no use** of the Einstein's equations. It just uses the scalar field equation defined thanks to hypothesis 2.

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$$S = \int \left[\frac{1}{16\pi\mathcal{G}} R - \nabla_\mu \phi^* \nabla^\mu \phi - V(\phi) + f(\phi)\mathcal{I}(g) \right] \sqrt{-g} \mathrm{d}^4 x.$$

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If we assume that both $V(\phi)$ and $f(\phi)$ are functions of $|\phi| = \sqrt{\phi^* \phi}$, the model possess a global $U(1)$ symmetry : $\phi \to e^{i\alpha}\phi$.

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We will focus on a coupling to the Gauss-Bonnet invariant :

$$\mathcal{I}(g) = \mathcal{L}_{GB} \equiv R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}.$$

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The functions V and f are choosen as :

$$V(\phi) = m^2|\phi|^2 + \lambda_4|\phi|^4 + \lambda_6|\phi|^6,$$

$$f(\phi) = \gamma_1|\phi| + \gamma_2|\phi|^2.$$

Equations of motion

For the metric function :

For the scalar field :

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$$G_{\mu\nu} = 8\pi\mathcal{G}\left(T^{(\phi)}_{\mu\nu} + T^{(\mathcal{I})}_{\mu\nu}\right),$$

where

$$T^{(\phi)}_{\mu\nu} = \nabla_{(\mu}\phi\nabla_{\nu)}\phi^* - \left(\nabla_\alpha\phi^*\nabla^\alpha\phi + V(\phi)\right)g_{\mu\nu},$$

and

$$T^{(\mathcal{I})}_{\mu\nu} = -(g_{\mu\rho}g_{\nu\sigma} + g_{\nu\rho}g_{\mu\sigma})\boldsymbol{\varepsilon}^{\rho\alpha\gamma\delta}\boldsymbol{\varepsilon}^{\beta\sigma\lambda\tau}R_{\gamma\delta\lambda\tau}\nabla_\alpha\nabla_\beta f(\phi),$$

with $\boldsymbol{\varepsilon}^{\rho\alpha\gamma\delta}$ the Levi-Civita tensor.

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For the scalar field :

$$-\Box\phi = -\frac{\partial V}{\partial\phi^*} + \frac{\partial f}{\partial\phi^*}\mathcal{I}(g),$$

with $\Box = \nabla^\mu\nabla_\mu$.

Ansatz

For the metric function :

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We will focus on a spherically symmetric space-time.
On an appropriate coordinate system (t, r, θ, φ), the metric read

$$\mathrm{d}s^2 = -N(r)\sigma^2(r)\mathrm{d}t^2 + \frac{1}{N(r)}\mathrm{d}r^2 + g(r)(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\varphi^2),$$

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For the scalar field :

In the same coordinate system, we choose a scalar field of the form

$$\phi(x^\mu) = e^{-i\omega t}\phi(r),$$

where ω is a constant real parameter.

Reduced equations

Within this ansatz, the field equations can be rewritten in the form

$$N' = F_1(N, \sigma, \phi, \phi';\ \omega),$$
$$\sigma' = F_2(N, \sigma, \phi, \phi';\ \omega),$$
$$\phi'' = F_3(N, \sigma, \phi, \phi';\ \omega),$$

where the functions F_1, F_2 and F_3 are involved algebraic functions of the fields N, σ, ϕ and ϕ'.

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where the functions F_1, F_2 and F_3 are involved algebraic functions of the fields N, σ, ϕ and ϕ'.

Note that we can reduce ourself to a **real** scalar field via a $\omega \to 0$ limit.

Boundary conditions

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$$\phi'(r_h) = \frac{-r_h^2 \pm \sqrt{\Delta}}{8r_h(\gamma_1 + 2\gamma_2\phi(r_h))},$$

where

$$\Delta = r_h^4 - 96\gamma_1^2 - 384(\gamma_2^2\phi(r_h)^2 + \gamma_1\gamma_2\phi(r_h)).$$

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- Finally, we require asymptotic flatness :

$$\sigma(r \to \infty) = 1, \quad \phi(r \to \infty) = 0.$$

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- The assymptotic flatness is ensured by setting

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The behaviour of the solutions is only due to the coupling function

$$f(\phi) = \gamma_1\phi + \gamma_2\phi^2.$$

Shift-symmetry ($\gamma_1 \neq 0$, $\gamma_2 = 0$)

The equation of motion for ϕ read

$$\Box\phi = -\gamma_1 \mathcal{I}(g).$$

The condition of regularity at the horizon reduces to

$$\phi'(r_h) = \frac{-r_h^2 \pm \sqrt{\Delta}}{8 r_h \gamma_1},\ \Delta = r_h^4 - 96\gamma_1^2.$$

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Consequently, the condition of positivity of the discriminant Δ constraint the accessible values of γ_1 :

$$\Delta \geq 0 \Leftrightarrow \gamma_1 \leq r_h^2 \sqrt{1/96} \approx r_h^2 \times 0.1021.$$

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- In the following, we will focus on solutions corresponding to the "$+$" sign.
  - $\rightarrow$ Solution corresponding to "$+$" sign $\leftrightarrow$ appproach regularily Schwarschild solution in the $\gamma_1 \rightarrow 0$ litmit.
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- On this branch, solutions exists for $\gamma_1 \in \left[0, r_h^2\sqrt{1/96}\right]$.
- Since $\phi'(r_h)$ does only depend on r_h and γ_1, for a fixed r_h, there is only one possible solution for each value of γ_1. (no excited solutions)

Spontaneous scalarization ($\gamma_1 = 0$, $\gamma_2 \neq 0$)

The equation of motion for ϕ read

$$\Box\phi = -2\gamma_2\phi\mathcal{I}(g) \Leftrightarrow \hat{D}\phi = \gamma_2\phi.$$

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In this case the pattern of solutions is very different :

- Solutions exists only for $\gamma_2 \in [\gamma_{2,c}, \gamma_{2,\max}]$, whith $\gamma_{2,c} > 0$.
- Excited solutions exists.

Spontaneous scalarization ($\gamma_1 = 0$, $\gamma_2 \neq 0$)

Origin of the critical values

The existence of regular solutions require 3 conditions :

$$\Delta \geq 0,\ \gamma_2 \neq 0 \text{ and } \phi(r_h) \neq 0$$

→ $\gamma_{2,c}$: Correspond to $\Delta \to 0$.

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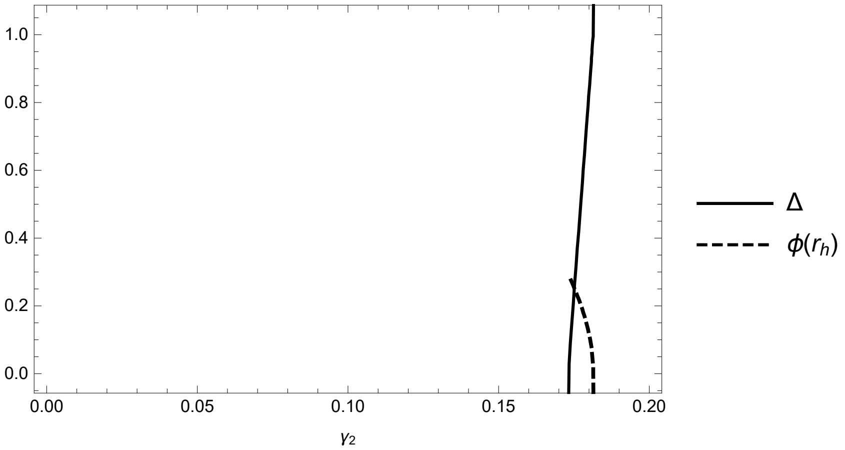
This pattern can be understood when examining the case of a test field :
On a fixed Schwarzschild background the equation for ϕ can be written as

$$\frac{r^4}{48M}\frac{\mathrm{d}}{\mathrm{d}r}\left[r^2\left(1-\frac{2M}{r}\right)\frac{\mathrm{d}}{\mathrm{d}r}\phi\right] = \gamma_2\phi \Leftrightarrow \hat{D}_{|_{Sch}}\phi = \gamma_2\phi.$$

$\Rightarrow \gamma_2$ must be an eigen value of the differential operator $\hat{D}_{|_{Sch}} \leftrightarrow \gamma_{2,\max}$.

Spontaneous scalarization ($\gamma_1 = 0$, $\gamma_2 \neq 0$)

unexcited solutions



New results ($\gamma_1 \neq 0$, $\gamma_2 \neq 0$)

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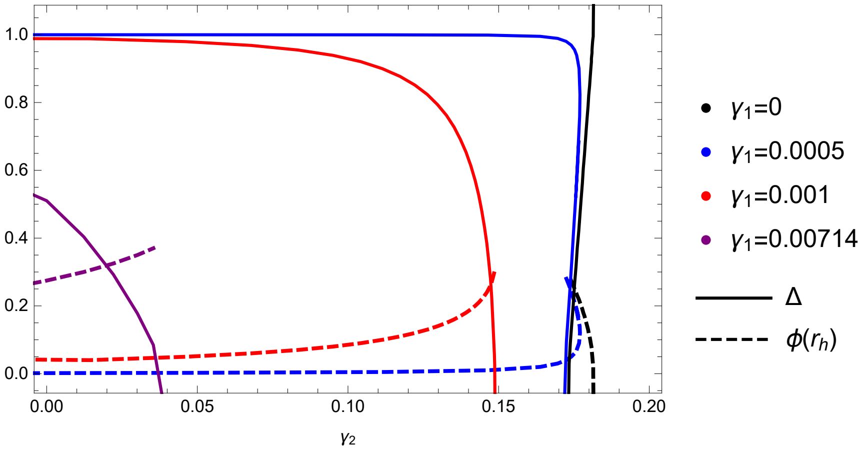

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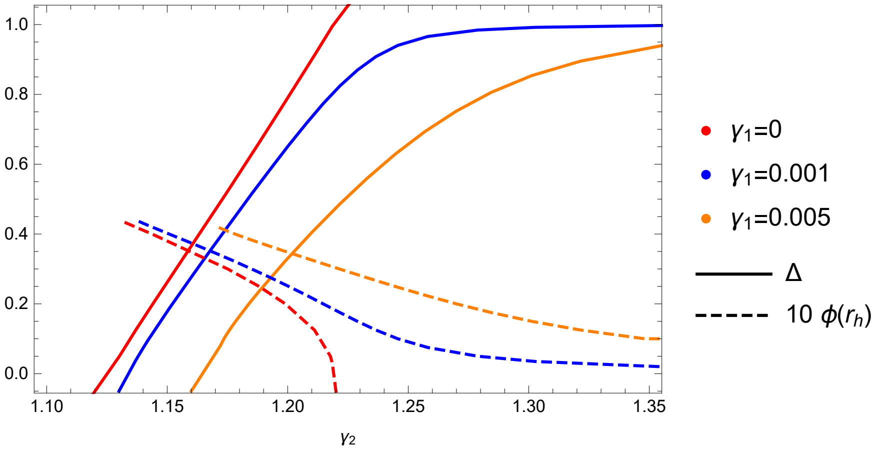

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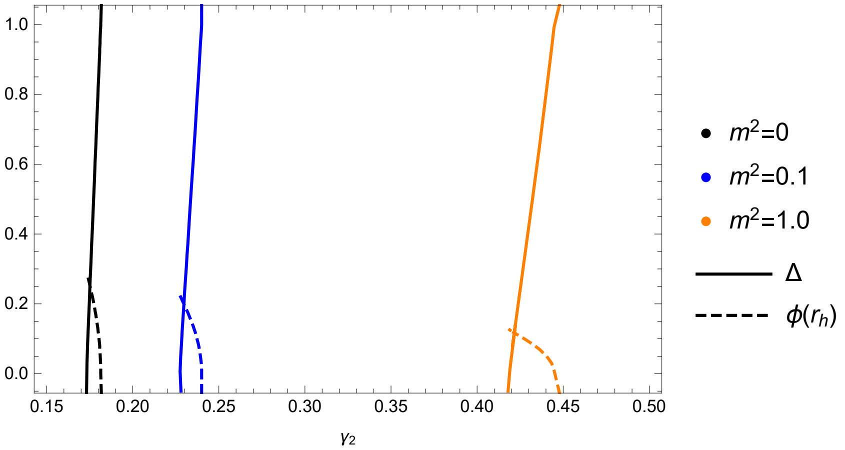
influence of a mass term



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- The potential is of the form $V(\phi) = m^2|\phi|^2 + \lambda_4|\phi|^4 + \lambda_6|\phi|^6$ and should contain at least a mass term, so $m > 0$.
  More precisely, we will concentrate our study to two cases :
  - no self-interaction : $m \neq 0,\ \lambda_4 = 0,\ \lambda_6 = 0,$
  - self-interaction : $m \neq 0,\ \lambda_4 = -2\frac{m^2}{\phi_c^2},\ \lambda_6 = \frac{m^2}{\phi_c^4}$. In this case, the potential is $V(\phi) = m^2\phi^2\left(1 - \frac{\phi^2}{\phi_c^2}\right)^2$. It possesses three degenerate minima located at $\phi = 0, \pm\phi_c$.

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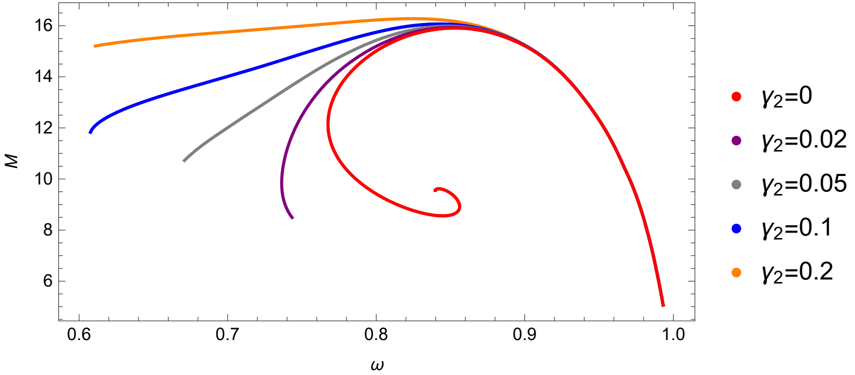
- The linear coupling to the Gauss-Bonnet term will be set to zero, so $\gamma_1 = 0$. In other words : $f(\phi) = \gamma_2\,|\phi|^2$.

Solutions without self-interaction



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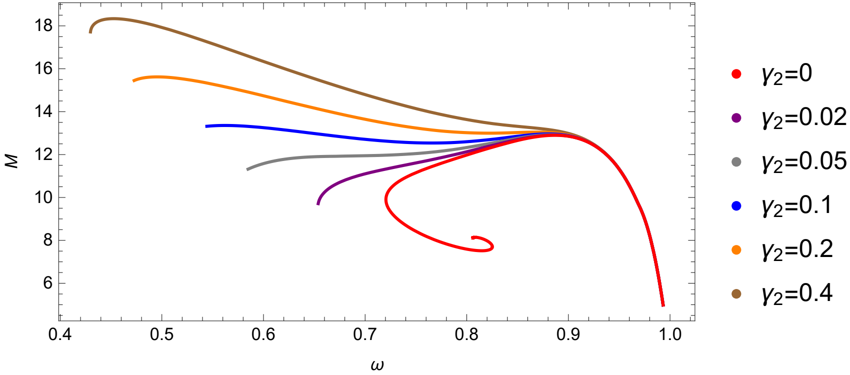

Classical stability

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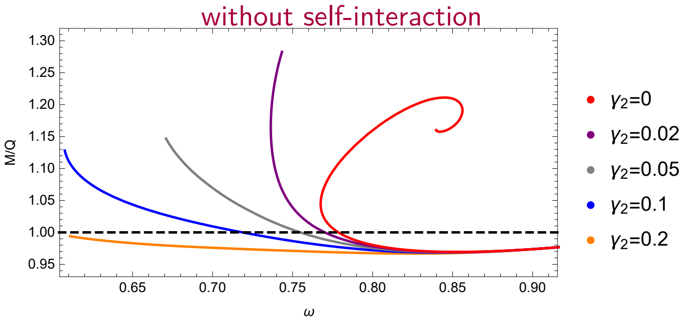
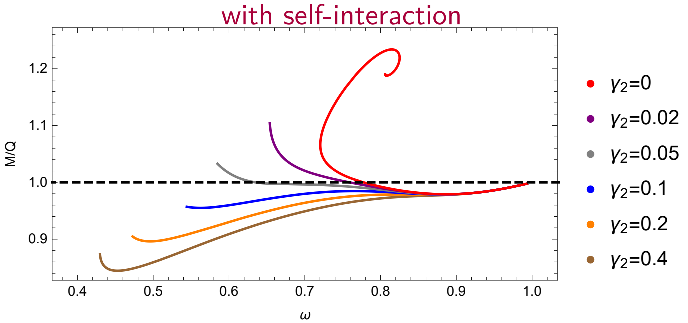
In the following, we will report our results in terms of the quantity $\frac{M}{mQ}$:

$$\frac{M}{mQ} \substack{> \\ <} 1 \Leftrightarrow \text{(un)stable.}$$


without self-interaction
1.30
1.25
1.20
1.15
1.10
1.05
1.00
0.95
M/Q
0.65
0.70
0.75
0.80
0.85
0.90
ω
γ2=0
γ2=0.02
γ2=0.05
γ2=0.1
γ2=0.2


with self-interaction
1.2
1.1
1.0
0.9
M/Q
0.4
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0.6
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ω
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Plan

Conclusions & outlooks

$$S = \int \left[\frac{1}{16\pi \mathcal{G}} R - \nabla_\mu \phi^* \nabla^\mu \phi - V(\phi) + f(\phi) \mathcal{I}(g) \right] \sqrt{-g} \mathrm{d}^4 x.$$

$$f(\phi) = \gamma_1 |\phi| + \gamma_2 |\phi|^2 , \ \mathcal{I}(g) = R^2 - 4 R_{\mu\nu} R^{\mu\nu} + R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma}.$$

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Conclusions :

- We have illustrated how $f(\phi)$ can lead to very different patterns when coupled to the Gauss-Bonnet invariant,
- In the case of black holes : our results link shift-symmetric theory to spontaneous scalarization,
- In the case of boson stars : we shown how a coupling function $\gamma_2\left|\phi\right|^2$ could enlarge the domain and improve the stability of the solutions.

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Outlooks :

- Charged scalar field : $\nabla_\mu \phi \to D_\mu \phi = (\partial_\mu + ieA_\mu)\phi$,
- Other types of coupling : $\nabla_\mu \phi^* g^{\mu\nu} \nabla_\nu \phi \to \nabla_\mu \phi^* \left(\alpha g^{\mu\nu} + \eta G^{\mu\nu} \right) \nabla_\nu \phi$,
- Influence on matter : $T_{\mu\nu} = (\rho + P)u_\mu u_\nu + P g_{\mu\nu}$.

Thank you for your attention !

References

Y. Brihaye & **L. Ducobu**,
"Hairy black holes, boson stars and non-minimal coupling to curvature invariants,"
Phys. Lett. B 795 (2019) 135 doi :10.1016/j.physletb.2019.06.006
[arXiv :1812.07438 [gr-qc]].