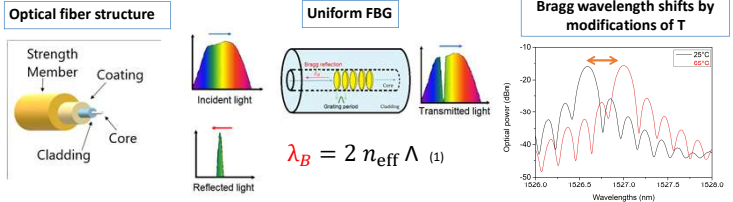


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Design of experiment is a scientific approach that provides sufficient information with a minimum number of experiments. Factorial design can be used when treatments are the combination of the levels of two or more factors that vary simultaneously.

Fiber Bragg Grating (FBG) sensor

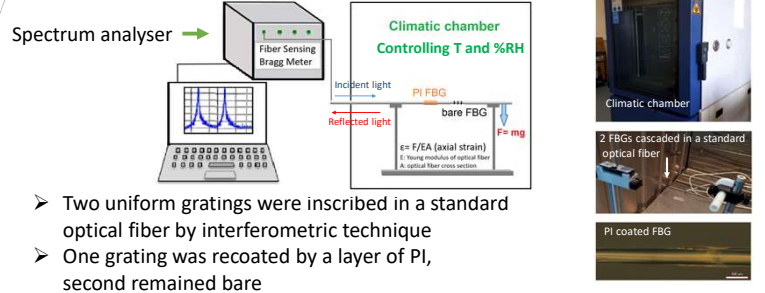


$$\lambda_B = 2 n_{eff} \Lambda \quad (1)$$

- FBG is a periodic modification of the refractive index in the optical fiber core.
- It can be achieved by exposure a short length of optical fiber to a UV light interference pattern
- FBG works as a wavelength selective mirror.
- Variation of temperature (T) and/or strain (ϵ) affect the λ_B , as both n_{eff} and Λ are T and ϵ dependent.
- A sensor is obtained by monitoring the $\Delta\lambda_B$ versus T and ϵ .
- For silica FBG temperature sensitivity is in order of 10 pm/°C, and strain sensitivity is in order of 1 pm/ $\mu\epsilon$.
- For humidity (%RH) sensing by FBG, it can be coated with an additional hygroscopic material like Polyimide (PI)
- Polyimide induces strain in the FBG by absorbing water and thus changes the grating period (Λ)
- Linear superposition of T, %RH, and ϵ effects

$$\Delta\lambda_B = S_T \Delta T + S_H \Delta H + S_\epsilon \Delta S \quad (2)$$

Experimental Setup and Factorial design



- Two uniform gratings were inscribed in a standard optical fiber by interferometric technique
- One grating was recoated by a layer of PI, second remained bare
- Factorial design collect data at the vertices of a hyper-cube in k-dimensions (number of factors)
- k factors with μ levels for each one, n^k measurement points
- 8 combination between 3 factors (X_T , X_H , and X_ϵ) with 2 levels (high and low) normalized to ± 1

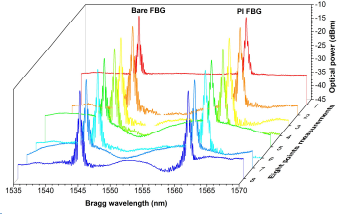
Trials	Factors	Interaction 2x2	Interaction 3x3	Meas.
	X_T X_H X_ϵ	$X_T X_H$ $X_T X_\epsilon$ $X_H X_\epsilon$	$X_T X_H X_\epsilon$	γ
1	-1 -1 -1	+1 +1 +1	-1	λ_{B1}
2	+1 -1 -1	-1 +1 +1	+1	λ_{B2}
3	-1 +1 -1	-1 +1 -1	+1	λ_{B3}
4	+1 +1 -1	-1 -1 -1	-1	λ_{B4}
5	-1 -1 +1	+1 -1 -1	+1	λ_{B5}
6	+1 -1 +1	+1 -1 -1	-1	λ_{B6}
7	-1 +1 +1	-1 +1 -1	-1	λ_{B7}
8	+1 +1 +1	+1 +1 +1	+1	λ_{B8}

$$y = a_0 + a_T X_T + a_H X_H + a_\epsilon X_\epsilon + a_{TH} X_T X_H + a_{TE} X_T X_\epsilon + a_{HE} X_H X_\epsilon + a_{THE} X_T X_H X_\epsilon \quad (3)$$

$$a = X^{-1} \cdot y \quad (4)$$

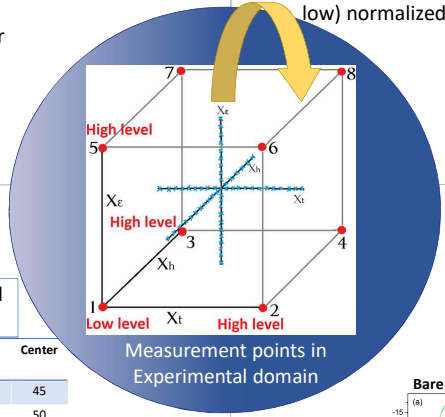
a: normalized coefficients
X: a matrix made from the above table

Experimental results



3 factors limited to 2 level

Factor	Low level	High level	Center
x_T (°C)	25	65	45
x_H (%RH)	30	70	50
x_ϵ ($\mu\epsilon$)	44.45	226.65	135.55



$$\Delta\lambda_B = S_T \Delta T + S_H \Delta H + S_\epsilon \Delta \epsilon + S_{TH} \Delta T \Delta H + S_{TE} \Delta T \Delta \epsilon + S_{HE} \Delta H \Delta \epsilon + S_{THE} \Delta T \Delta H \Delta \epsilon \quad (5)$$

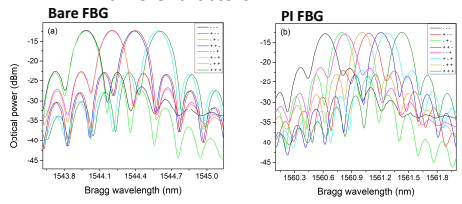
Coefficients	Bare FBG	PI FBG
S_0 (nm)	1544.28	1561.03
S_T (pm/°C)	10.18	11.18
S_H (pm/%RH)	-0.18	3.68
S_ϵ (pm/ $\mu\epsilon$)	1.16	1.14
S_{TH} (pm/°C%RH)	-0.9×10^{-2}	-3.12×10^{-2}
S_{TE} (pm/°C $\mu\epsilon$)	-6.86×10^{-4}	-9.19×10^{-4}
S_{HE} (pm/%RH $\mu\epsilon$)	-3.43×10^{-5}	-1.37×10^{-3}
S_{THE} (pm/°C%RH $\mu\epsilon$)	-1.67×10^{-4}	-2.19×10^{-4}

- Reflection spectra from 2 FBGs modifications in 8 measurement points
- By applying Eq. (4), all the coefficients (a) calculated, then de-normalized (S)
- All sensitivities respect to the central point calculated as well as cross interactions sensitivities, only by 8 measurements.
- Temperature sensitivity for PI FBG is slightly higher than bare FBG due to Polyimide coating.
- Strain sensitivity is in good range for both FBGs and Bare FBG is not sensitive to humidity, while PI is good transducer material for it (compare with classic measurement).
- Result of center point measurements confirms the factorial amount for S_0

bare FBG: 1544.29 (nm) PI FBG: 1561.027 (nm)

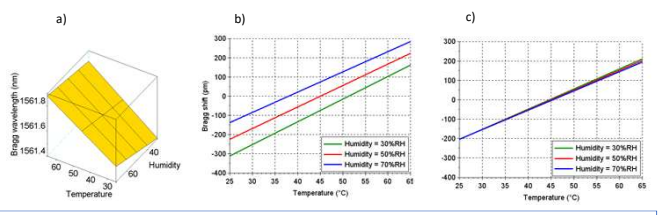
Advantages of Factorial design

- In classic experimental methods, each experimental variable is studied separately at a time
- Factorial design allows estimation of sensitivity to each factor as well as the effect of interaction between different factors.



λ_B in the PI FBG shifted with humidity modifications, while it did not in the bare FBG.

- One of the advantages of experimental design is the easy access to cross-sensitivity coefficients.
- For e.g. temperature sensitivity slightly depends on humidity. Hence, humidity correction could be done for temperature measurements with PI FBG.



Interaction of T and %RH in stable strain for PI FBG in a)3D, b)2D, and c)bare FBG

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