BMS-like symmetries in field theory

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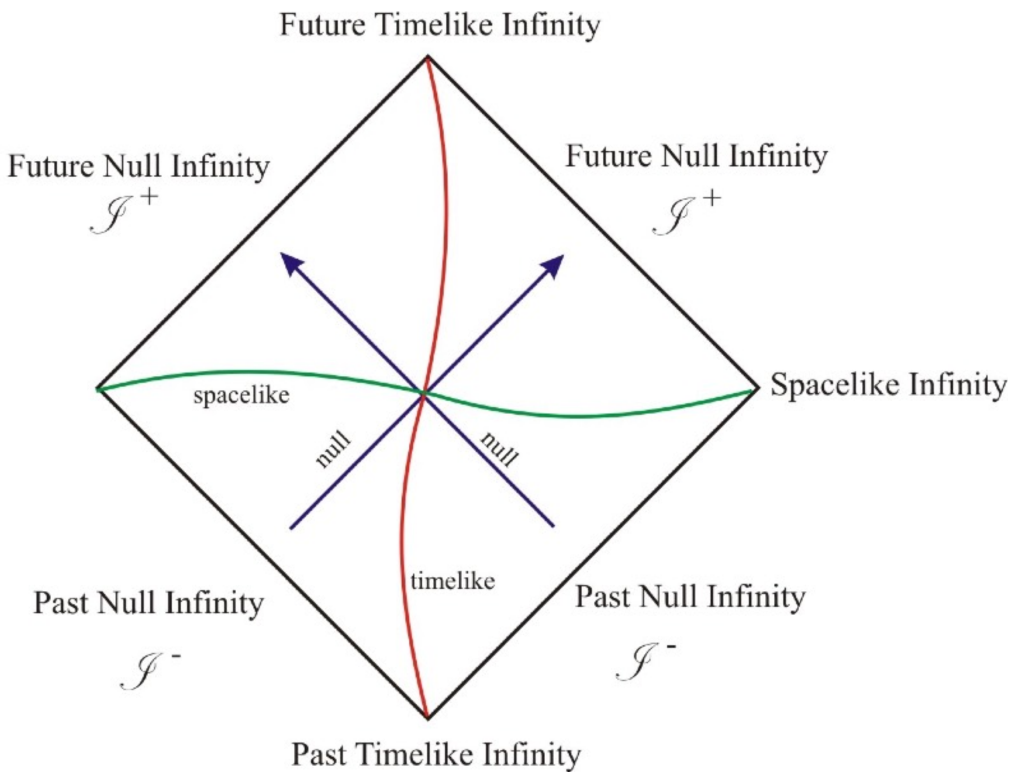
Gravitational scattering: two perspectives

Goal: describe *scattering in flat space* including gravity



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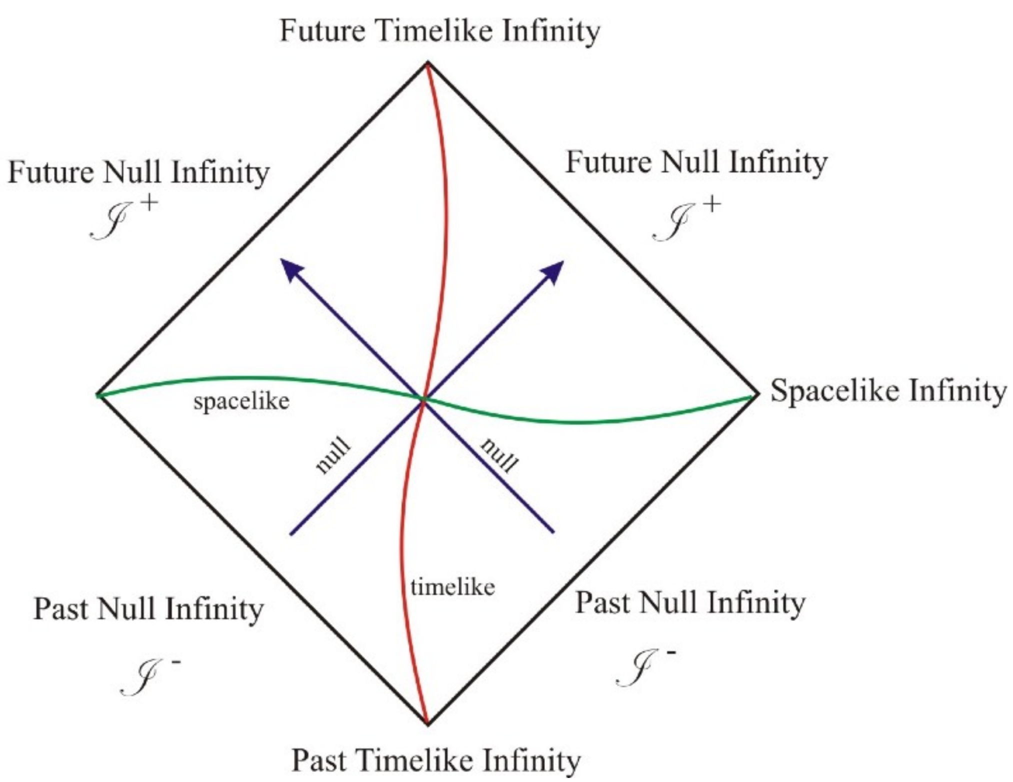

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  - Universal result: Weinberg's factorisation theorem

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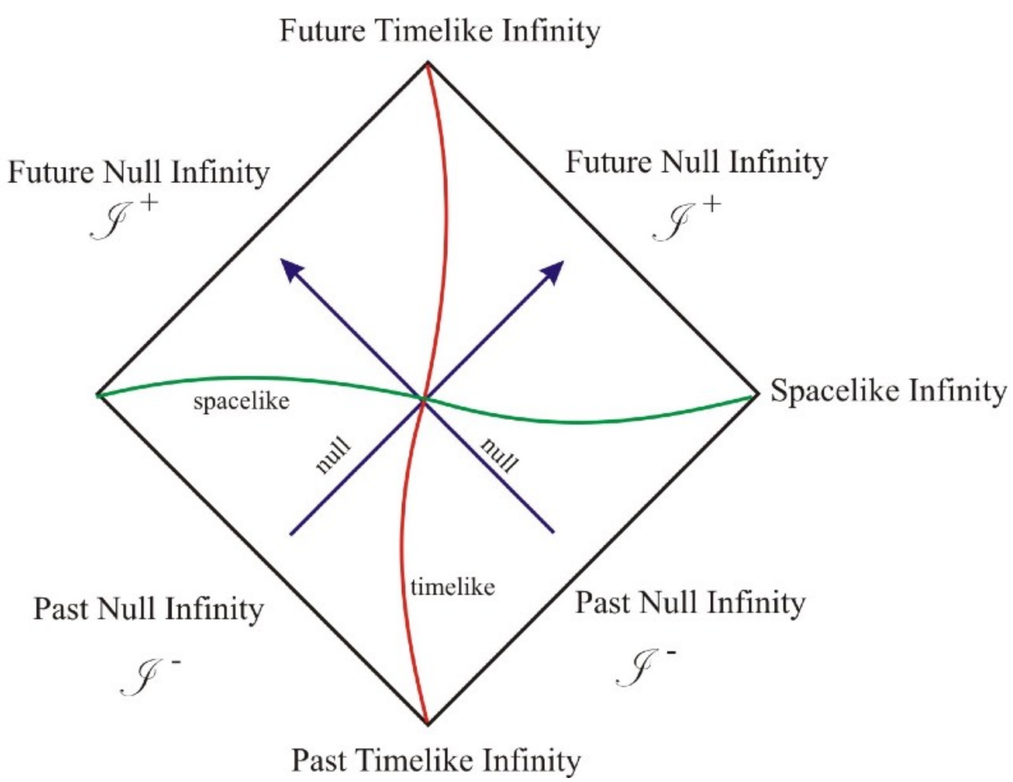

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- **General relativity perspective**
  - identify asymptotically flat manifolds and learn how to distinguish them
  - Universal result: BMS symmetry

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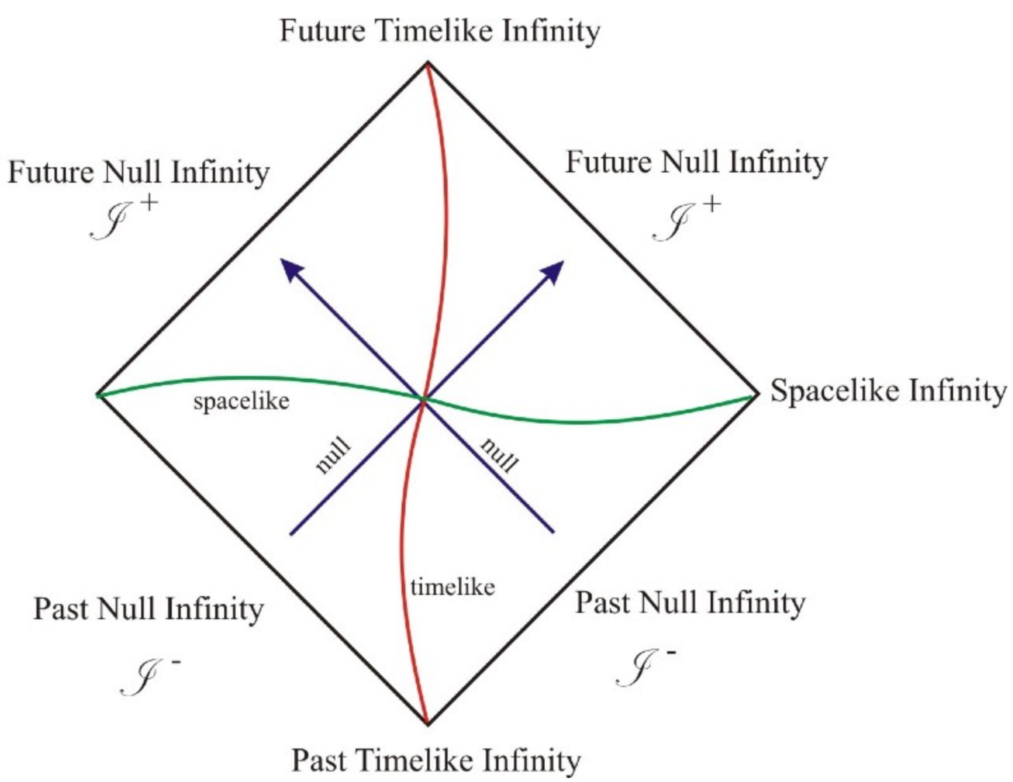

a precise link between the two setups has been proposed recently!

- **Field theory perspective**
  - compute the (tree-level) S-matrix
  - Universal result: Weinberg's factorisation theorem

- **General relativity perspective**
  - identify asymptotically flat manifolds and learn how to distinguish them
  - Universal result: BMS symmetry

Soft theorems & BMS symmetries

- Results that have been linked:

Strominger (2013); He, Lysov, Mitra, Strominger (2014); see also the 2017 review by Strominger

  - *1962*: **B**ondi, van der Burg, **M**etzner, **S**achs (**BMS**) *infinite-dimensional* group of asymptotic symmetries in general relativity
  - *1964*: **Weinberg**'s soft theorems

Weinberg's soft graviton theorem	$\Longleftrightarrow$	Ward identity for BMS symmetry

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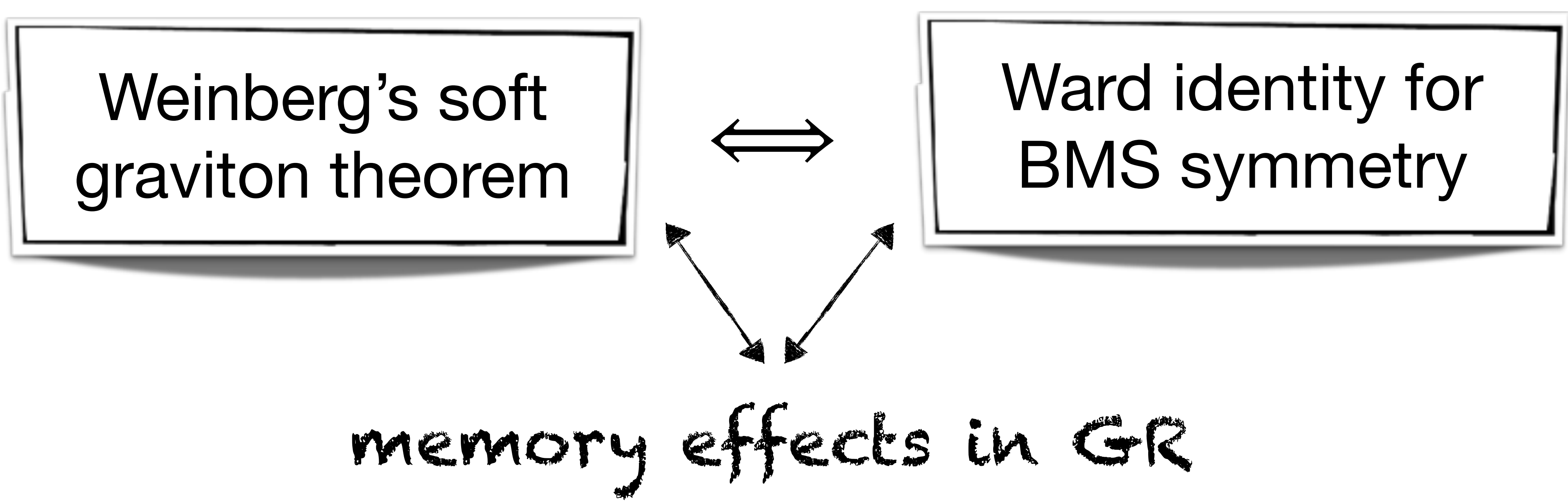

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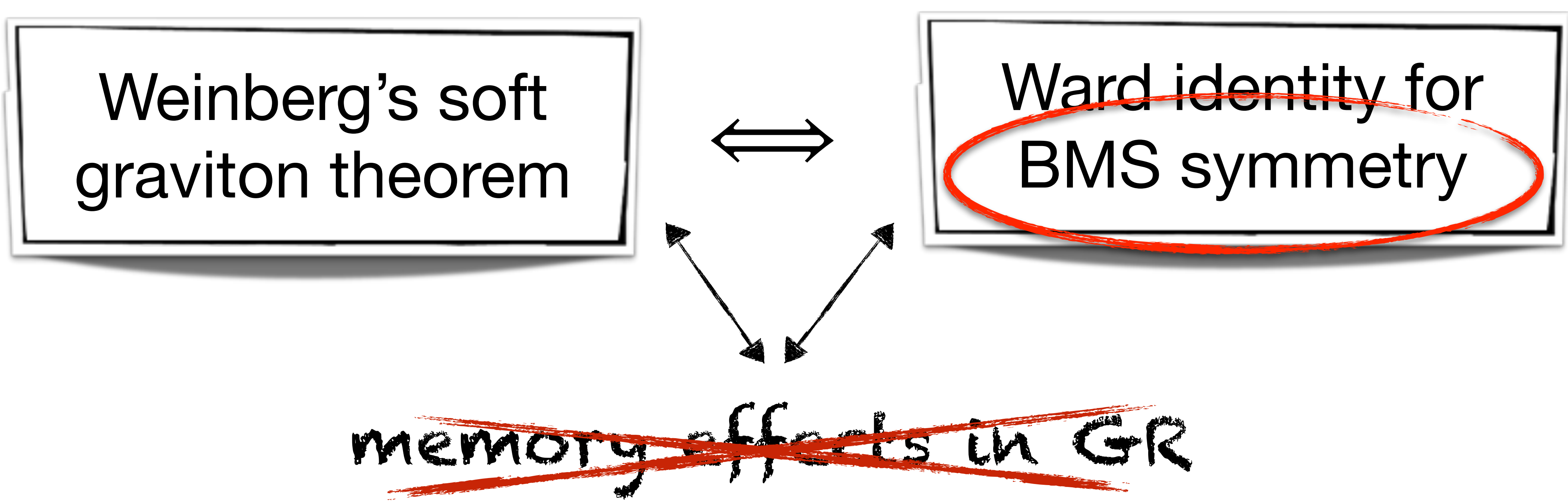
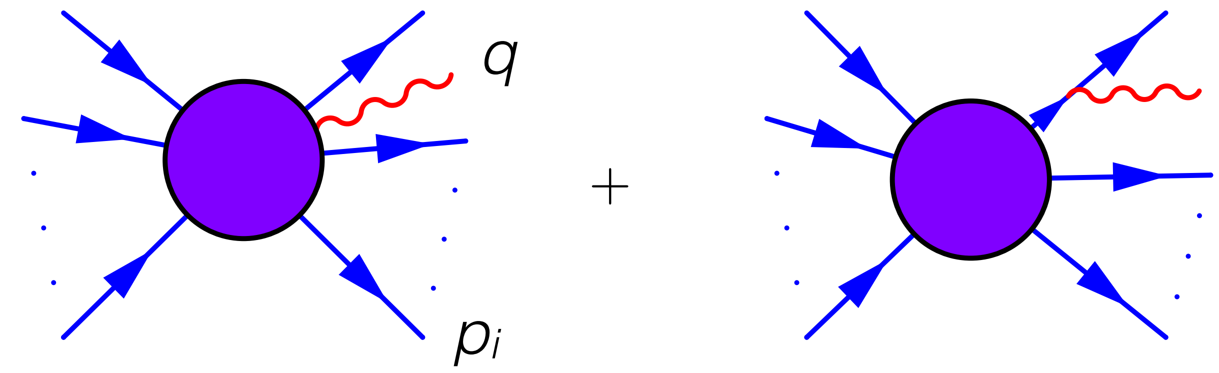


Weinberg's soft theorem(s)

- Amplitude for N−1 scalars and one "soft" particle
  Weinberg (1964)



- In the limit $q \to 0$ **the amplitude factorises:**

$$A_{tot}(1,\ldots,N) \sim A(1,\ldots,N-1) \times \sum_{i=1}^{N-1} g_i \, \frac{p_i^\mu p_i^\nu h_{\mu\nu}(q)}{2\, p_i \cdot q}$$

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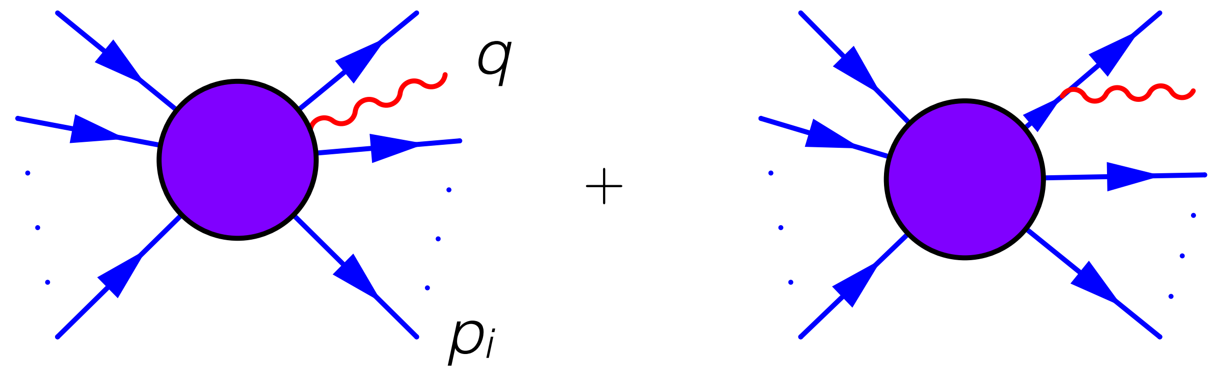

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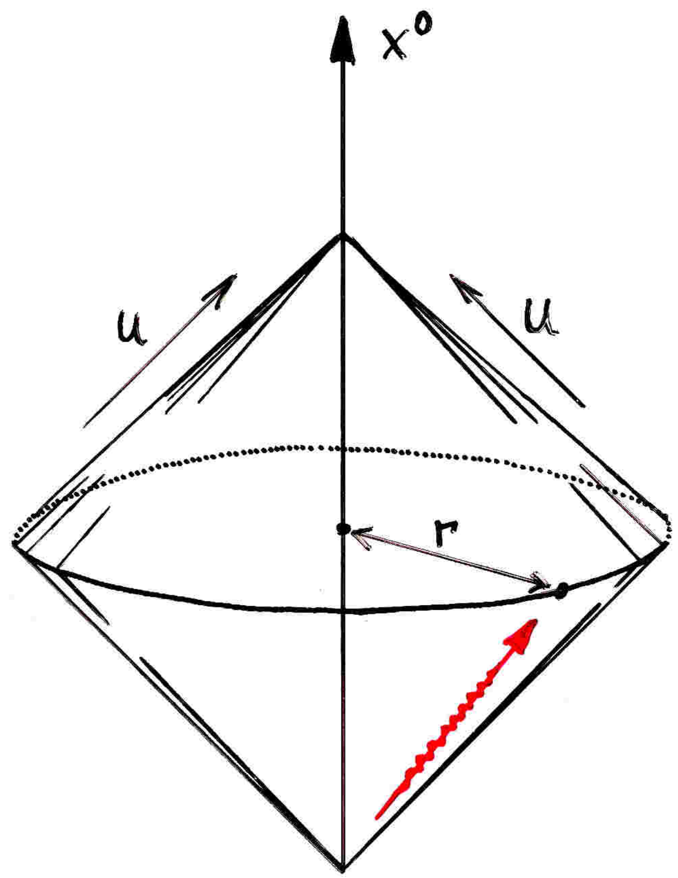
- On-shell gauge transformations $h_{\mu\nu}(q) \to h_{\mu\nu}(q) + i\, q_{(\mu}\Lambda_{\nu)}(q)$
  - QED: $\sum_{i=1}^{N-1} g_i = 0$ (charge conservation)
  - Gravity: $g_i = g_j \; \forall\, i,j \;\Rightarrow\; \sum_{i=1}^{N-1} p_i^\mu = 0$ (equivalence principle)

BMS symmetry in a nutshell

- (Retarded) Bondi coordinates
  - start from Minkowski space: $ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu ,$
  - change coord.: $r \equiv \left[x^i x^i\right]^{1/2}, \qquad z \equiv \frac{x^1 + i x^2}{r + x^3}, \qquad u \equiv x^0 - r.$
  - Minkowski metric in the new coordinates:

$$ds^2 = -\,du^2 - 2\,du dr + \frac{4\,r^2 dz d\bar{z}}{(1+z\bar{z})^2}$$



BMS symmetry in a nutshell

- Boundary conditions

  - Minkowski: $$ds^2 = -du^2 - 2\,dudr + \frac{4\,r^2 dz d\bar{z}}{(1+z\bar{z})^2}$$

  - Asymptotically flat:

Bondi, van der Burg, Metzner; Sachs (1962);
see also Barnich, Troessaert (2010);
He, Lysov, Mitra, Strominger (2014)

$$\begin{aligned} ds^2 \quad = \quad & -du^2 - 2dudr + 2r^2\gamma_{z\bar{z}}dzd\bar{z} \\ & + \frac{2m_B}{r}du^2 + rC_{zz}dz^2 + rC_{\bar{z}\bar{z}}d\bar{z}^2 - 2U_z dudz - 2U_{\bar{z}}dud\bar{z} \\ & + \ldots \quad , \end{aligned}$$

Bondi mass aspects

Outgoing Bondi news: $N_{zz} \equiv \partial_u C_{zz}$

- Next task: classify diffeos preserving the asymptotic form of the metric (aka *asymptotic symmetries*)

How to link the two approaches?

- Gravity as a field theory in Minkowski space:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$I \sim \int d^4x \Big\{ h^{\mu\nu} \left(\Box h_{\mu\nu} - 2\,\partial_\mu \partial \cdot h_\nu \right) - h' \left(\Box h' - 2\,\partial \cdot \partial \cdot h \right) \Big\} + \mathcal{O}(h^3)$$

- Right language to compare with Weinberg's soft theorems!

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- The geometry of spacetime is not manifest anymore, but asymptotic symmetries are still visible

- Extra bonus: the same analysis applies to other relativistic field theories, like QED, Yang-Mills, sugra and… more!

BMS symmetry in *linearised* gravity

Asymptotic symmetries of linearised gravity

- Boundary conditions in the *linearised theory* ($g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$)

$$h_{\mu\nu}dx^\mu dx^\nu = \frac{2\,m_B}{r}\,du^2 - 2\,U_z du dz - 2\,U_{\bar{z}} du d\bar{z} + r\,C_{zz} dz^2 + r\,C_{\bar{z}\bar{z}} d\bar{z}^2$$

- Look for *linearised diffeomorphisms that preserve them*

$$\delta h_{\mu\nu} = \partial_{(\mu}\epsilon_{\nu)} := \partial_\mu \epsilon_\nu + \partial_\nu \epsilon_\mu$$

- **Idea:** asymptotically the gravitational field is weak

- **Observation:** soft theorems can be recovered within the linearised theory

Supertranslations

- *u*-independent *linearised* diffeos preserving the bnd conds

$$\epsilon^\mu \partial_\mu = T\partial_u + D^z D_z T\,\partial_r - \frac{1}{r}\left(D^z T\,\partial_z + D^{\bar z} T\,\partial_{\bar z}\right)$$

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Symmetries generated by an arbitrary function $T(z, \bar{z})$ on the celestial sphere

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Symmetries generated by an arbitrary function $T(z,\bar z)$ on the celestial sphere

- Poincaré translations generated by special *T*

$$1\,, \qquad \frac{1-z\bar z}{1+z\bar z}\,, \qquad \frac{z+\bar z}{1+z\bar z}\,, \qquad \frac{i(z-\bar z)}{1+z\bar z}\,.$$

notation: D_z = covariant derivative on the celestial sphere with metric $\gamma_{z\bar z}$

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Supertranslations: infinite-dimensional enhancement of the translation symmetry!

Superrotations

- Generic asymptotic symmetry

$$\epsilon = \left(T + \frac{u}{2} D \cdot Y\right) \partial_u + \left(D_z D^z T - \frac{1}{2} (u + r) D \cdot Y\right) \partial_r$$
$$+ \left(Y^z - \frac{1}{r} D^z Y - \frac{u}{2r} D^z D \cdot Y\right) \partial_z + \left(Y^{\bar{z}} - \frac{1}{r} D^{\bar{z}} Y - \frac{u}{2r} D^{\bar{z}} D \cdot Y\right) \partial_{\bar{z}}$$

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- Additional symmetries generated by

Y^i satisfying $D^{(i}Y^{j)} - \eta^{ij} D \cdot Y = 0$

- When D = 4 (i.e. when the celestial sphere is S^2) *locally* the conformal Killing equation is solved by $Y^z(z)$ and $Y^{\bar{z}}(\bar{z})$

Spin-s Weinberg theorem & … and what?

- Weinberg's soft theorem extends to particles of all spin
  - *Universal* formula relating S-matrix elements with and without a soft higher-spin massless particle

Is it possible to recover Weinberg's formula from the **Ward identity** of a proper **asymptotic symmetry**?

Spin-s Weinberg theorem & … and what?

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  - *Universal* formula relating S-matrix elements with and without a soft higher-spin massless particle

> Is it possible to recover Weinberg’s formula from the **Ward identity** of a proper **asymptotic symmetry**?

YES!

- Gauge theories in flat space display *infinite dimensional asymptotic symmetry algebras* (at least when D = 3 and 4)

for the case of $s>2$ see Afshar, Bagchi, Fareghbal, Grumiller, Rosseel (2013); Gonzalez, Matulich, Pino, Troncoso (2013); A.C., Francia, Heissenberg (2017)

Massless particles of any spin and their asymptotic symmetries

Free massless particles of arbitrary spin

- Example I: Maxwell
  - Field equations: $\partial^\lambda F_{\lambda\mu} = 0 \quad \Rightarrow \quad \Box A_\mu - \partial_\mu \partial \cdot A = 0$
  - Gauge symmetry: $\delta A_\mu = \partial_\mu \xi$

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- Example II: linearised gravity
  - Field equations: $\Box h_{\mu\nu} - \partial_\mu \, \partial \cdot h_\nu - \partial_\nu \, \partial \cdot h_\mu + \partial_\mu \partial_\nu \, h_\lambda{}^\lambda = 0$
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Particle of arbitrary spin s:

Fronsdal (1978)

$$\mathcal{F}_{\mu_1 \cdots \mu_s} \equiv \Box \varphi_{\mu_1 \cdots \mu_s} - \partial_{(\mu_1} \partial \cdot \varphi_{\mu_2 \cdots \mu_s)} + \partial_{(\mu_1} \partial_{\mu_2} \varphi_{\mu_3 \cdots \mu_s)\lambda}{}^\lambda = 0$$

- Gauge symmetry: $$\delta \varphi_{\mu_1 \cdots \mu_s} = \partial_{(\mu_1} \xi_{\mu_2 \cdots \mu_s)}$$
- Gauge invariance requires: $$\xi_{\mu_1 \cdots \mu_{s-3} \lambda}{}^\lambda = 0$$

Massless higher-spin (s > 2) particles

- **Weinberg** (1964): no long-range interactions mediated by higher-spin particles (vanishing couplings in the *soft limit*)

$$A_s(1,\dots,N) \sim A(1,\dots,N-1) \times \sum_{i=1}^{N-1} g_i^{(s)} \frac{p_i^{\mu_1} \cdots p_i^{\mu_s} \varphi_{\mu_1 \cdots \mu_s}(q)}{2 p_i \cdot q}$$

On shell gauge symmetry → $\displaystyle\sum_{i=1}^{N-1} p_i^{\mu_1} \cdots p_i^{\mu_{s-1}} = 0$

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→ *No interacting higher-spin gauge theories in flat space*

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Higher-spin (HS) theories: the status quo

- Weinberg's results apply to *massless* particles in *flat space*
- Two known examples of interacting HS theories:
  - *String (field) theory*: **massive** HS excitations in flat space
  - *Vasiliev's equations*: massless HS fields in **(A)dS** Fradkin, Vasiliev (1987); Vasiliev (1990)
  - Support for HS gauge theories from **AdS/CFT** Sundborg (2001); Sezgin, Sundell (2002); Klebanov, Polyakov (2002); Giombi, Yin (2009) etc.
- Higher-spin interactions seem to require an **infrared regulator** (mass or cosmological constant)
  - Difficulties emerge when one tries to remove the regulator

Spin-3 supertranslations…

- Linearised gauge transformations: $\varphi_{\mu\nu\rho} \sim \varphi_{\mu\nu\rho} + \partial_{(\mu}\epsilon_{\nu\rho)}$
- Boundary conditions on the Fronsdal field A.C., Francia, Heissenberg (2017)

$$\varphi_{uuu} = \frac{B}{r}\,, \qquad \varphi_{uuz} = U_z\,, \qquad \varphi_{uzz} = r\,C_{zz}\,, \qquad \varphi_{zzz} = r^2 B_{zzz}$$

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- *u*-independent residual “gauge” symmetry

$$\begin{aligned}\epsilon_{\mu\nu}dx^\mu dx^\nu = &-\left(\frac{3}{4}T + D^zD_zT + \frac{1}{4}(D^zD_z)^2T\right)du^2 - 2\left(\frac{3}{4}T + \frac{1}{4}D^zD_zT\right)dudr\\ &- 2r\left(\frac{3}{4}D_zT + \frac{1}{4}D_z^2D^zT\right)dudz - 2r\left(\frac{3}{4}D_{\bar z}T + \frac{1}{4}D_{\bar z}^2D^{\bar z}T\right)dud\bar z - Tdr^2\\ &- r\left(D_zTdz + D_{\bar z}Td\bar z\right)dr - \frac{r^2}{2}\left(D_z^2Tdz^2 + D_{\bar z}^2Td\bar z^2\right) - \frac{r^2}{2}\gamma_{z\bar z}\left(T + D^zD_zT\right)dzd\bar z\,,\end{aligned}$$

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…and higher-spin superrotations!

- Full residual "gauge" symmetry

$$\epsilon^{ij} = \left[K^{ij} + \frac{u}{r}\,\mathcal{T}_2^{ij}(K) + \left(\frac{u}{r}\right)^2 \mathcal{T}_4^{ij}(K)\right] + \frac{1}{r}\left[\mathcal{U}_1^{ij}(\rho) + \frac{u}{r}\,\mathcal{U}_3^{ij}(\rho)\right] + \frac{1}{r^2}\,\mathcal{V}_2^{ij}(T)\,,$$

$$\epsilon^{ui} = \frac{u}{n+2}\left[D\cdot K^i - \frac{u\,r^{-1}}{2(n+1)}\,D^i D\cdot D\cdot K\right] - \left[\rho^i - \frac{u\,r^{-1}}{n+1}\,D^i D\cdot\rho\right] + \frac{1}{2\,r}\,D^i T\,,$$

$$\epsilon^{uu} = \frac{u^2}{(n+1)(n+2)}\,D\cdot D\cdot K - \frac{2\,u}{n+1}\,D\cdot\rho - T\,.$$

and similar expressions for ϵ^{ri}, ϵ^{ru} and ϵ^{rr}

Asymptotic symmetries generated by T, ρ^i and K^{ij}

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$$\epsilon^{uu} = \frac{u^2}{(n+1)(n+2)}\,D\cdot D\cdot K - \frac{2\,u}{n+1}\,D\cdot\rho + T\,.$$

and similar expressions for ϵ^{ri}, ϵ^{ru} and ϵ^{rr}

- $T(z,\bar{z})$
- $\rho^z = \alpha(z)\,\partial_z k(z,\bar{z}) + \beta(z)\,, \qquad \rho^{\bar{z}} = \tilde{\alpha}(\bar{z})\,\partial_{\bar{z}} k(z,\bar{z}) + \tilde{\beta}(\bar{z})\,,$
- $K^{zz} = K(z)\,, \qquad K^{\bar{z}\bar{z}} = \tilde{K}(\bar{z})\,, \qquad K^{z\bar{z}} = 0\,.$

Summary and outlook

- **HS theories in 4D Minkowski space admit an ∞-dimensional algebra of asymptotic symmetries**
  - Generalisations of gravitational super-translations and -rotations
- **Weinberg's theorem $\Longleftrightarrow$ supertranslation Ward identity**
- **Further research directions**
  - Non-abelian extension of BMS-like HS symmetries?
  - Relics of HS symmetries in string amplitudes? Flat-space holography?
  - *Higher-dimensional gravity; modified theories of gravity*

Interpretation of the new symmetries

- Global solutions are in one to one correspondence with
  - Gravity: $T \rightarrow \square \simeq P_a \qquad v^i \rightarrow \begin{array}{|c|}\hline \; \\ \hline \; \\ \hline \end{array} \simeq J_{ab}$
  - HS: $T \rightarrow \begin{array}{|c|c|}\hline \; & \; \\ \hline \end{array} \simeq P_{(a}P_{b)} \qquad \rho^i \rightarrow \begin{array}{|c|c|}\hline \; & \; \\ \hline \; \\ \cline{1-1} \end{array} \simeq P_{(a}J_{b)c} \qquad K^{ij} \rightarrow \begin{array}{|c|c|}\hline \; & \; \\ \hline \; & \; \\ \hline \end{array} \simeq J_{a(c}J_{d)b}$
- Same structure as in AdS Vasiliev's algebra!
  - Gravity: $\mathcal{A} = e^a P_a + \omega^{ab} J_{ab} = \omega^{AB} M_{AB}$, $\qquad M_{AB}$ span $\mathfrak{so}(2, D-1)$
  - HS: $\mathcal{A} = \sum_{s=0}^{\infty} \Omega^{A_1 \cdots A_{s-1} | B_1 \cdots B_{s-1}} M_{A_1 \cdots A_{s-1} | B_1 \cdots B_{s-1}}$
- Hints of a <u>Carrollian Vasiliev's algebra</u>