Generalized Derivatives of the Optimal Value of a Linear Program with respect to Matrix Coefficients.

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Abstract

In the framework of linear programming, we consider the problem of estimating the variation of the objective function resulting from changes in some matrix coefficients. Our objective is to extend results already available for the right-hand-side to this more general problem.

The interpretation of the dual variables as derivatives of the optimal value of the objective function with respect to the elements of the right-hand-side is well known in mathematical programming. This result can be extended to the case of multiple dual solutions. The set of all dual solutions is then the subdifferential of the optimal value of the objective function, seen as a convex function of the right-hand side.

The object of this paper is to extend these well known results to the derivative of the optimal value of the objective function with respect to matrix coefficients.

It is easy to show on a simple example that the objective function value of a linear program is not a convex function of the matrix coefficients. The subdifferential concept is thus inappropriate here. One must therefore resort to Clarke's notion of a generalized derivative.

We present here in a not (too) mathematical manner the generalized derivative of the optimal value of the objective function of a linear

program as a function of matrix coefficients. We generalize the result of Freund (1985) to the cases where derivatives may not be defined because of the existence of multiple primal or dual solutions.

Subject classification : Parametric Linear Programming, Linear Programming Theory.

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References

Clarke F.H., (1990), "Optimization and nonsmooth analysis" (Classics in Applied Mathematics 5, SIAM).

Freund R.M., (1985), "Postoptimal analysis of a linear program under simultaneous changes in matrix coefficients", in: Cottle, ed., *Mathematical Programming Study 24,* pp 1-13.

Gal T., (1995), "Postoptimal Analyses, Parametric Programming and Related Topics", Walter de Gruyter, Berlin.

Lemaréchal C., (1989), "Nondifferentiable optimization", in: G.L. Nemhauser and al., eds., (Handbooks in Operations Research and Management Science 1 : Optimization, Elsevier Science Publishers).

Renegar, J., (1994), "Some perturbation theory for linear programming", *Mathematical Programming 65,* pp 73-91..