

Turán's problem [1]

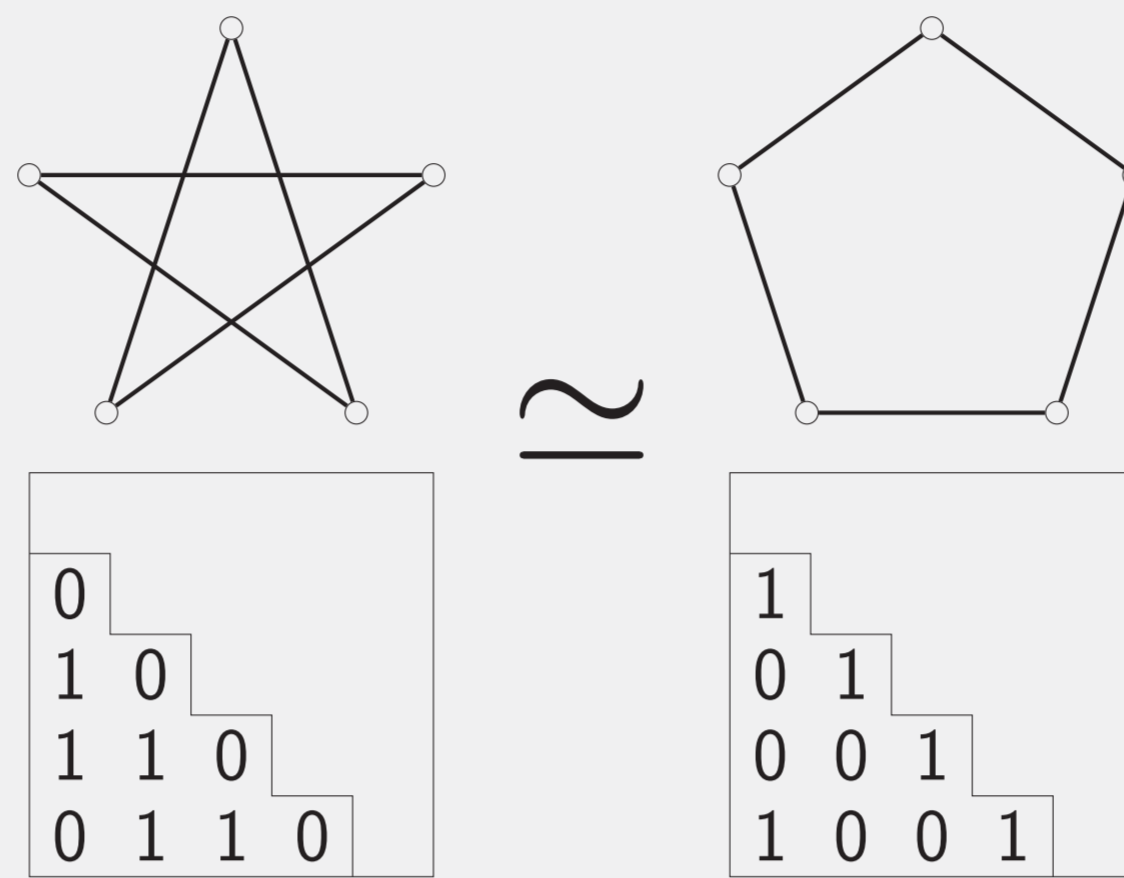
At most how many edges can have a K_k -free graph of order n ?

Invariants DB

Canonical Graphs	#edges	CliqueMax
	13	5
	21	3
	10	4

- ▶ All (non isomorphic) graphs up to order 10 and some of higher order.
- ▶ Dynamic programming helps computing invariants faster.
- ▶ Fast and simple (for the user) queries on graphs invariants.
- ▶ Implementation: PostgreSQL Database.

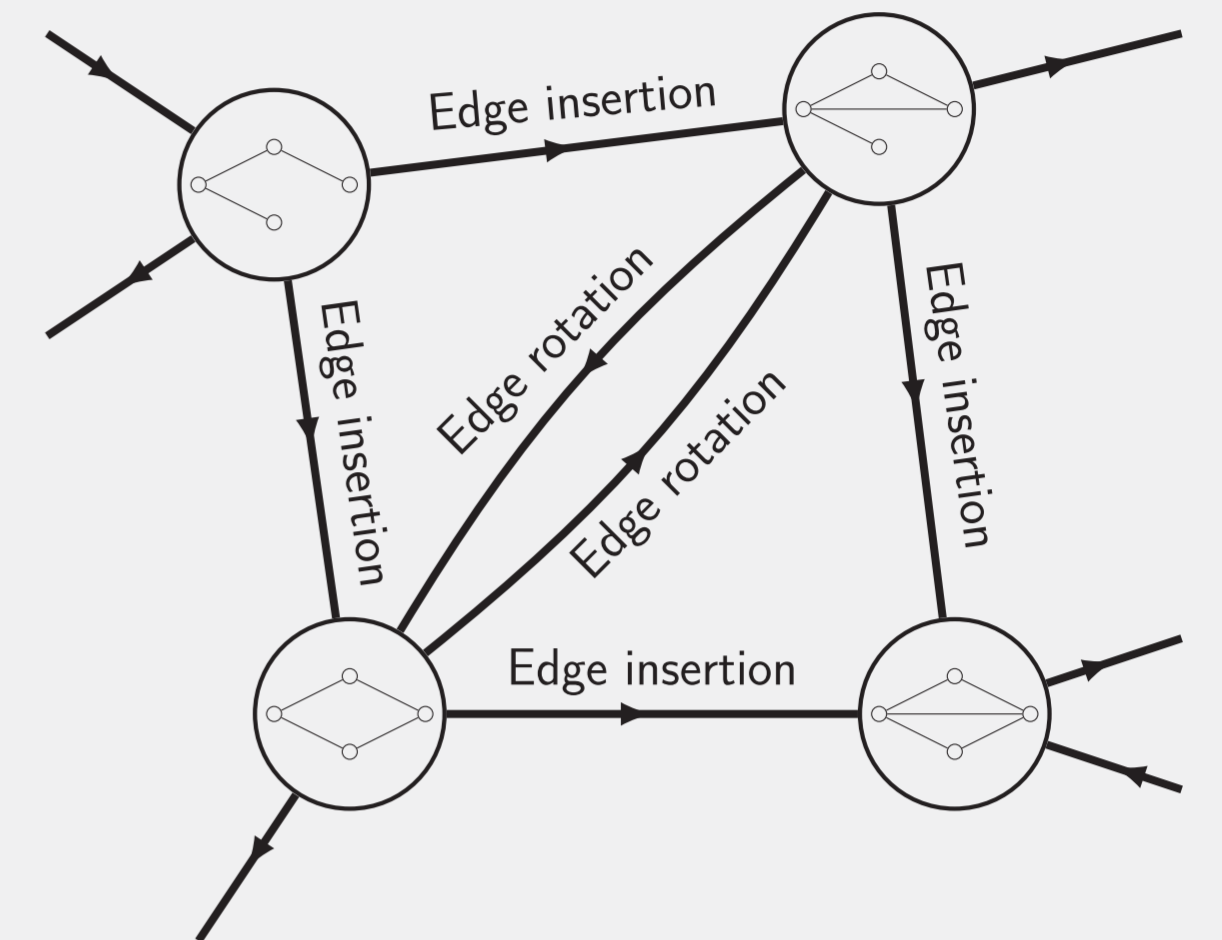
Nauty Isomorphisms [2]



Use McKay's Nauty tool to:

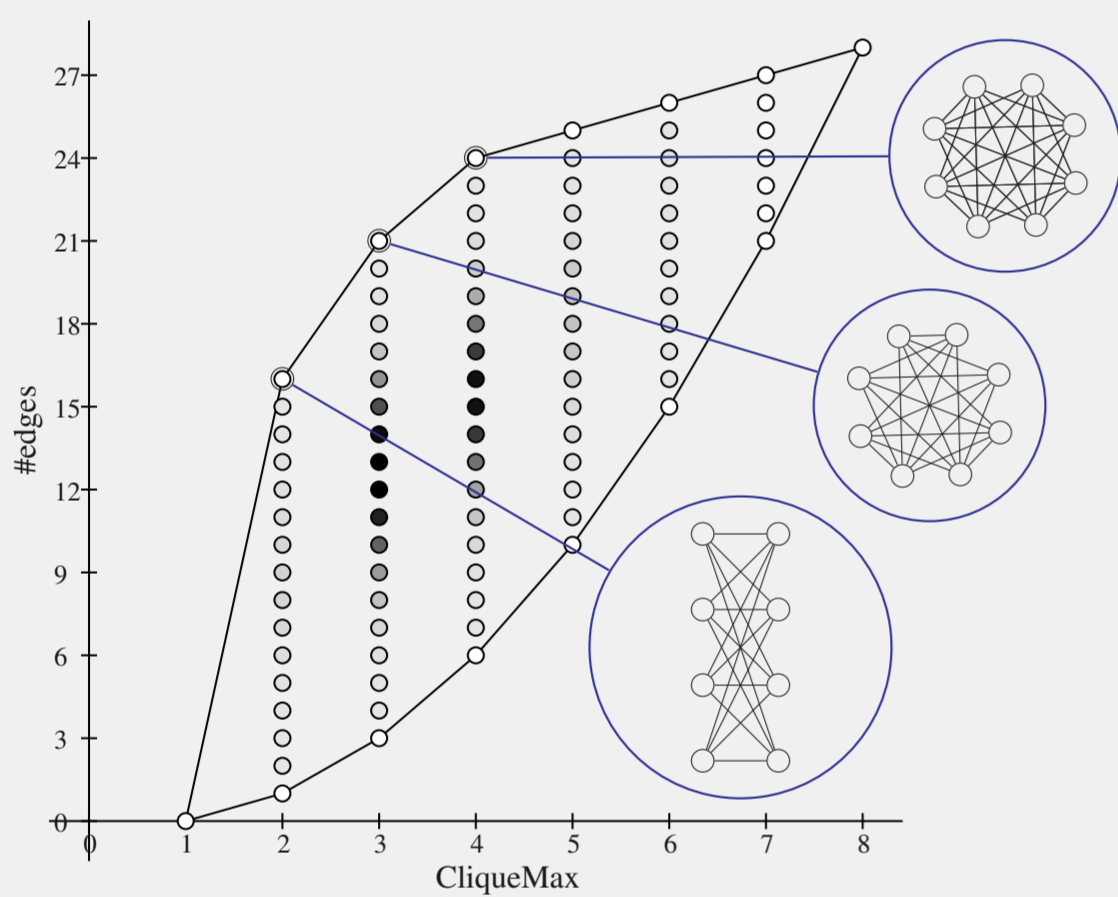
- ▶ compute canonical representations of graphs;
- ▶ generate all non isomorphic graphs for a given order.

Transformations Graph DB



- ▶ Graph database with graphs as vertices and transformations transitions as arcs.
- ▶ Fast and simple (for the user) queries on graphs transformations.
- ▶ Implementation: Neo4j.
- ▶ `MATCH (u),(v) WHERE NOT EXISTS((u)-[]-(v))`
`RETURN COUNT(u) = 0`

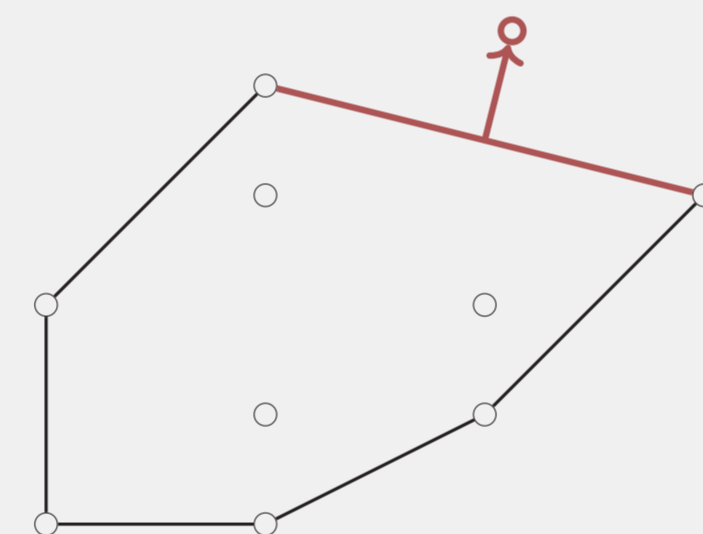
GraPHedron [3]



- ▶ Graphs as points in invariants space.
- ▶ Convex hull facets give bounds on invariants relations.
- ▶ Exact approach.

Up in the order ?

- ▶ # graphs of order 10: 12 005 168
- ▶ # graphs of order 11: 1 018 997 864



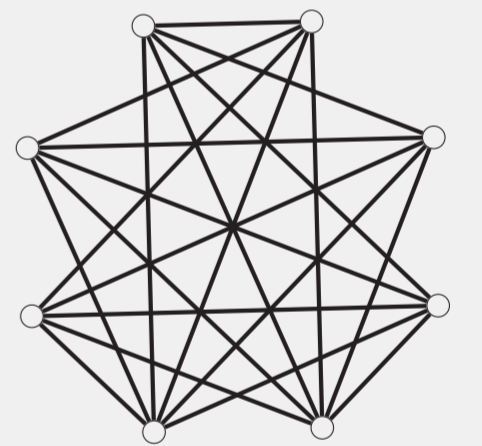
- ▶ Build convex hull of a sparse sampling of higher order graphs using metaheuristics (VNS).
- ▶ Optimize with cost function in facets direction in order to grow the convex hull approximation.

Conjecture

Bound and extremal graphs for Turán's problem:

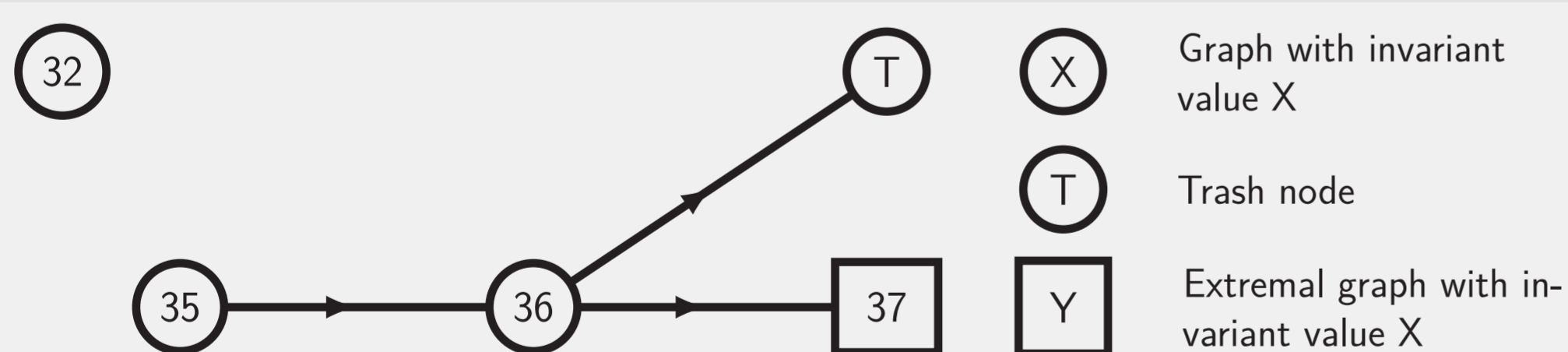
$$|E| \leq \frac{(k-2)n^2}{2(k-1)}$$

Extremal graphs:
 $T(n, k)$



- ▶ Establish conjectures from previous data (convex hull, invariants DB).

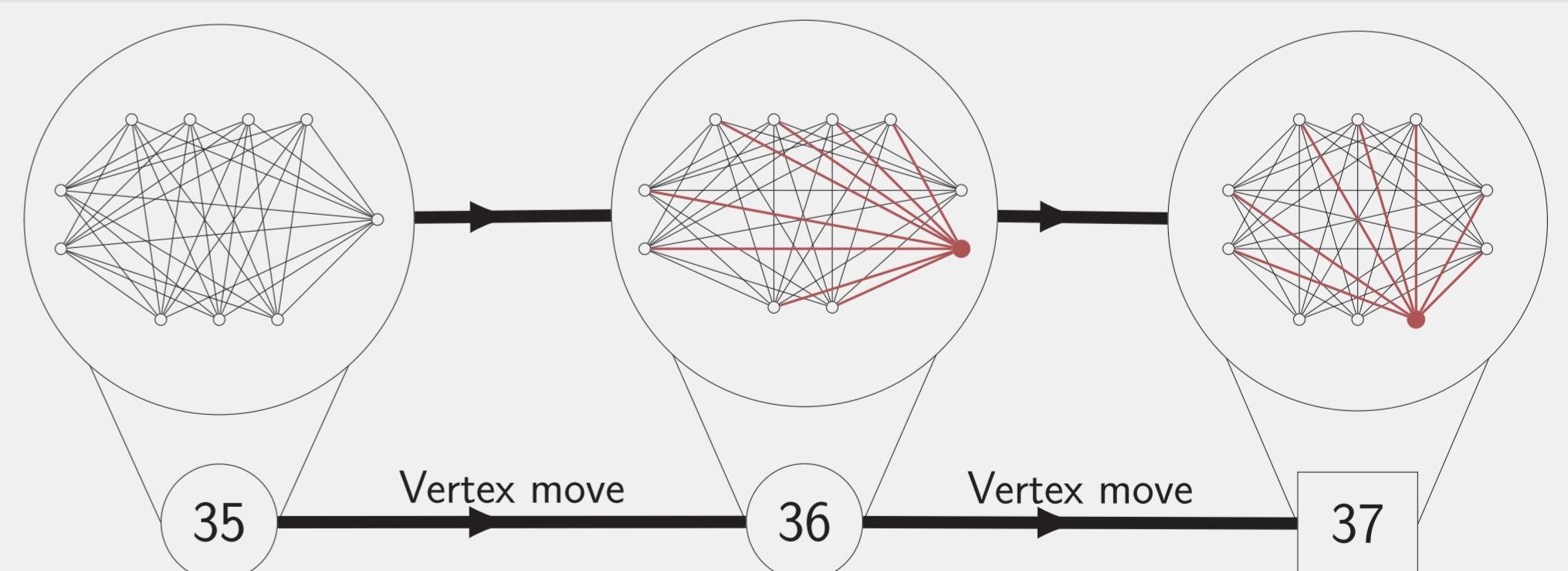
Transproof



The transformation graph shows if:

- ▶ there is a path between every non extremal graph and an extremal graph;
- ▶ the transformation only gives graphs in the studied class (e.g., connected);
- ▶ no transformation transition changes the invariant the wrong way.

Transformations path



- ▶ Study effects of transformations on a path.
- ▶ Help refine transformations to avoid problematic cases (e.g., disconnecting the graph).

References

- [1] Pál Turán. On an extremal problem in graph theory. *Mat. Fiz. Lapok*, 48(436-452):137, 1941.
- [2] Brendan D. McKay and Adolfo Piperno. Practical graph isomorphism, {II}. *Journal of Symbolic Computation*, 60(0):94 – 112, 2014.
- [3] Hadrien Mélot. Facet defining inequalities among graph invariants: the system graphedron. *Discrete Applied Mathematics*, 156(10):1875–1891, 2008.