Deformed black strings in 5-dimensional Einstein-Yang-Mills theory

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We construct the first examples of deformed non-abelian black strings in a 5-dimensional Einstein-Yang-Mills model. Assuming all fields to be independent of the extra coordinate, we construct deformed black strings, which in the 4-dimensional picture correspond to axially symmetric non-abelian black holes in gravity-dilaton theory. These solutions thus have *deformed* $S^2 \times \mathbb{R}$ horizon topology. We study fundamental properties of the black strings and find that for all choices of the gravitational coupling two branches of solutions exist. The limiting behaviour of the second branch of solutions however depends strongly on the choice of the gravitational coupling.

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I. INTRODUCTION

Higher dimensional black holes have gained a lot of interest in recent years. This is mainly due to the on-going study of theories in higher dimensions such as Kaluza-Klein theories [1, 2], (super)string theories [3] and brane world models [4]. In higher dimensions, horizon topologies other than those known in 4 dimensions are possible. While the first examples of higher-dimensional black holes, namely the higher dimensional generalisations of Schwarzschild-, Reissner-Nordström- [5] and Kerr solutions [6] have horizon topology S^{d-2} in d dimensions, so-called black string solutions [7] (in its simplest version a 4-dimensional Schwarzschild black hole extended into one extra dimension) with horizon topology $S^2 \times \mathbb{R}$ as well as black ring solutions [8] with horizon topology $S^2 \times S^1$ have been constructed. The black strings (in their simplest version) are -like the 5-dimensional Schwarzschild black holes - static solutions of the vacuum Einstein equations. They have been mainly investigated with respect to their stability [9]. Since the black strings have entropy proportional to M^2/y_0, where M denotes the mass of the black string and y_0 is the extension in the extra dimension, hyperspherical black holes have entropy proportional to $M^{3/2}$. Thus for large enough y_0, one would expect an instability -thereafter called the "Gregory-Laflamme instability" - of the black string which was confirmed analytically in [9]. For recent reviews on black strings and black holes in space-times with compact extra dimensions see [10, 11].

The higher dimensional Kerr solutions, to which in the following we refer to as the Myers-Perry solutions, as well as the Emparan-Reall black ring solutions are stationary solutions of the vacuum Einstein equations and thus carry angular momentum. While the Myers-Perry solutions exist only up to a maximal value of the angular momentum, black rings are balanced against gravitational collapse by rotation and thus exist only above a critical value of the angular momentum.

All existing higher dimensional black holes have been studied intensively with respect to their uniqueness. While uniqueness theorems for a variety of static black hole solutions have been well establised [12, 13, 14], stationary black holes seem to violate uniqueness. In $d = 5$, Myers-Perry solutions as well as black ring solutions exist for the same values of the mass and angular momentum and are thus *not* uniquelly characterised by these latter parameters. Interestingly, the situation changes if 5-dimensional supersymmetric black holes and black rings are studied [15, 16].

While most black hole solutions in higher dimensions have been constructed in different (dilaton-) gravity theories without additional matter fields, the first examples of black holes in a 5-dimensional SO(4)-Einstein-Yang-Mills model have been constructed [17]. These black holes are hyperspherically symmetric generalisations (horizon topology S^3) of the coloured black hole solutions in 4-dimensional SU(2) Einstein- Yang-Mills theory [18]. These solutions show that - like in 4 dimensions - the uniqueness theorems for higher dimensional static black holes cannot be extended to models involving non-abelian gauge fields.

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In [19] 5-dimensional black strings in an SU(2) Einstein-Yang-Mills model which has been introduced in [20] have been constructed. These are 4-dimensional, spherically symmetric non-abelian black holes extended trivially into one extra dimension and have thus horizon topology $S^2 \times \mathbb{R}$.

In this paper, we extend these latter results and discuss deformed black strings. These black strings are axially symmetric black holes in 4 dimensions extended trivially into one extra dimension. They thus have *deformed* $S^2 \times \mathbb{R}$ topology. Our solutions are translationally invariant - in contrast to the recently constructed translationally non-uniform black strings [21]. To distinguish the case studied here from the (non-) uniform black strings, we call our solutions with rotational symmetry in 4 dimensions "undeformed" black strings and the solutions with axial symmetry in 4 dimensions "deformed" black strings, respectively. Note also that the solutions studied here are the black hole analogues of the deformed vortex-type solutions studied in [22].

Our paper is organised as follows: In section II, we give the model, the ansatz, the equations of motion and the boundary conditions. In Section III, we introduce fundamental properties of the black strings. In Section IV, we discuss our numerical results and in Section V, we give our conclusions.

II. THE MODEL

We study the model introduced in [20]. This is an SU(2) Einstein-Yang-Mills model in 5 dimensions, where all fields are assumed to be independent of the extra dimension. We take the length of the extra dimension to be equal to unity.

The Einstein-Yang-Mills Lagrangian in $d = (4 + 1)$ dimensions then reads:

$$S = \int \left(\frac{1}{16\pi G_5} R - \frac{1}{4} F^a_{MN} F^{aMN} \right) \sqrt{g^{(5)}} d^5x \tag{1}$$

with the SU(2) Yang-Mills field strengths $F^a_{MN} = \partial_M A^a_N - \partial_N A^a_M + e\epsilon_{abc} A^b_M A^c_N$, the gauge index $a = 1, 2, 3$ and the space-time index $M = 0, ..., 4$. G_5 and e denote respectively the 5-dimensional Newton's constant and the coupling constant of the gauge field theory. G_5 is related to the Planck mass M_{pl} by $G_5 = M_{pl}^{-3}$ and e^2 has the dimension of [length].

Both the metric and matter fields are assumed to be independent of the extra coordinate y. The gauge fields can then be parametrized as follows [20]:

$$A^a_M dx^M = A^a_\mu dx^\mu + A^a_y dy \ , \tag{2}$$

while we give the generalized Ansatz for the metric below.

A. The Ansatz

Our aim is to construct non-abelian black strings, which are axially symmetric in 4 dimensions and extended trivially into one extra dimension. We thus have three Killing vectors $\frac{\partial}{\partial y}$, $\frac{\partial}{\partial \varphi}$, $\frac{\partial}{\partial t}$ associated with the black string solutions. Due to the fact that we will choose the components of A_y and A_φ to point in the same direction of the internal space, off-diagonal components of the energy-momentum tensor appear. We thus choose the following ansatz for the metric tensor [22]:

$$g^{(5)}_{MN} dx^M dx^N = e^{-\xi} \left[-f dt^2 + \frac{m}{f} dr^2 + \frac{mr^2}{f} d\theta^2 + \frac{l}{f} r^2 \sin^2\theta \left(d\varphi + J dy\right)^2 \right] + e^{2\xi} dy^2 \ , \tag{3}$$

where $f = f(r, \theta)$, $m = m(r, \theta)$, $l = l(r, \theta)$, $J = J(r, \theta)$ and $\xi = \xi(r, \theta)$ are functions of r and θ only. We have parametrized the metric such that the determinant of the metric g becomes independent of J:

$$\sqrt{-g} = e^{-\xi} \frac{mr^2}{f} \sqrt{l} \sin\theta \tag{4}$$

Note that our parametrisation here differs from that used in [22], however can be obtained by a simple transformation of the fields.

For the gauge fields, the Ansatz reads [22]:

$$
\begin{aligned}
A_\mu dx^\mu &= \frac{1}{2er}\left[\tau_\varphi^n\left(H_1 dr + (1-H_2) r d\theta\right) - n\left(\tau_r^n H_3 + \tau_\theta^n (1-H_4)\right) r\sin\theta d\varphi\right. \\
&+ \left.\left(H_5\tau_r^n + H_6\tau_\theta^n\right) r dy\right] \ ,
\end{aligned} \tag{5}
$$

where $H_i = H_i(r,\theta)$, $i = 1,..,6$ and τ_r^n, τ_θ^n and τ_φ^n denote the scalar product of the vector of Pauli matrices $\vec{\tau} = (\tau_1,\tau_2,\tau_3)$ with the unit vectors $\vec{e}_r^{\,n} = (\sin\theta\cos n\varphi, \sin\theta\sin n\varphi, \cos\theta)$, $\vec{e}_\theta^{\,n} = (\cos\theta\cos n\varphi, \cos\theta\sin n\varphi, -\sin\theta)$, $\vec{e}_\varphi^{\,n} = (-\sin n\varphi, \cos n\varphi, 0)$. n corresponds to the winding number of the configuration.

We fix the residual gauge invariance by imposing the gauge condition $r\partial_r H_1 - \partial_\theta H_2 = 0$ [22, 23].

B. Equations of motion

The matter Lagrangian in terms of the field strength tensor reads:

$$
\begin{aligned}
\mathcal{L}_M &= -\frac{1}{4}\mathrm{trace}\left(F_{MN}F^{MN}\right) = -\frac{1}{2}\ \mathrm{trace}\ \left[F_{r\theta}^2 e^{2\xi}\frac{f^2}{m^2r^2} + F_{r\varphi}^2\left(e^{2\xi}\frac{f^2}{ml\sin^2\theta r^2} + e^{-\xi}\frac{J^2 f}{m}\right)\right. \\
&+ F_{ry}^2 e^{-\xi}\frac{f}{m} + F_{\theta\varphi}^2\left(e^{2\xi}\frac{f^2}{ml\sin^2\theta r^4} + e^{-\xi}\frac{J^2 f}{mr^2}\right) + F_{\theta y}^2 e^{-\xi}\frac{f}{mr^2} \\
&+ \left. F_{\varphi y}^2\left(e^{-\xi}\frac{f}{\sin^2\theta l r^2} + J^2 e^{-4\xi}\right) - 2F_{r\varphi}F_{ry}\frac{e^{-\xi}Jf}{m} - 2F_{\theta\varphi}F_{\theta y}\frac{e^{-\xi}Jf}{mr^2} - F_{\varphi y}^2 e^{-4\xi}J^2\right]
\end{aligned} \tag{6}
$$

where the non-vanishing parts of the field strength tensor are given by:

$$
\begin{aligned}
F_{r\theta} &= -\frac{1}{r}\left(H_{1,\theta} + rH_{2,r}\right)\frac{\tau_\varphi^n}{2} \ , \\
F_{r\varphi} &= -n\frac{\sin\theta}{r}\left(rH_{3,r} - H_1H_4\right)\frac{\tau_r^n}{2} + n\frac{\sin\theta}{r}\left(rH_{4,r} + H_1H_3 + \cot\theta H_1\right)\frac{\tau_\theta^n}{2} \ , \\
F_{\theta\varphi} &= -n\sin\theta\left(H_{3,\theta} - 1 + H_2H_4 + \cot\theta H_3\right)\frac{\tau_r^n}{2} + n\sin\theta\left(H_{4,\theta} - H_2H_3 - \cot\theta(H_2 - H_4)\right)\frac{\tau_\theta^n}{2} \ , \\
F_{ry} &= \left(H_{5,r} + \frac{H_1H_6}{r}\right)\frac{\tau_r^n}{2} + \left(H_{6,r} - \frac{H_1H_5}{r}\right)\frac{\tau_\theta^n}{2} \ , \\
F_{\theta y} &= \left(H_{5,\theta} - H_2H_6\right)\frac{\tau_r^n}{2} + \left(H_{6,\theta} + H_2H_5\right)\frac{\tau_\theta^n}{2} \ , \\
F_{\varphi y} &= n\left(H_3H_6\sin\theta + H_6\cos\theta + H_4H_5\sin\theta\right)\frac{\tau_\varphi^n}{2} \ .
\end{aligned} \tag{7}
$$

The energy-momentum tensor

$$
T_{MN} = 2\ \mathrm{trace}\ \left(g^{AB}F_{MA}F_{NB} - \frac{1}{4}g_{MN}F_{AB}F^{AB}\right) \tag{8}
$$

has non-vanishing components T_{MM}, $M = 0,..,4$ and $T_{r\theta}$, $T_{\varphi y}$.

The Euler-Lagrange equations $\nabla_M F^{MN} + i[A_M, F^{MN}] = 0$ are obtained by varying the Lagrangian with respect to the matter fields $H_i(r,\theta)$, while the Einstein equations read: $G_{MN} = 8\pi G_5 T_{MN}$. We thus obtain a system of 11 coupled partial differential equations to be solved subject to appropriate boundary conditions.

Note that in the 4-dimensional picture our system corresponds to an SU(2) Einstein-Yang-Mills-Higgs-dilaton system with an additional U(1) potential given in terms of $J(r,\theta)$ [22]. The A_y-component of the gauge field then plays the role of a Higgs field, while ξ can be interpreted as a dilaton.

C. Boundary conditions

Due to the requirement of asymptotic flatness, we have for the metric functions at infinity:

$$
f(r=\infty) = 1 \ , \ m(r=\infty) = 1 \ , \ l(r=\infty) = 1 \ , \ \xi(r=\infty) = 0 \ , \ J(r=\infty) = 0 \ , \tag{9}
$$

while for the gauge field functions, we have:

$$H_i(r=\infty)=0\ ,\ i=1,2,3,4,6\ ,\ H_5(r=\infty)=1\ . \tag{10}$$

At the regular horizon, the boundary conditions read:

$$f(r=r_h)=0\ ,\ m(r=r_h)=0\ ,\ l(r=r_h)=0\ ,\quad \partial_r\xi|_{r=r_h}=0\ ,\ \partial_r J|_{r=r_h}=0 \tag{11}$$

for the metric functions and

$$H_1(r=r_h)=0\ ,\ \partial_r H_i|_{r=r_h}=0\ ,\ i=2,3,4,5,6 \tag{12}$$

for the gauge field. Finally, the boundary conditions on the ρ- and z-axis read (due to symmetry requirements):

$$\partial_\theta f|_{\theta=\theta_0}=\partial_\theta m|_{\theta=\theta_0}=\partial_\theta l|_{\theta=\theta_0}=\partial_\theta\xi|_{\theta=\theta_0}=\partial_\theta J|_{\theta=\theta_0}=0\ ,\ \theta_0=0,\frac{\pi}{2} \tag{13}$$

for the metric fields and

$$H_1(\theta=\theta_0)=H_3(\theta=\theta_0)=H_6(\theta=\theta_0)=\partial_\theta H_2|_{\theta=\theta_0}=\partial_\theta H_4|_{\theta=\theta_0}=\partial_\theta H_5|_{\theta=\theta_0}=0\ ,\ \theta_0=0,\frac{\pi}{2} \tag{14}$$

for the gauge fields.

III. FUNDAMENTAL PROPERTIES OF THE BLACK STRINGS

With the introduction of the new variable $x=re$ (with $x_h\equiv r_h e$) the equations of motion depend only on the coupling constant

$$\alpha^2=4\pi G_5\ . \tag{15}$$

The entropy S of the black strings is given by:

$$S=\frac{A}{4}=\frac{1}{4}\int_0^{y_0}\int_0^{\pi}\int_0^{2\pi}\sqrt{g_{yy}}\ \sqrt{g_{\theta\theta}}\ \sqrt{g_{\varphi\varphi}}\ d\varphi\ d\theta\ dy=y_0\frac{\pi}{2}\left(\int_0^{\pi}d\theta\sin\theta\sqrt{l\ m}\frac{x^2}{f}\right)\Bigg|_{x=x_h}\ , \tag{16}$$

where y_0 is the length of the extra dimension, which we set to one, $y_0=1$.

The parameter entering our boundary conditions is the horizon parameter x_h. However, the interpretation of this parameter is not as straightforward as in the case of Schwarzschild-like coordinates. We thus use the area parameter x_Δ [23] with

$$x_\Delta=\sqrt{\frac{A}{4\pi y_0}} \tag{17}$$

to characterise the solutions.

The mass M (per unit length of the extra dimension) is given by [23]:

$$M=\frac{1}{2\alpha^2}\lim_{x\to\infty}x^2\partial_x f\ . \tag{18}$$

Furthermore, we study the ratio of the circumference along the equator L_e and that along the poles L_p to have a measure for the deformation of the horizon of the black strings [23]:

$$L_e=\left(\int_0^{2\pi}d\varphi\sqrt{\frac{l}{f}}\ \sin\theta\ e^{-\xi/2}\ x\right)\Bigg|_{x=x_h,\theta=\pi/2}\ ,\quad L_p=2\left(\int_0^{\pi}d\theta\ \sqrt{\frac{m}{f}}\ e^{-\xi/2}\ x\right)\Bigg|_{x=x_h,\varphi=0} \tag{19}$$

A further important property of black holes and black strings is the temperature of the solutions, which here is given by:

$$T=\frac{f_2(\theta)}{2\pi x_h\sqrt{m_2(\theta)}}\ , \tag{20}$$

where we have used the expansion of the metric functions at x_h [23]:

$$f(x,\theta) = f_2(\theta)\left(\frac{x-x_h}{x_h}\right)^2 + O\left(\frac{x-x_h}{x_h}\right)^3 ,$$
$$m(x,\theta) = m_2(\theta)\left(\frac{x-x_h}{x_h}\right)^2 + O\left(\frac{x-x_h}{x_h}\right)^3 . \quad (21)$$

The zeroth law of black hole mechanics states that the temperature is constant at the horizon, i.e. $\partial_\theta T = 0$, which requires $2m_2\ \partial_\theta f_2 - f_2\ \partial_\theta m_2 = 0$ (this follows directly from (20)).

In the isolated horizon framework for 4-dimensional black hole solutions, it has been stated [24] and in fact confirmed for 4-dimensional non-abelian black holes in SU(2) Einstein-Yang-Mills-Higgs theory [23] that a non-abelian black hole is a bound system of a Schwarzschild black hole and the corresponding non-abelian regular solution. To test whether this also holds true here, we have studied the binding energy E_b (per unit length of the extra dimension) of the black string solutions:

$$E_b = M - M_{reg} - M_s \quad , \quad M_s = \frac{x_\Delta}{2\alpha^2} \quad (22)$$

where M_s is the mass of the Schwarzschild black string (per unit length of the extra dimension) and M_{reg} is the mass of the corresponding non-abelian regular solution which has been studied in [22]. Note that for M_{reg}, we have used the mass of the fundamental solution, i.e. the solution on the 1. branch of regular solutions, which we believe to be stable.

In the study of (non-) uniform black strings and black holes in space-times with extra compact dimensions, a further quantity, namely the tension along the extra dimensions has been studied [10, 11]. The phase diagram in the mass-tension plane gives good indication about the properties of the solutions. The detailed study of these diagrams has been done in a follow-up publication by the present authors [25].

IV. NUMERICAL RESULTS

We have solved numerically the system of partial differential equations subject to the above given boundary conditions for several values of the coupling constant α and of the horizon parameter x_h, respectively. Here, we report on our analysis of the cases $n = 1$, $n = 2$ and $\alpha = 0.5$. More details for other parameter values will be presented elsewhere [25].

Before we discuss the numerical results, let us recall the results for the regular case. It turns out that these are crucial for the understanding of the qualitative features of the black hole solutions. The regular case for $n = 1$ was studied in detail in [20, 26]. It has been found that several branches of solutions (which in the following we refer to as α-branches) for varying α exist. α-branch refers here to a curve giving one of the quantities of the solution (e.g. the energy) as function of α. Typically, several distinct curves ("branches") appear in an energy-α-plot, such that for a fixed value of α different solutions (with different energies) exist. In [26] four branches have been constructed such that for $\alpha \in [0 : 0.312[$, $\alpha \in]0.419 : 1.268]$, $\alpha \in [0.312 : 0.395[$ and $\alpha \in [0.395 : 0.419]$ one, two, three and four solutions, respectively, exist. It is likely that for $\alpha \in [0.395 : 0.419]$ further branches exist, however, these have not been constructed so far.

The regular $n = 2$ case was studied in [22] and only one branch of solutions in α has been constructed. Corresponding to the $n = 1$ case we believe, though, that further branches also exist for $n = 2$, the numerical construction of which however seems very involved.

A. Undeformed black strings for $n = 1$

In [19], the existence of several branches for a fixed area parameter x_Δ and varying α has been demonstrated for the $n = 1$ black strings. The behaviour of the solutions for fixed x_Δ and varying α is thus similar to that observed for regular solutions [20]. Here, we observe a new phenomenon for α fixed and varying x_Δ. Since this case has not been studied in [19], we have reconsidered the case $n = 1$ here. Note that for $n = 1$, the black strings are 4-dimensional spherically symmetric black holes extended trivially into the extra dimension. The solutions thus depend only on the radial coordinate x. For the functions we have: $H_2(x) = H_4(x)$, $H_1(x) = H_3(x) = H_6(x) = J(x) = 0$.

Our numerical results for $\alpha = 0.5$ are shown in Fig.1 and Fig.2. In Fig.1, we give the values of the gauge field functions $H_2(x) = H_4(x)$, $H_5(x)$ and of the metric function $\xi(x)$ at x_Δ, $H_2(x_\Delta)$, $H_5(x_\Delta)$, $\xi(x_\Delta)$, as functions of the area parameter x_Δ. Clearly, two branches of solutions exist. The first branch (denoted by "1") exists for $x_\Delta \in [0 : x_\Delta^{(max)}]$ with $x_\Delta^{(max)} \approx 0.633$. The limit $x_\Delta \to 0$ on this branch of solutions corresponds to the fundamental regular solution, i.e. the solution on the first α-branch [20]. Clearly $H_2(x_\Delta \to 0) \to 1$ and $H_5(x_\Delta \to 0) \to 0$, which corresponds to the boundary conditions for the globally regular solutions at $x = 0$. At the same time $\xi(x_\Delta \to 0) \to 0.07216$, which is the numerically determined value for the fundamental regular solution [20]. Similarly the mass on the first (lower) branch tends to the mass of the fundamental regular solution for $x_\Delta \to 0$ (see Fig. 2).

The second branch of solutions (denoted by "2" in Fig.1) similarly exists for $x_\Delta \in [0 : x_\Delta^{(max)}]$. Clearly, the solutions on this second branch are distinct from those on the first branch having higher mass (see Fig.2) and different values of $H_2(x_\Delta)$, $H_5(x_\Delta)$ and $\xi(x_\Delta)$ (see Fig.1). In the limit $x_\Delta \to 0$, we find again that $H_2(x_\Delta \to 0) \to 1$ and $H_5(x_\Delta \to 0) \to 0$, while $\xi(x_\Delta \to 0) \to -1.262$ tends to the value of the corresponding solution of the second α-branch of regular solutions. Strong evidence for this is also given by inspection of the mass curve in Fig.2, where in the limit $x_\Delta \to 0$, the mass tends to that of the regular solution of the second α-branch.

The binding energy E_b (see Fig.2) is negative on the first branch, indicating that indeed non-abelian black strings are bound systems of a Schwarzschild black string and the corresponding regular non-abelian vortex solution. On the second branch, the binding energy becomes positive, indicating an instability of the solutions. Note that for both branches, we have used the mass of the fundamental regular solution, which we believe to be the stable regular solution. If we had used the mass of the regular solution on the second α-branch to obtain the binding energy for our black string solutions on the second branch, the binding energy would have been negative.

Results for different values of α will be given elsewhere [25]. However, we believe that for all values of α two branches of black string solutions will exist. The critical behaviour, however, will strongly depend on the value of α. For $\alpha = 0.5 \in]0.419 : 1.268]$ two regular solutions exist and the second branch terminates in the corresponding regular solution of the second α-branch. We believe that if $\alpha \in [\alpha_1^{(n)} : \alpha_2^{(n)}]$, where n indicates the number of regular solutions available for this range of α, i.e. the number of branches, the black string solutions on the second branch will tend to the nth regular solution, i.e. to the solution on the nth branch, for $x_\Delta \to 0$. E.g. the black strings on the second branch of solutions for $\alpha = 0.35 \in [0.312 : 0.395[$ would tend to the third solution, i.e. the solution on the third α-branch of regular solutions. For $\alpha \in]0 : 0.312]$ for which only the fundamental regular solution exists, we expect that the second branch terminates at some *finite* $x_\Delta > 0$. At this point, the solution bifurcates with the branch of Einstein-Maxwell-dilaton solution with $H_2(x) = H_4(x) \equiv 0$, $H_5(x) \equiv 1$ and $\xi(x)$ given by the corresponding function of the Einstein-Maxwell-dilaton solution.

B. Deformed black strings for $n = 2$

Our results for the deformed black string solutions ($n = 2$) are given in Fig.2, Fig.3 and Fig.4.

The first branch exists for $x_\Delta \in [0 : x_{\Delta,max}]$ with $x_{\Delta,max} \approx 1.31$. In the limit $x_\Delta \to 0$, the solution approaches the corresponding regular solution which has been constructed in [22]. When increasing x_Δ, our results show that both the mass M (see Fig.2) and the corresponding value of the horizon parameter x_h increase, while the temperature T decreases (see Fig.3). We have confirmed numerically that the temperature on the horizon is constant and our solutions thus fulfill the zeroth law of black hole mechanics. The deformation parameter L_e/L_p decreases, but stays very close to one indicating that the horizon is only deformed slightly. On this branch, the absolute value of the new function $|J(x,\theta)|$ stays small. In Fig.4 we show the values of J at the horizon, $J(x_\Delta, \theta = 0)$ together with the values of $\xi(x_\Delta, \theta = 0)$. These latter values are positive on the first branch of solutions. The values $H_2(x_\Delta, \theta)$ and $H_4(x_\Delta, \theta)$ decrease from one, while $H_5(x_\Delta, \theta)$ increases from zero for increasing x_Δ. We demonstrate the x_Δ-dependence for $H_2(x_\Delta, \theta = 0)$ and $H_5(x_\Delta, \theta = 0)$ in Fig.4.

Like in the $n = 1$ case the limit $x_\Delta \to 0$ corresponds to the fundamental regular deformed vortex solution [22].

We managed to construct the second branch of solutions for $x_\Delta \in [x_{\Delta,end} : x_{\Delta,max}]$ with $x_{\Delta,end} \approx 0.2$. At $x_\Delta = x_{\Delta,max}$ the branches merge into a single solution. The mass of the solutions on the second branch is higher than that of the corresponding solution on the first branch for the same value of x_Δ and thus same entropy (see Fig.2).

The other features of the second branch are that, when x_Δ decreases from $x_{\Delta,max}$, the parameter x_h and the mass decrease, while the temperature T increases and stays higher than the temperature of the corresponding solution on the first branch. As compared to the solutions on the first branch, the solutions on the second branch have a much stronger deformed horizon. This can be noted by observing the curve L_e/L_p in Fig.3 and the data plotted in Fig.4. We also notice that the value of the metric function ξ at the horizon becomes negative on the second branch and that the value of J at the horizon now deviates significantly from zero.

Finally, let us mention that the values of $H_2(x_\Delta, \theta)$ and $H_4(x_\Delta, \theta)$ start to increase again, while $H_5(x_\Delta, \theta)$ decreases for decreasing x_Δ. The detailed curves for $H_2(x_\Delta, \theta = 0)$ and $H_5(x_\Delta, \theta = 0)$ are shown in Fig.4.

We strongly believe that this second branch extends all the way back to $x_\Delta = 0$ similar to the $n = 1$ case. Then

$H_2(x_\Delta \to 0, \theta) \to 1$, $H_5(x_\Delta \to 0, \theta) \to 0$, $\xi(x_\Delta \to 0, \theta) \to \xi_0 < 0$, $J(x_\Delta \to 0, \theta) \to J_0 < 0$, where ξ_0 and J_0 have not been determined so far. Correspondingly, $x_h \to 0$ in this limit, $L_e/L_p \to 1$, (like for the first branch) and $T \to \infty$, since T is not defined for regular solutions.

In Fig.2, we also show (the negative of) the binding energy per winding number $-E_b/n$ for $\alpha = 0.5$. Clearly, the binding energy on the first branch of solutions (note the inversion of the branches with respect to the plot of the mass) is negative and the non-abelian black strings on the first branch are thus *bound systems* of the corresponding regular non-abelian vortex solutions of [22] and the Schwarzschild black string. On the second branch, the situation changes. The binding energy becomes positive. This signals that the non-abelian black strings are unstable to decay into the non-abelian, globally regular vortex solutions and a Schwarzschild black string on this second branch of solutions.

Comparing the $n = 1$ and $n = 2$ solutions for the same value of α, we find that the extension of the branches in x_Δ is bigger for $n = 2$ as compared to $n = 1$. Furthermore, when comparing the respective first and second branches for $n = 1$ and $n = 2$, the mass (per winding number) of the $n = 2$ solution is always lower than that of the $n = 1$ solution.

Results for different values of α will be given elsewhere. However, we believe that the scenario is similar to the $n = 1$ case, such that the second branch terminates in the corresponding regular solution of the nth α-branch for $x_\Delta \to 0$ if more than one α-branch exist, or in an Einstein-Maxwell-dilaton solution for finite x_Δ with $H_1(x, \theta) = H_2(x, \theta) = H_3(x, \theta) = H_4(x, \theta) = H_6(x, \theta) = 0$, $H_5(x, \theta) \equiv 1$, $J(x, \theta) = 0$ and $\xi(x, \theta) = \xi(x)$ given by the corresponding function of the Einstein-Maxwell-dilaton solution if only one α-branch exists for the regular solutions.

V. CONCLUSIONS

In this paper, we have constructed black strings as solutions of a 5-dimensional Einstein-Yang-Mills model. We have presented our results for $n = 1$, $n = 2$ and fixed $\alpha = 0.5$. We find that two branches of solutions exist in both cases, which in the limit of $x_\Delta \to 0$ tend to the corresponding regular solutions of the first, respectively second α-branch. We believe that this is a generic feature of the system. For other values of α, the second branch of black string solutions will terminate in the nth available regular solution. This means that in specific parameter ranges, the black string solutions will start to show an oscillatoric behaviour in the gauge field functions. For those values of α, for which only one globally regular solution exists, we believe that the second branch will terminate at a finite value of the area parameter in an Einstein-Maxwell-dilaton solution. More details will be given elsewhere [25].

The question whether the non-abelian black strings are unstable to decay to hyperspherically symmetric non-abelian black holes, i.e. would have an instability corresponding to the "Gregory-Laflamme instability" for Schwarzschild black strings is beyond the scope of this paper. A first step to answer this question would be the numerical construction of hyperspherically symmetric non-abelian black holes of the SU(2) Einstein-Yang-Mills system in 5 dimensions.

We remark that next to the "fundamental" solutions constructed here, also excited solutions could exist-similar to what is known for 4-dimensional spherically symmetric black hole solutions in SU(2) Einstein-Yang-Mills-Higgs theory [27]. These solutions would have a number m, $m \in \mathbb{N}$, of zeros of the gauge field functions. The construction of these solutions has not been done so far - neither in the case of globally regular vortices nor black strings - and is left to a future publication.

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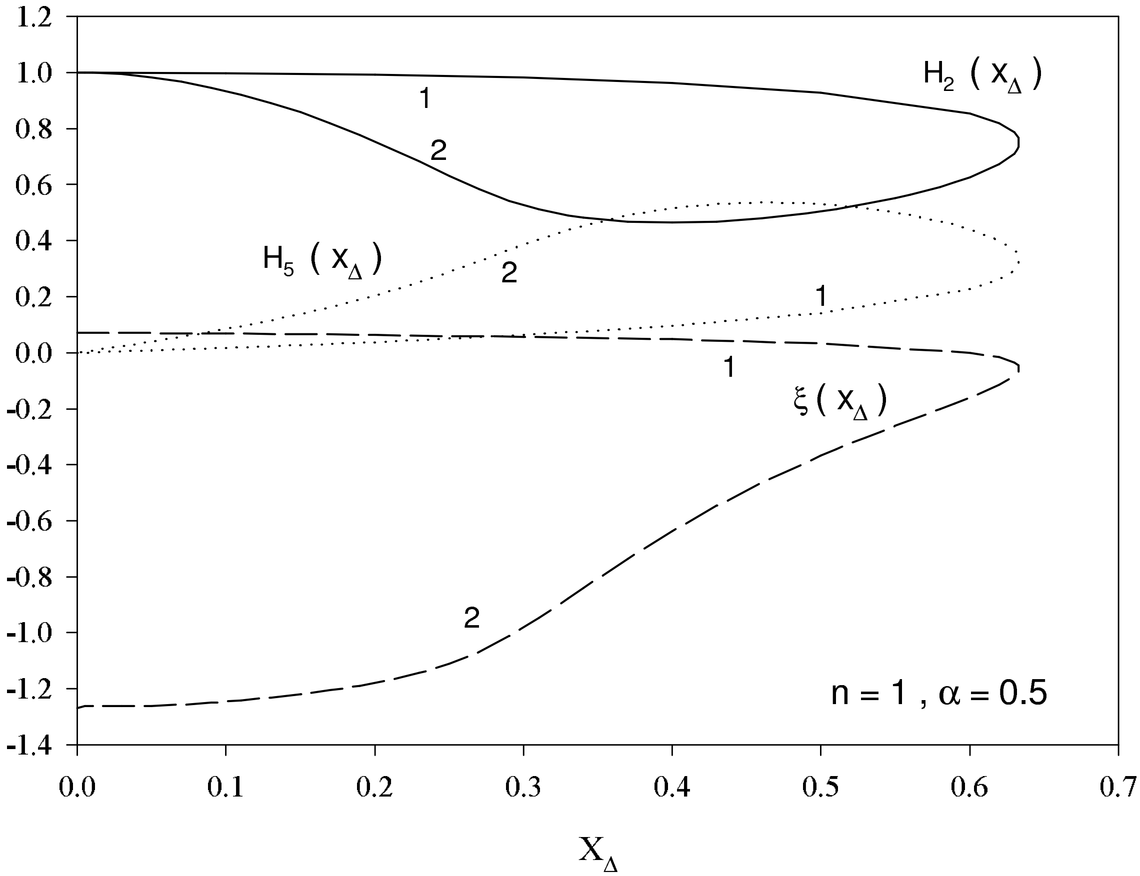

FIG. 1: The value of the gauge field functions $H_2 = H_4$, H_5 and of the metric function ξ at x_Δ, $H_2(x_\Delta)$, $H_4(x_\Delta)$, $\xi(x_\Delta)$, on the first (1) and second branch (2) of solutions are shown as function of the area parameter x_Δ for the $n = 1$ black strings with $\alpha = 0.5$.

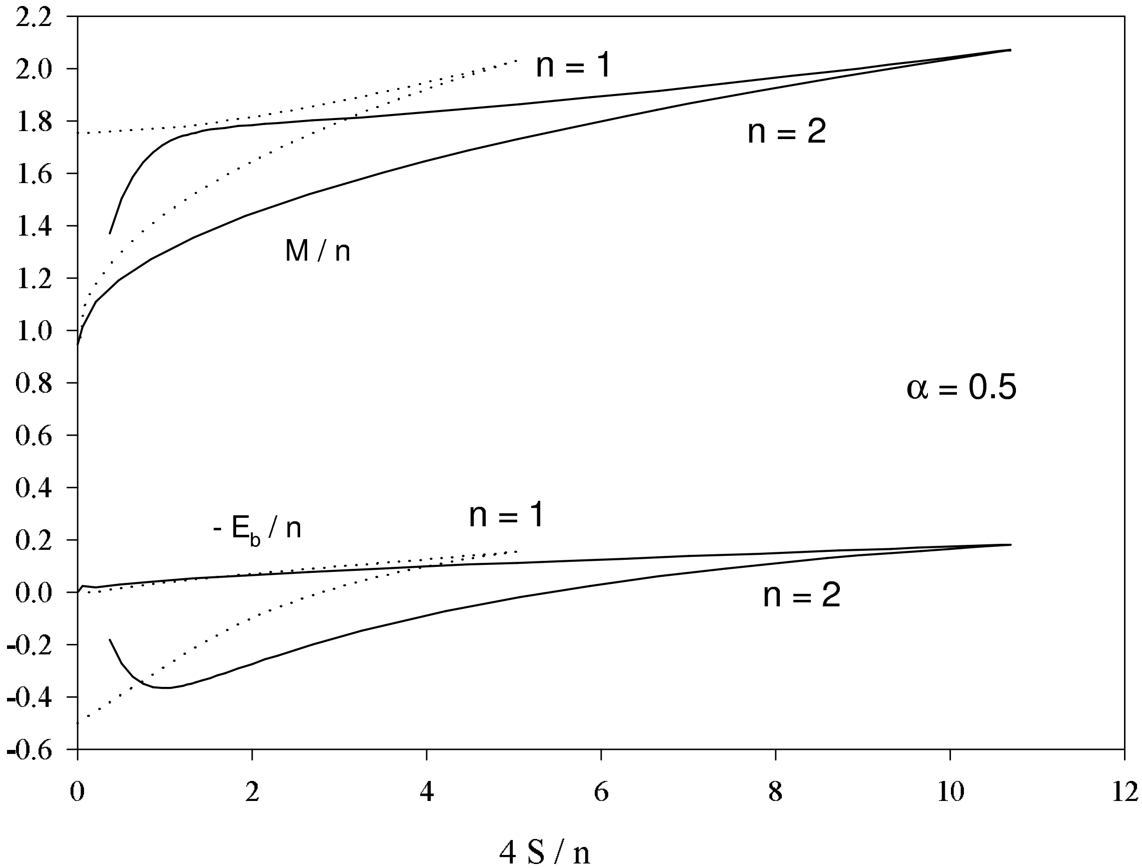

FIG. 2: The mass per winding number M/n of the solutions is shown as function of (4 times) the entropy per winding number $4S/n$ for the $n = 1$ and $n = 2$ black string solutions with $\alpha = 0.5$. Also shown is (the negative of) the binding energy per winding number $-E_b/n$ as function of $4S/n$. For this latter values, note the inversion of the branches with respect to the plot of the mass.

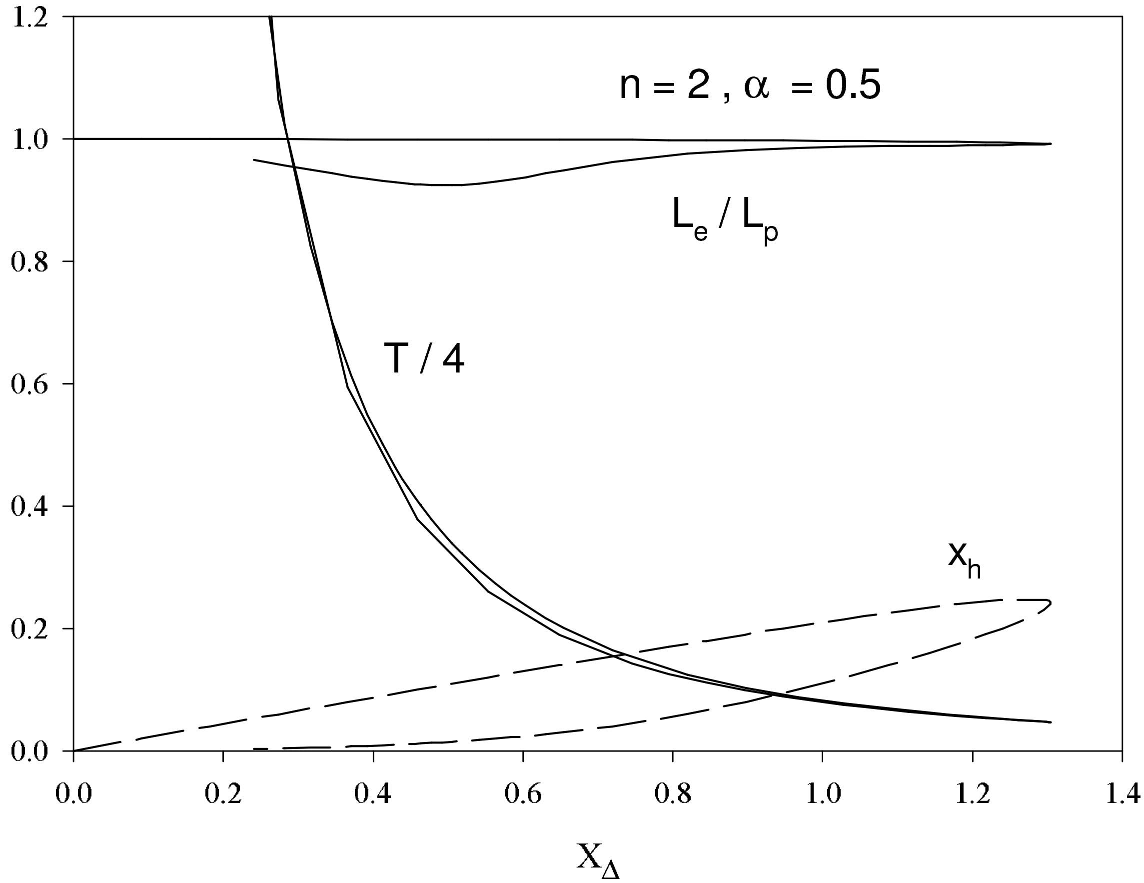

FIG. 3: The ratio of the circumference almong the equator and the circumference along the poles L_e/L_p, the temperature T as well as the horizon parameter x_h are shown as functions of the area parameter x_Δ for the deformed black string solutions with $n = 2$ and $\alpha = 0.5$.

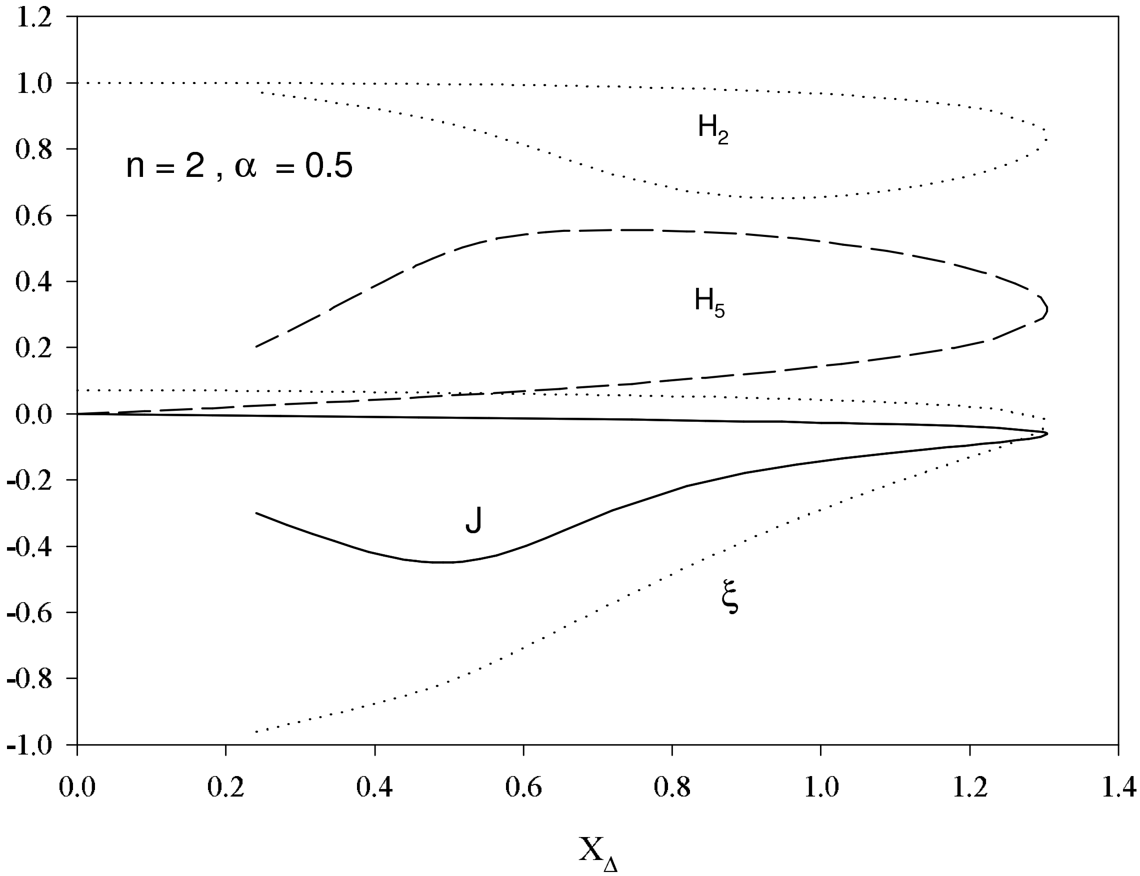

FIG. 4: The values of the metric functions ξ and J and of the gauge field functions H_2 and H_5 at x_Δ, $\xi(x_\Delta, \theta = 0)$, $J(x_\Delta, \theta = 0)$, $H_2(x_\Delta, \theta = 0)$,$H_5(x_\Delta, \theta = 0)$, are shown as functions of the area parameter x_Δ for the black string solutions with $n = 2$ and $\alpha = 0.5$.