Scattering amplitudes via multi-particle higher-spin charges

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Motivation

Why conserved multiparticle higher-spin (HS) charges $\mathcal{Q}$ (of **free classical** HS fields) can be related to scattering amplitudes $\mathcal{A}$ (of **interacting quantum** fields)? Both $\mathcal{Q}$ and $\mathcal{A}$ are:

- constructed from **classical on-shell** data,
- non-local objects, *i.e.*

$$\mathcal{Q} \sim \int_{\mathbf{S}\text{'urface}} J, \quad \mathcal{A} \sim \exp i \int_{\mathcal{M}_4\text{'inkowski}} \mathcal{L},$$

- ∞-many $\mathcal{Q}$'s (functional ambiguity) $\Rightarrow$ probably large enough space to embed expressions for scattering amplitudes of QFT's.

Paradox alert!

How can it happen that a single free classical HS theory can produce scattering amplitudes of various interacting QFT's?

Resolution: the approach in question does not reconstruct amplitudes from non-linear dynamics, but rather provides us with a large kinematical space where (probably all) amplitudes of (probably all) QFT's can be embedded.

Example of MHV amplitudes

We aim at reproducing Parke-Taylor formula for color-ordered tree MHV amplitudes for n gluons with i^-, j^-

$$\mathcal{A}^{ij,12\ldots i-1,i+1\ldots j-1,j+1\ldots n}_{\mu,\widetilde{\mu}} \propto$$

$$\propto \frac{\langle \mu_i \mu_j \rangle^4}{\langle \mu_1\mu_2\rangle\langle \mu_2\mu_3\rangle \ldots \langle \mu_{n-1}\mu_n\rangle\langle \mu_n\mu_1\rangle}\, \delta^{(4)}\left(\sum_{I=1}^{n} \mu_{I a}\widetilde{\mu}_{I\dot{a}}\right),$$

where μ_a, $\widetilde{\mu}_{\dot{a}}$ are $SL(2,\mathbb{C})$ momentum spinors such that $p_{a\dot{a}} = \mu_a \widetilde{\mu}_{\dot{a}}$ ($p^2 = \frac{1}{2}\det p_{a\dot{a}} = 0$) and $\langle \mu, \mu' \rangle = \varepsilon^{ab}\mu_a \widetilde{\mu}'_{\dot{a}}$.

MHV amplitudes as HS charges*

To reproduce a given amplitude there are "free parameters" on the HS side to adjust: $\mathbf{r} = m + \overline{m}$ and HS symmetry **parameter** η. For 3-particle MHV amplitude we take $\mathbf{r} = 3$ with $m = 2$, $\overline{m} = 1$ and $\eta = \eta^{(12,3)} := \prod_{\mathrm{i}=1,2,\bar{\mathrm{j}}=\bar{3}} \operatorname{sign}(\rho_{\mathrm{i}\bar{\mathrm{j}}})$. Then

$$\mathcal{Q}^{(2,1)}_{\mu,\widetilde{\mu}}\left[\eta^{(12,\bar{3})}\right] \propto \mathcal{A}^{(12,3)}_{\mu,\widetilde{\mu}}.$$

The result is independent of the integration surface $\mathbf{S} \subset \mathbf{Cor}_4^{(2,1)}$. All n-particle MHV amplitudes can be reproduced as well by taking $\mathbf{r} = n$ with $m = 2$, $\overline{m} = n-2$ and a particular HS symmetry parameter $\eta^{(12,3\ldots n)}$.

*See the right column for technical details.

Concluding remarks

- Scattering amplitudes are related to HS multiparticle charges of classical free HS theory. We expect that our result extends beyond tree level and QCD gluon amplitudes. This means, in particular, that amplitudes are embedded in the HS multiparticle symmetry.
- Various approaches for amplitude calculations can be expected to be related via a properly extended space **Cor** and an on-shell-closed differential form by different choices of integration surface.

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Unfolded HS dynamics

Higher-spin fields are represented by 0-forms $\mathcal{C}^1(y,\widetilde{y}|x)$ obeying rank-1 unfolded equation

$$\left(\frac{\partial}{\partial x^{a\dot{a}}} + i\,\frac{\partial^2}{\partial y^a \partial \widetilde{y}^{\dot{a}}}\right)\mathcal{C}^1(y,\widetilde{y}|x) = 0,$$

where $a, \dot{a} = 1,2$ are $SL(2,\mathbb{C})$-spinor indices, $x^{a\dot{a}}$ are coordinates of the 4-dimensional Minkowski $4d$ spacetime $\mathcal{M}_4$, y^a, $\widetilde{y}^{\dot{a}}$ are auxiliary spinor variables. When

$$\mathcal{C}^1(y|x) = \sum_{k,m=0}^{\infty} c_{a_1\ldots a_k,\dot{a}_1\ldots\dot{a}_m}(x)\frac{y^{a_1}\ldots y^{a_k}\,\widetilde{y}^{\dot{a}_1}\ldots\widetilde{y}^{\dot{a}_m}}{k!m!}$$

is a formal power series, then unfolded equation describes on-shell gauge-invariant field strengths of all half-integer spins: spin-0 field $c(x)$, spin-1/2 fields (Weyl fermions) $c_a(x)$, $c_{\dot{a}}(x)$, spin-1 field (self-duality components of Maxwell tensor) $c_{aa}(x)$, $c_{\dot{a}\dot{a}}(x)$, etc. Mixed components $c_{a_1\ldots a_l,\dot{a}_1\ldots\dot{a}_m}(x)$ are auxiliary and represent all on-shell derivatives fields.

We deal with distributions which do not admit projection to a particular spin, for example plane-wave solutions

$$\chi_{\mu,\widetilde{\mu}}(y,\widetilde{y}|x) = \exp\left[i\left(\mu_a\widetilde{\mu}_{\dot{a}}x^{a\dot{a}} + \mu_a y^a + \widetilde{\mu}_{\dot{a}}\widetilde{y}^{\dot{a}}\right)\right]$$

with light-like momenta $i\frac{\partial}{\partial x^{a\dot{a}}} \propto \mu_a\widetilde{\mu}_{\dot{a}}$.

Multiparticle states = tensor product

Multiparticle states of HS fields are described by their products $\mathcal{C}^1(y_1|x)\ldots\mathcal{C}^1(y_2|x)$ which are particular case of rank-$\mathbf{r}$ fields satisfyig rank-$\mathbf{r}$ unfolded equation

$$\left(\frac{\partial}{\partial x^{a\dot{a}}} + i\,\delta_{IJ}\frac{\partial^2}{\partial y_I^a \partial \widetilde{y}_J^{\dot{a}}}\right)\mathcal{C}^{\mathbf{r}}(y,\widetilde{y}|x) = 0,$$

where $I, J = 1\ldots\mathbf{r}$. The case $\mathbf{r} = 2$ (bilinear fields) corresponds to conserved HS currents, higher ranks $\mathbf{r} \geqslant 3$ appear to correspond to scattering amplitudes.

For a partition $\mathbf{r} = m + \overline{m}$ half-Fourier transform in variables y^a_{i} ($\mathrm{i} = 1\ldots m$) and $\widetilde{y}^{\dot{a}}_{\bar{\mathrm{j}}}$ ($\bar{\mathrm{j}} = m+1\ldots\mathbf{r}$) gives a first-order PDE

$$\left(\frac{\partial}{\partial x^{a\dot{a}}} - \lambda_{\mathrm{i}\,a}\frac{\partial}{\partial \widetilde{y}^{\dot{a}}_{\mathrm{i}}} - \widetilde{\lambda}_{\bar{\mathrm{j}}\,\dot{a}}\frac{\partial}{\partial y^{a}_{\bar{\mathrm{j}}}}\right) g^{(m,\overline{m})}\left(\lambda_{\mathrm{i}}, y_{\bar{\mathrm{i}}}; \widetilde{y}_{\mathrm{j}}, \widetilde{\lambda}_{\bar{\mathrm{j}}}\Big|x\right) = 0$$

with characteristics $\lambda_{\mathrm{i}\,a}$, $\widetilde{y}^{\dot{a}}_{\mathrm{i}} + x^{a\dot{a}}\lambda_{\mathrm{i}\,a}$ and $\widetilde{\lambda}_{\bar{\mathrm{j}}\,\dot{a}}$, $y^a_{\bar{\mathrm{j}}} + x^{a\dot{a}}\widetilde{\lambda}_{\bar{\mathrm{j}}\,\dot{a}}$. The solutions of our interest are $\eta(\lambda_{\mathrm{i}}, \widetilde{\lambda}_{\bar{\mathrm{i}}}, \rho_{\mathrm{i}\bar{\mathrm{j}}})\chi_{\mu,\widetilde{\mu}}\left(\lambda_{\mathrm{i}}, y_{\bar{\mathrm{i}}}; \widetilde{y}_{\mathrm{j}}, \widetilde{\lambda}_{\bar{\mathrm{j}}}\Big|x\right)$, where $\rho_{\mathrm{i}\bar{\mathrm{j}}} = \lambda_{\mathrm{i}\,a}y^a_{\bar{\mathrm{j}}} - \widetilde{\lambda}_{\bar{\mathrm{j}}\,\dot{a}}\widetilde{y}^{\dot{a}}_{\mathrm{i}}$ and η is an **arbitrary** functional (HS-symmetry) **parameter**.

On-shell-closed differential form

Fields $g^{(m,\overline{m})}\left(\lambda_{\mathrm{i}}, y_{\bar{\mathrm{i}}}; \widetilde{y}_{\mathrm{j}}, \widetilde{\lambda}_{\bar{\mathrm{j}}}\Big|x\right)$ live in the extended space $\mathbf{Cor}_4^{(m,\overline{m})} = \mathcal{M}_4 \times \mathbb{R}^{4\mathbf{r}}$ and allow us to build an on-shell-closed differential form

$$\Omega^{(m,\overline{m})}_{\mu,\widetilde{\mu}}[\eta] = \mathrm{d}^{2m}\lambda\,\mathrm{d}^{2\overline{m}}\widetilde{\lambda}\,\mathrm{d}^{2\overline{m}}(y + x\widetilde{\lambda})\,\mathrm{d}^{2m}(\widetilde{y} + x\lambda)\,\eta\,\chi_{\mu,\widetilde{\mu}},$$

such that $\mathrm{d}_{\mathbf{Cor}}\Omega^{(m,\widetilde{m})}_{\mu,\widetilde{\mu}}[\eta] = 0$. This gives a charge

$$\mathcal{Q}^{(m,\overline{m})}_{\mu,\widetilde{\mu}}[\eta] := \int_{\mathbf{S}} \Omega^{(m,\overline{m})}_{\mu,\widetilde{\mu}}[\eta]$$

which is conserved in a sense that it does not depend on local variations on integration surface $\mathbf{S} \subset \mathbf{Cor}_4^{(m,\overline{m})}$. In particular, one gets the **same** answer by integrating equally over both **spacetime** or **spinor variables**.