HIGHER-SPIN GRAVITY

An overview

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Mostly based on [X. Bekaert, N.B. and P. Sundell, Rev. Mod. Phys. 84 (2012) "How higher-spin gravity surpasses the spin two barrier"].

PLAN

1. Origins ; some free HS

2. Nonlinear higher-spin equations : Vasiliev (1989 & 2003)

3. AdS/CFT & Open problems

① ORIGINS

- At dawn of QFT : Majorana (1932), Dirac (1936), Fierz-Pauli (1939), and most notably Wigner's 1939 classification of UIR's of Poincaré group $ISO(3,1)$.

- Relativistic, linear & covariant equations : Bargmann-Wigner (1948)

  ⟶ massless, helicity particles characterized by

  - Mass $m = 0$; • helicity $s \in \{0, 1/2, 1, 3/2 \dots\}$

[• Rem : In the $m = 0$ case, there are also the " continuous " or " infinite " spin UIR's ⇝ $\vec{\mu} \neq \vec{0}$ in $\mathbb{R}^{D-2}$.]

- Appearance of NO-GO results

↳ Problems with :

- Minimal $u(1)$ coupling for $s \geq 3/2$ (1961)
- Minimal Lorentz coupling for $s \geq 5/2$ (1964)
- Infinite-component Majorana-like equations (1968) (tachyons)

↳ Together with the observation of high-spin hadronic resonances

Belief that consistent high-spin interactions require infinitely-many fields of unbounded spin.

Once the HS representations have been seen to exist in the sense of UIR's of spacetime isometry algebra, i.e. first quantization, then standard second quantization naturally requires a covariant Lagrangian.

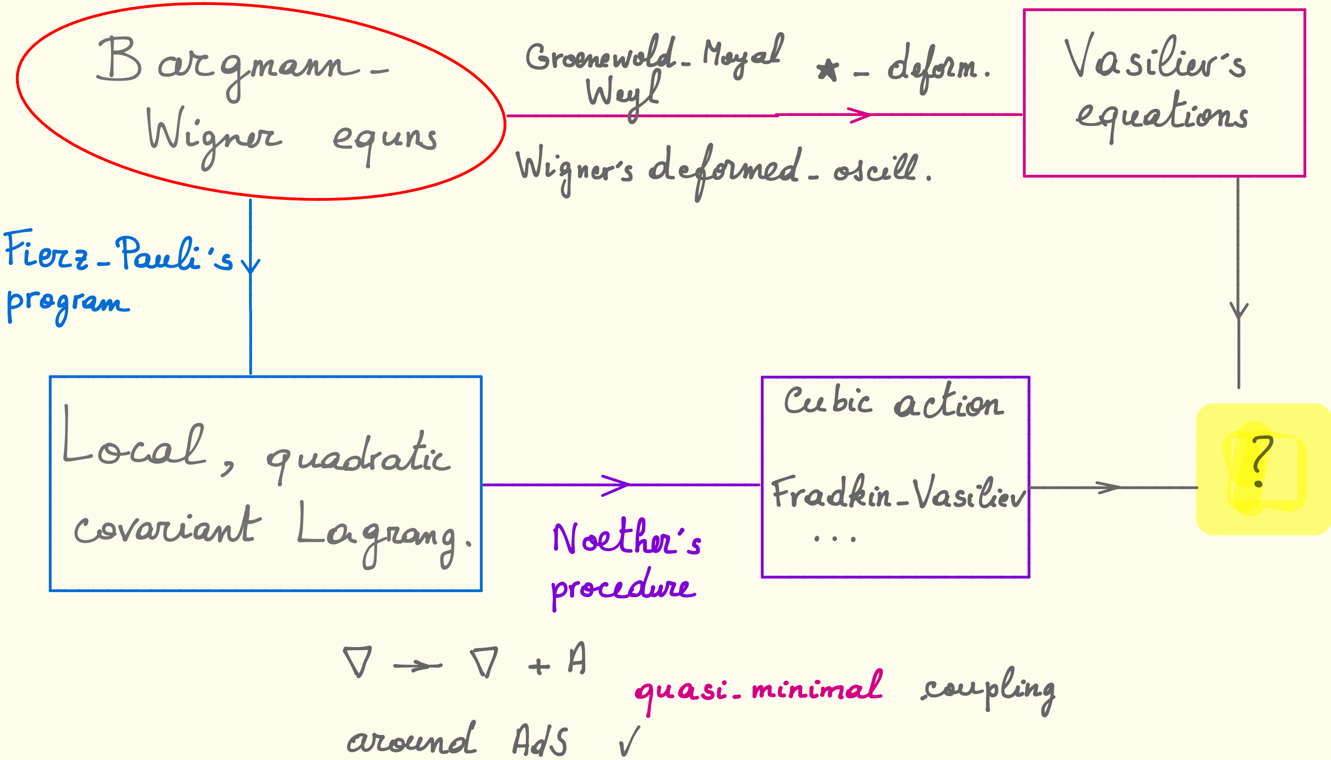
Fierz-Pauli program :

> Associate a quadratic, local and covariant Lagrangian to every UIR of maximally-symetric spacetime-isometry algebra.

- Initiated by F.P. in 1939 for massive, spin-2 particle in $\mathbb{R}^{1,3}$. Then, notably [Chang (67), Schwinger (70), Singh-Hagen (74)]

- In 1978, Fronsdal and Fang gave Lagrangian for $m=0$ helicity-s field around $\mathbb{R}^{1,3}$ and $(A)dS_4$ by taking the $m \to 0$ limit of Singh-Hagen's $\mathcal{L}$ and introducing (bosons) $\varphi_{\mu_1 \dots \mu_s}(x)$ and gauge parameter $\epsilon_{\mu_1 \dots \mu_{s-1}}(x)$ subject to trace constraints:

$$\bar{g}^{\mu\nu} \bar{g}^{\rho\sigma} \varphi_{\mu\nu\rho\sigma \dots} \equiv 0 \equiv \bar{g}^{\mu\nu} \epsilon_{\mu\nu\rho \dots}$$


Bargmann-Wigner equns
Groenewold-Moyal-Weyl ★-deform.
Wigner's deformed-oscill.
Vasiliev's equations
Fierz-Pauli's program
Local, quadratic covariant Lagrang.
Noether's procedure
Cubic action
Fradkin-Vasiliev
...
?
∇ → ∇ + A
quasi-minimal coupling
around AdS ✓

• WAVE EQUATIONS ⇝ BARGMANN-WIGNER IN AdS [Fronsdal in 70's].

Conventions and notation for so(2,d)

↳ Lie algebra so(2,d) with generators $M_{AB} = M_{AB}^\dagger$

$$[M_{AB}, M_{CD}] = i\left(\eta_{BC} M_{AD} - \eta_{AC} M_{BD} - \eta_{BD} M_{AC} + \eta_{AD} M_{BC}\right)$$

$A, B, \dots = 0', 0, 1, \dots, d$ $\qquad a, b, \dots = 0, 1, \dots, d$

$$\eta_{AB} = \mathrm{diag}(\underset{0'}{-}, \underset{0}{-}, \underset{1}{+}, \underset{\dots}{\dots}, \underset{d}{+}) \qquad \eta_{ab} = \mathrm{diag}(\underset{0}{-}, \underset{1}{+}, \underset{\dots}{\dots}, \underset{d}{+})$$

$$\boxed{P_a := M_{0'a}}\,, \qquad \text{in particular} \quad E = P_0 = M_{0'0}$$

$$\Rightarrow [P_a, P_b] = [M_{0'a}, M_{0'b}] = i\,\sigma\, M_{ab}\,, \qquad \sigma = \begin{cases} +1 & AdS_{d+1} \\ -1 & dS_{d+1} \end{cases}$$

$$\boxed{L^{\pm}_i := M_{i0} \mp i\, M_{i0'}} \qquad i = 1, \dots, d \quad so(d) \text{ index}$$

- $so(2,d) \supset \underbrace{so(2)}_{E} \oplus \underbrace{so(d)}_{M_{ij}}$ maximal compact

$$\boxed{\begin{aligned} &[M_{ij}, M_{kl}] = 4i\,(\delta_{jk} M_{il} + 3 \text{ terms}) \\ &[E, L^{\pm}_i] = \pm L^{\pm}_i\,, \qquad [M_{ij}, L^{\pm k}] = 2i\, \delta^k_{[j} L^{\pm}_{i]} \\ &[L^-_i, L^+_j] = 2\,\delta_{ij}\, E + 2i\, M_{ij} \end{aligned}}$$

• $C_2[so(2,d)] := \frac{1}{2} M^{AB} M_{AB}$, $P^2 := P_a P_b \eta^{ab} = M_{0'a} M_{0'b} \eta^{ab}$

$$\Leftrightarrow P^2 = -\frac{1}{2} M^{AB} M_{AB} + \frac{1}{2} M^{ab} M_{ab} = - C_2[so(2,d)] + C_2[so(1,d)] .$$

• On the other hand, using the decomposition $M_{AB} \sim \{ M_{ij} , E , L^{\pm}_i \}$,

one finds $C_2[so(2,d)] = E(E-d) - L^+_i L^{-i} + C_2[so(d)]$, so that

$$\boxed{C_2[so(2,d) | \mathcal{D}(e_0,\vec{s})] = -e_0(-e_0+d) + s_1(s_1+d-2) + \dots + s_r(s_r+d-2r)}$$

upon evaluation on Lowest-weigth state $|e_0, \vec{s}\rangle_{i\dots j\dots}$ obeying

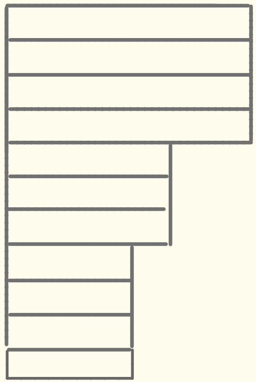
$$(E-e_0)|e_0,\vec{s}\rangle_{i\dots j\dots} = 0 = L^-_k |e_0,\vec{s}\rangle_{i\dots j\dots}$$

Where the vacuum $|e_0, \vec{s}\rangle_{i\dots j\dots}$ of the generalized Verma module

$$\mathcal{g}(e_0, \vec{s}) := \left\{ L^+_{i_1} \dots L^+_{i_n} \, |e_0, \vec{s}\rangle_{i\dots j\dots} \right\}_{n = 0, 1, 2, \dots}$$

with highest $so(2,d)$- weight $\Lambda = (-e_0, \vec{s})$

transforms in the $so(d)$- irrep $\mathcal{R}^{so(d)}_{\vec{s}}$ associated with Young diagram

$\vec{s} =$  of $so(d)$.

• Taking the wave-equation representation $\rho(P_a) = -i\,\ell\,\nabla_a$

with $\ell^2 = \lambda^{-2} \propto$ square radius of AdS_{d+1},

$$\left[\; P_a = \left(-\frac{E}{c}, \vec{p}\right) = -i\hbar\,\overbrace{\left(\tfrac{1}{c}\partial_t , \vec{\nabla}\right)}^{\nabla_a}, \quad \Box = \eta^{ab}\nabla_a\nabla_b \rightsquigarrow \left(\Box - \frac{c^2}{\hbar^2} m^2\right)\phi = 0 \;\right]$$

Measure masses in unit of λ : $m \to \frac{\hbar}{c}\lambda\,\bar{m}$ where $\bar{m}$ dimensionless.

$L = M^{-1}\cdot(\hbar/c)$

$$\rho(P^2) = -\ell^2\,\nabla_a\nabla_b\,\eta^{ab}\,, \quad \text{hence} \quad (\Box - \lambda^2\,\bar{m}_h^2)\,\phi_h = 0$$

entails $-P^2 = \ell^2\,\bar{m}_h^2$, so that

$$\boxed{\ell^2\,\bar{m}_h^2 = C_2\left[so(2,d)\,\middle|\,\mathcal{D}(e_0,\vec{s})\right] - C_2\left[so(1,d)\,\middle|\,\underset{\sim}{h}\right]}$$

where $\underset{\sim}{h}$ denotes the $so(1,d)$ Lorentz type of the tensor ϕ_h in the Lorentz-covariant realisation of the abstract UIR $\mathcal{D}(e_0, \vec{s})$.

Wigner	$\longrightarrow$	Bargmann - Wigner
$\mathcal{D}(e_0, \vec{s})$	$\longrightarrow$	$\{ \phi_{h_\alpha} \quad \} \quad (\Box - \lambda^2 \bar{m}^2_{h_\alpha}) \, \phi_{h_\alpha} = 0 \}$.

<u>Example</u> : Spin-s UIR of $so(2,3) \rightsquigarrow AdS_4$ [Fronsdal, 70's]

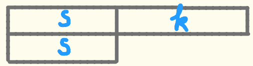
$\vec{s}$ of $so(3)$: [s] $\rightsquigarrow$ $|e_0, \vec{s}\rangle_{i_1 \dots i_s}$ vacuum.

$C_2[so(3) \mid \vec{s}\,] = s(s+d-2) = s(s+1)$

$e_0 = s+1$ leading to $L^{+i} |s+1, s\rangle_{i\dots}$ being nul. $(s>0)$

- $C_2\left[so(2,3) \mid \mathcal{D}(s+1,s)\right] = -e_0(-e_0+3) + s(s+1) = 2(s+1)(s-1)$

↳ The set of Lorentz tensors carrying this UIR $\mathcal{D}(s+1,s)$ is

$$\{ \phi_{h_\alpha} \} = \{ C^{a(s+k),\, b(s)} \sim$$  $$\}$$

with

$$C^{a(s+k),\, b(s)} := C^{a_1 a_2 \cdots a_{s+k},\, b_1 b_2 \cdots b_s} \equiv C^{(a_1 a_2 \cdots a_{s+k}),\, (b_1 b_2 \cdots b_s)}$$

s.t.

- $C^{(a_1 a_2 \cdots a_{s+k},\, b_1) b_2 \cdots b_s} \equiv 0$.
- $C^{a(s+k),\, b(s)}$ $\quad \eta^{ab}$ – traceless

• The Lorentz tensors $\{ C^{a(s+k), b(s)} \}_{k=0,1,\dots}$ obey the linear, relativistic wave equations (⇝ BW program)

$$\left(\Box - \lambda^2 M^2_{(s,k)} \right) C^{a(s+k), b(s)}(x) = 0$$

where

$$M^2_{(s,k)} = -\sigma \left[4\epsilon_0 + 2s + k(k + 2s + 2\epsilon_0 + 1) \right]$$

$(\sigma = +1 : AdS)$

$\epsilon_0 := \frac{(d-2)}{2}$

• In particular, for $d=3$, $k=0$, the primary Weyl tensors obey

$$\left(\Box + 2\lambda^2 (s+1) \right) C_{a(s), b(s)} = 0 \, .$$

Check: $M^2 = C_2[SO(2,3)] - C_2[SO(1,3) \,|\, (s,s)] = 2(s+1)(s-1) - \left(s(s+4-2) + s(s+4-4) \right)$

$= -2(s+1) \, .$

Note: In the scalar case $s = 0$, the primary Weyl tensor

$\phi(x)$ obeys $(\Box + 2\lambda^2)\,\phi(x) = 0$ in AdS_4, where

$$\bar{M}^2_{(0,0)} = -2 = C_2[so(2,3)\,|\,\mathcal{D}(e_0,0)] = -e_0(-e_0+3)$$

leaving 2 possibilities compatible with unitarity:

↳ $e_0 = 1$ (Dirichlet) or 2 (Neuman) BC's.

. So, in the zoology of "massless" UIR's

⇝ (bosonic) fields propagating in AdS_4, we have

$\mathcal{D}(s+1,s)$ $s = 0, 1, 2, \dots$ and $\mathcal{D}(2,0)$: Fronsdal on-shell fields

Dirac singletons and Flato-Fronsdal

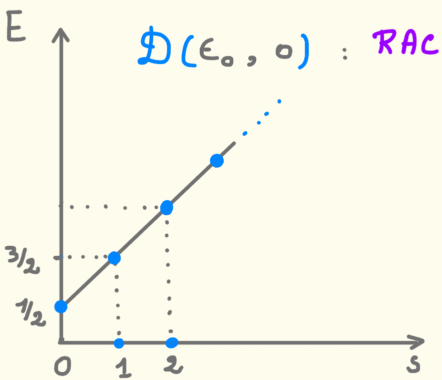
$[\epsilon_0 = \frac{d-2}{2}]$.

Two remarkable $so(2,d)$-UIRs : $\mathcal{D}(\epsilon_0, 0)$ & $\mathcal{D}(\epsilon_0 + \frac{1}{2}, \frac{1}{2})$

Not propagating inside AdS_{d+1} but at ∂AdS_{d+1} .

↳ single line in compact weight space

$\mathcal{D}(\epsilon_0, 0)$: RAC & $\mathcal{D}(\epsilon_0 + \frac{1}{2}, \frac{1}{2})$: Di



Flato - Fronsdal theorem ($d=3$)

- $\mathcal{D}(\tfrac{1}{2},0) \otimes \mathcal{D}(\tfrac{1}{2},0) \simeq \bigoplus_{s=0}^{\infty} \mathcal{D}(s+1,s)$

- $\mathcal{D}(1,\tfrac{1}{2}) \otimes \mathcal{D}(1,\tfrac{1}{2}) \simeq \mathcal{D}(2,0) \oplus \bigoplus_{s=1}^{\infty} \mathcal{D}(s+1,s)$

Consequence : Compositeness of massless particles in AdS_4

RAC : $\Box_3 \, \phi(x) = 0$ (*) with $\dim(\phi) = \frac{1}{2}$. $\left(\int d^3x \, \partial\phi . \partial\phi \right)$

conformal scalar

- Symmetries of (*) : $\dfrac{\mathcal{U}(so(2,d))}{\mathrm{Annih}(\mathrm{RAC})} \simeq \mathcal{A}$ associative algebra

$\downarrow$ $[\cdot , \cdot]$

$hs(d+1)$

Unfolded version and extension of BW → First-order differential equations

- $\Omega := (h^a, \varpi^{ab})$ $so(2,d)$-valued 1-form $\left(\Omega = dx^\mu \Omega_\mu{}^{AB} \tfrac{1}{2} M_{AB}\right)$
- AdS_{d+1} : $R_0 := d\Omega + \Omega\Omega = 0$ & invertibility of $h^a{}_\mu$
- $\nabla = d + \varpi$ Lorentz-cov. derivative $\left(\nabla^2 V^a = -\lambda^2 h^a{}_\wedge h_b V^b\right)$

$$(S)\quad \begin{cases} \nabla \overset{(0)}{C}_{a(s),b(s)} = h^c \overset{(1)}{C}_{c\{a(s),b(s)\}} \\ \nabla \overset{(1)}{C}_{a(s+1),b(s)} = h^c \overset{(2)}{C}_{c\{a(s+1),b(s)\}} + \lambda_1\, h_{\{a} \overset{(0)}{C}_{a(s),b(s)\}} \\ \nabla \overset{(2)}{C}_{a(s+2),b(s)} = h^c \overset{(3)}{C}_{c\{a(s+2),b(s)\}} + \lambda_2\, h_{\{a} \overset{(1)}{C}_{a(s+1),b(s)\}} \\ \quad\vdots \qquad\qquad\qquad \vdots \end{cases}$$

λ_i's $\propto \lambda^2$.

- Due to the symmetry properties of the zero-forms $\overset{(*)}{C}$'s, the system (S)

  (i) reproduces the wave equations

$$\left(\Box - \lambda^2 M^2_{(s,k)}\right) C^{a(s+k),\, b(s)}(x) = 0 \; ;$$

  (ii) Can be integrated so as to give a <u>1-form module</u>

$$\nabla e^{a(s-1)} + h_b\, \omega^{a(s-1),b} = 0$$

$$\nabla \omega^{a(s-1),b} + h_b\, X^{a(s-1),bb} + \bar{\lambda}_1\, h^{\{b}\, e^{a(s-1)\}} = 0$$

$$\vdots$$

$$\nabla X^{a(s-1),b(s-1)} + \bar{\lambda}_{s-1}\, h^{\{b}\, X^{a(s-1),b(s-2)\}} = h_a \wedge h_b\, C^{a(s-1),b(s-1)}$$

· Gauge symmetries : differential and algebraic

↳ Minimal set of fields & gauge parameters ⇝ Fronsdal.

$$-2\mathcal{L}(\varphi,\nabla\varphi) = \nabla_\nu \varphi_{\mu(s)} \nabla^\nu \varphi^{\mu(s)} - \frac{s(s-1)}{2} \nabla_\nu \varphi'_{\mu(s-2)} \nabla^\nu \varphi'^{\mu(s-2)}$$

$$+ s(s-1) \nabla_\nu \varphi'_{\mu(s-2)} \nabla_\rho \varphi^{\rho\nu\mu(s-2)} - s \nabla_\nu \varphi^\nu{}_{\mu(s-1)} \nabla_\rho \varphi^{\rho\mu(s-1)}$$

$$- \frac{s(s-1)(s-2)}{2} \nabla_\nu \varphi'^\nu{}_{\mu(s-3)} \nabla_\lambda \varphi'^{\lambda\mu(s-3)}$$

$$+ m_e^2 \varphi^{\mu(s)} \varphi_{\mu(s)} + m_e'^2 \varphi'^{\mu(s-2)} \varphi'_{\mu(s-2)} .$$

$$m_e = \lambda^2 \left(s^2 + (D-6)s - 2D + 6 \right)$$

$$\cdot\ \bar{R}_{\mu\nu\rho\sigma} = -\lambda^2 \left(\bar{g}_{\mu\rho} \bar{g}_{\nu\sigma} - \bar{g}_{\nu\rho} \bar{g}_{\mu\sigma} \right) , \qquad \lambda^2 = \frac{-2\Lambda}{(D-1)(D-2)}$$

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 0$$

- **Maxwell's theory**: $A_\mu(x) := \varphi_\mu(x)$, $\delta_\epsilon A_\mu(x) = \partial_\mu \epsilon(x)$

  - $S[A_\mu] = -\frac{1}{4}\int d^4x\, F^{\mu\nu} F_{\mu\nu}$, $F_{\mu\nu} := 2\,\partial_{[\mu} A_{\nu]}$

  - $\delta_\epsilon S[A_\mu] = 0 \iff \partial^\mu F_{\mu\nu} \equiv 0$ (Noether id.)

- **Fierz-Pauli in metric-like notation**:

  $h_{\mu_1\mu_2}(x) =: \varphi_{\mu(2)}(x)$, $\delta_\epsilon \varphi_{\mu(2)} = 2\,\partial_\mu \epsilon_\mu$ $\left(\delta_\epsilon h_{\mu\nu} = 2\,\partial_{(\mu}\epsilon_{\nu)}\right)$

  - $S_0[\varphi_{\mu(2)}] = -\frac{1}{2}\int d^4x \left[\partial^\nu \varphi^{\mu(2)}\, \partial_\nu \varphi_{\mu(2)} + \dots\right]$

  - $\delta_\epsilon S_0[\varphi_{\mu(2)}] = 0 \iff \partial^\mu \overset{(1)}{G}_{\mu\nu}(x) \equiv 0$, $\overset{(1)}{G}_{\mu\nu} := \overset{(1)}{R}_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}\overset{(1)}{R}$.

• Fronsdal's formulation

• $\varphi_{\mu_1 \ldots \mu_s} = \varphi_{(\mu_1 \ldots \mu_s)} = \varphi_{\mu(s)}$,

↳ Gauge transformation : $\delta_\epsilon \varphi_{\mu_1 \ldots \mu_s} = s\, \bar{\nabla}_{(\mu_1} \epsilon_{\mu_2 \ldots \mu_s)}$

Constr. : $\bar{g}^{\mu\nu} \bar{g}^{\rho\sigma} \varphi_{\mu\nu\rho\sigma \ldots} \equiv 0 \quad (s \geqslant 4)$, $\bar{g}^{\mu\nu} \epsilon_{\mu\nu \ldots} \equiv 0 \quad (s \geqslant 3)$

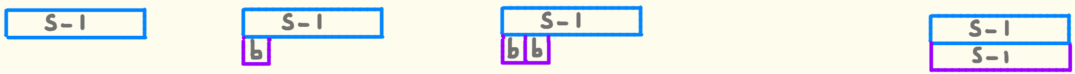
• $S^{Fr}[\varphi] = \int \mathcal{L}(\varphi, \bar{\nabla}\varphi)$, $\dfrac{\delta S^{Fr}}{\delta \varphi_{\mu(s)}} =: G^{\mu(s)} \approx 0$

$\nabla^{\mu_1} G_{\mu_1 \mu_2 \ldots \mu_s} \propto \bar{g}_{(\mu_2 \mu_3} \nabla^{\alpha} G'_{\mu_4 \ldots \mu_s)\alpha}$ Noether identity

Back to frame-like, Lopatin-Vasiliev's formulation :

↳ Familly of connection 1-forms for spin-s :

$$\left\{ e^{a(s-1)} \,,\; \omega^{a(s-1),b} \,,\; X^{a(s-1),b(2)} \,,\; \dots \,,\; X^{a(s-1),b(s-1)} \right\} .$$



All are Lorentz-valued $so(1,d)$-tensors.

↳ Packed up into a single $so(2,d)$-valued 1-form

$$W^{(s-1,\,s-1)} = dx^{\mu}\, W_{\mu}{}^{A(s-1),\,B(s-1)}$$

1. INTRO TO UNFOLDING ; VASILIEV'S EQUNS

- Weyl's Gauge Principle : HS theories contain gravity. ∞-dim gauge algebra ;
- Vasiliev's unfolding : A geometric, Cartan-like approach to field theory ;
- AdS/CFT dualities for Vasiliev's theory : AdS_4/CFT_3 [Sezgin-Sundell, Klebanov-Polyakov] and AdS_3/CFT_2 [Gaberdiel-Gopakumar]. Relations with statistical physics, non-commutative field theory, strings.

THE GAUGE PRINCIPLE [H. WEYL, 1929]

M. A. Vasiliev : *fully nonlinear field equations* for higher-spin gauge fields in 4D [Vasiliev, 1990 – 1992] and in D space-time dimensions [hep-th/0304049]. Salient features :

- Manifest diffeomorphism invariance, no explicit d^2s ;
- Cartan integrability $\Rightarrow$ *gauge invariance* under $\mathfrak{hs}_D$;
- Two ∞-dim $\mathfrak{so}(2, D-1)$ modules : *adjoint* and *twisted-adjoint* representations $\rightsquigarrow$ master 1-form and master zero-form. Uses unfolding in terms of FDA.

A free (graded commutative, associative) differential algebra $\mathfrak{R}$ is set $\{X^\alpha\}$ of *a priori* independent variables, locally-defined differential forms obeying first-order equations of motion

$$\mathscr{R}^\alpha = \mathrm{d}X^\alpha + Q^\alpha(X) = 0 \;, \quad Q^\alpha(X) = \sum_n f^\alpha_{\beta_1 \dots \beta_n} X^{\beta_1} \cdots X^{\beta_n} \;.$$

Nilpotency of d and integrability condition $\mathrm{d}\mathscr{R}^\alpha = 0$ require

$$Q^\beta \, \frac{\partial Q^\alpha}{\partial X^\beta} \equiv 0 \quad .$$

For $X^\alpha_{[p_\alpha]}$ with $p_\alpha > 0$, gauge transformation preserving $\mathscr{R}^\alpha \approx 0$:

$$\delta_\epsilon X^\alpha = \mathrm{d}\epsilon^\alpha - \epsilon^\beta \frac{\partial^L}{\partial X^\beta} Q^\alpha \quad .$$

The principle of unfolding [Vasiliev, 1988 –]

- The concepts of spacetime, dynamics and observables are *derived* from infinite-dimensional FDA's.
- Unfolded dynamics is an inclusion of local d.o.f. into field theories described *on-shell* by flatness conditions on generalized curvatures.
- The local, perturbative d.o.f. are contained in the zero-forms ;
- Lorentz-covariant derivative, minimal coupling.

- A master 1-form $A = \sum_{s=2,4,...} A_{(s)}$ where

$$A_{(s)} = -i \sum_{t=0}^{s-1} dx^{\mu}\, A_{\mu}^{a(s-1),b(t)}(x) M^{a_1 b_1} \dots M^{a_t b_t} P^{a_{t+1}} \dots P^{a_s - 1} \; ;$$

- A master zero-form $\Phi = \sum_{s=0,2,4,...} \Phi_{(s)}$ where

$$\Phi_{(s)} = \sum_{k=0}^{\infty} \tfrac{1}{k!}\, \Phi^{a(s+k),b(s)}(x) M^{a_1 b_1} \dots M^{a_s b_s} P^{a_{s+1}} \dots P^{a_s + k} \; ;$$

Vasiliev's eqns : $\quad F + \sum_{n=1}^{\infty} J_{(n)}(A, A; \Phi, \dots, \Phi) = 0 \; ,$

$$D_x \Phi + \sum_{n=2}^{\infty} P_{(n)}(A; \Phi, \dots, \Phi) = 0 \; ,$$

$$F := \mathrm{d}_x A + A \star A \; , \; D_x \Phi := \mathrm{d}_x \Phi + [A, \Phi]_{\pi} \; , \; \pi(P, M) = (-P, M) \; ,$$

Master fields of the *minimal bosonic model* :

$$\star\ \underline{\text{adjoint}}\quad A \;=\; A_x + A_z\ ,$$

$$A_x \;=\; \mathrm{d}x^M\, A_M(x, Z; Y)\ , \qquad\qquad A_z \;=\; \mathrm{d}Z^{\underline{\alpha}}\, A_{\underline{\alpha}}(x, Z; Y)\ ,$$

and a

$$\star\ \underline{\text{twisted-adjoint zero-form}}\ \Phi \;=\; \Phi(x, Z; Y)\ ,$$

where the x^M's are commuting coordinates, while

$(Y^{\underline{\alpha}}, Z^{\underline{\alpha}}) = (y^{\alpha}, \bar{y}^{\dot{\alpha}}; z^{\alpha}, -\bar{z}^{\dot{\alpha}})$ are non-commutative.

Minimal bosonic higher-spin gravity :

$$F + \Phi \star J \;=\; 0\;, \qquad D\,\Phi \;=\; 0\;, \qquad \mathrm{d}J = 0\;,$$

$$F \;:=\; \mathrm{d}A + A \star A\;, \qquad D\Phi \;:=\; \mathrm{d}\Phi + [A,\Phi]_\pi\;,$$

$$\tau(A,\Phi) \;=\; (-A,\pi(\Phi))\;, \qquad (A,\Phi)^\dagger \;=\; (-A,\pi(\Phi))\;,$$

$$\hookrightarrow [A,J]_\pi = 0 = [\Phi,J]_\pi\;.$$

[The integrability of $F + \Phi \star J = 0$ implies that $D\Phi \star J = 0$, that is, $D\Phi = 0\,$, where the twisted-adjoint covariant derivative $D\Phi = d\Phi + A \star \Phi - \Phi \star \pi(A)\,$.

This constraints is integrable since

$D^2\Phi \;=\; F \star \Phi - \Phi \star \pi(F) \;=\; -\Phi \star J \star \Phi + \Phi \star \pi(\Phi) \star J$ gives zero, using the constraint on F and $0 = [\Phi,J]_\pi$ with $\pi(J) = J\,$.]

$\hookrightarrow$ Integrability implies invariance under Cartan gauge transformations

$$\delta_\epsilon A = D\epsilon \ , \qquad \delta_\epsilon \Phi = -[\epsilon, \Phi]_\star^\pi \ ,$$

for zero-form gauge parameters $\epsilon(x, Z; Y)$ obeying the same kinematic constraints as the master one-form, *i.e.* $\tau(\epsilon) = -\epsilon$ and $(\epsilon)^\dagger = -\epsilon$.

$\hookrightarrow$ The closure of the gauge transformations reads

$$[\delta_{\epsilon_1}, \delta_{\epsilon_2}] = \delta_{\epsilon_{12}} \ , \qquad \epsilon_{12} = [\epsilon_1, \epsilon_2]_\star \ ,$$

defining the algebra $\mathfrak{hs}(4)$.

③ AdS / CFT & open problems

HS_4 / CFT_3 [Sezgin-Sundell, Klebanov-Polyakov]

BC on φ	type A	type B
$\Delta = 1$	UV fixed-pt Free singlet theory CFT_3	Gross-Neveu model critical
$\Delta = 2$	Critical $O(N)$ model	Free Fermions CFT_3

$R \ll l_s$, $N \to \infty$

$\updownarrow$

$\lambda \ll 1$, $N \to \infty$

$$\frac{G}{R^2} \sim \frac{1}{N}$$

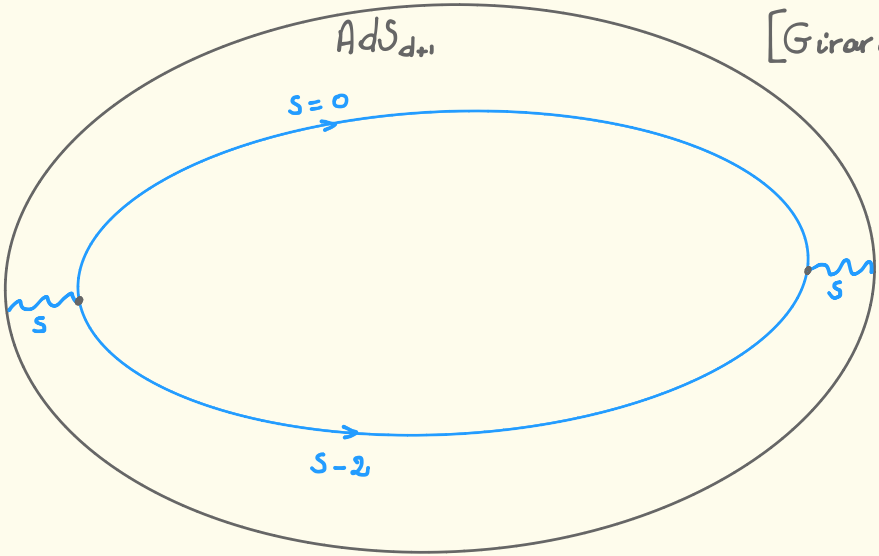
where G = Newton's .

HS_3 / CFT_2 Prokushkin-Vasiliev $\longleftrightarrow$ Minimal model CFT_2

[Gaberdiel-Gopakumar]

When bulk scalar field in $\Delta = 2$ BC,

[Girardello - Porrati - Zaffaroni]



- Boundary CFT: $\partial^\mu J^{(s)}_{\mu\nu(s-1)} = \frac{1}{\sqrt{N}} \partial_\nu J^{(0)} \cdot J^{(s-2)}_{\nu(s-2)}$

- Bulk: Gives mass to $s > 2$ fields, perturbatively. Spin fields $s \leq 2$ protected.

· Exact solution ·

- Amplitude computation (correspondance space or Witten diagram)
- Conformal HS
- Higher-dim HSGRA's, generalized HS algebras
- Mixed-symmetry and strings
- Multiparticle algebras
- Action principles & amplitude ; Effective action
- HS symmetry breaking

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