Université de Mons
Faculté Polytechnique – Service de Mécanique Rationnelle, Dynamique et Vibrations
31, Bld Dolez - B-7000 MONS (Belgique)
065/37 42 15 – georges.kouroussis@umons.ac.be

B. Olivier, G. Kouroussis, Multibody simulation of vehicles equipped with an automatic transmission, *Journal of Physics: Conference Series*, 744(1): 012210, 2016.

Multibody simulation of vehicles equipped with an automatic transmission

B Olivier and G Kouroussis

University of Mons - UMONS, Faculty of Engineering, 7000 Mons, Belgium

E-mail: `bryan.olivier@umons.ac.be`

Abstract. Nowadays automotive vehicles remain as one of the most used modes of transportation. Furthermore automatic transmissions are increasingly used to provide a better driving comfort and a potential optimization of the engine performances (by placing the gear shifts at specific engine and vehicle speeds). This paper presents an effective modeling of the vehicle using the multibody methodology (numerically computed under EasyDyn, an open source and in-house library dedicated to multibody simulations). However, the transmission part of the vehicle is described by the usual equations of motion computed using a systematic matrix approach: del Castillo's methodology for planetary gear trains. By coupling the analytic equations of the transmission and the equations computed by the multibody methodology, the performances of any vehicle can be obtained if the characteristics of each element in the vehicle are known. The multibody methodology offers the possibilities to develop the vehicle modeling from 1D-motion to 3D-motion by taking into account the rotations and implementing tire models. The modeling presented in this paper remains very efficient and provides an easy and quick vehicle simulation tool which could be used in order to calibrate the automatic transmission.

1. Introduction

Like any computer-aided design tool, the purpose of a numerical simulation is to provide information about a physical system without having to physically build it. Practically, simulation replaces the physical prototype with a numerical (or virtual) one on which the dynamic behavior can be investigated. As a virtual prototype is far cheaper than a physical one, the use of simulations is rapidly increasing. This paper investigates the dynamic simulations used in automotive engineering and, more specifically, the modeling and simulation of an automatic gearbox. This powertrain element has one of the highest impacts on vehicle driving. To simulate gear shifts, one option consists in building a series of models for each gear ratios and the simulation jumps from model to model according to some continuity conditions [1]. Another option is to build a complete mechanical system, including the actuators [2]. Several commercial software packages offer an efficient way to assess the dynamic performances of such models. On the other hand, industry needs only simple models which are sufficiently versatile and easy to use in order to to evaluate the efficiency of a powertrain during the design stage. This offers the possibility for industry to evaluate, in real-time, the gear transmissions [3]. In contrast to these approaches, Kouroussis et al. proposed an intermediate model which allows for the simulation of the complete acceleration process of a vehicle based on an analytical theory of gearbox kinematics [4]. Other models are more specific, focusing only on internal elements such as clutches, brakes, mechanical synchronizing elements (e.g. [5]) or on vibration problems of planetary gears [6]. Despite the abundance of prediction tools available, any new tool must be a simple model to meet industry preference, particularly as insufficient details are available at the design stage of a powertrain. The purpose of this paper is to develop a hybrid formulation of automatic transmission dynamics including a multibody model of the vehicle and an analytic representation of an automatic transmission. It is based upon work in [4]. A description of the approach is presented in this paper. An analysis is presented, based on the Aisin Warner 55-50 SN transmission, by comparing the numerical results with experimental data.

2. Transmission modeling

The vehicle transmission is the mechanical part through which the power generated by the engine flows. Generally speaking, the transmission is composed of: the torque converter, the hydraulic pump (which pressurizes the oil in the part of the transmission dedicated to the gear shifts), the gearbox, the differential and the shafts linking the different rotating parts. Figure 1 illustrates the power flow going through the vehicle and the overall working of the automatic gearbox. The gear shifts are automatically controlled by the transmission's electronic control unit (ECU), determining gear shifts based on the engine speed and throttle position. In order to determine the power transformation, all the components cited in Figure 1 have to be described.

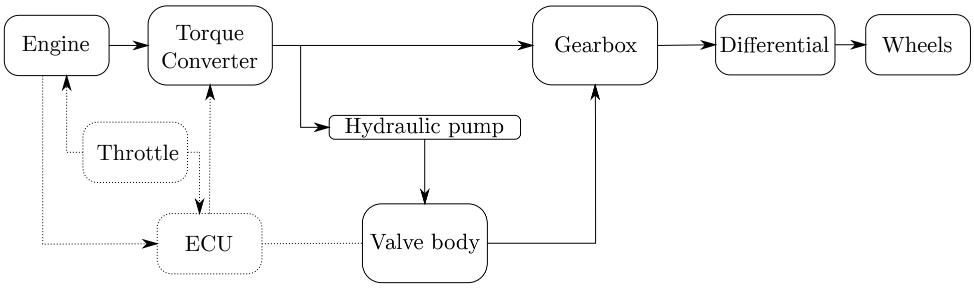


Figure 1. Powerflow in the vehicle transmission (——) and transmission control (······).

2.1. Engine and gearbox definition

The engine can be represented as the torque delivered to the engine shaft with respect to the speed of the engine output shaft. In this case, a simple law is used to describe the torque:

$$T_e = m_2\omega_e^2 + m_1\omega_e + m_0 \tag{1}$$

where ω_e is the engine output speed and m_i are coefficients depending on the engine characteristics.

The torque converter can be defined by two equations linking the input/output speeds and input/output torques where the f_i and t_i are the characteristics of the torque converter itself.

$$\frac{T_{T/C,out}}{T_{T/C,in}} = t_1\frac{\omega_{T/C,out}}{\omega_{T/C,in}} + t_0 \tag{2}$$

$$\text{C-factor} = \frac{T_{T/C,in}}{\omega_{T/C,in}^2} = f_2\left(\frac{\omega_{T/C,out}}{\omega_{T/C,in}}\right)^2 + f_1\left(\frac{\omega_{T/C,out}}{\omega_{T/C,in}}\right) + f_0 \tag{3}$$

Otherwise, the influence of the hydraulic pump is simplified into the determination of a resistive torque, which can be computed linearly with respect to the gearbox input speed (also the torque converter output speed) if the valve body pressure is assumed to be constant.

The simulated transmission is an Aisin 55-50SN Automatic Transmission (including the torque converter, the hydraulic pump, the gearbox and the differential) used in front wheel drive vehicles. The gearbox included in this type of transmission is composed of two planetary gear trains as shown in the global transmission construction in Figure 2. In Figure 3, the kinematic scheme of the gearbox and its actuators is presented.

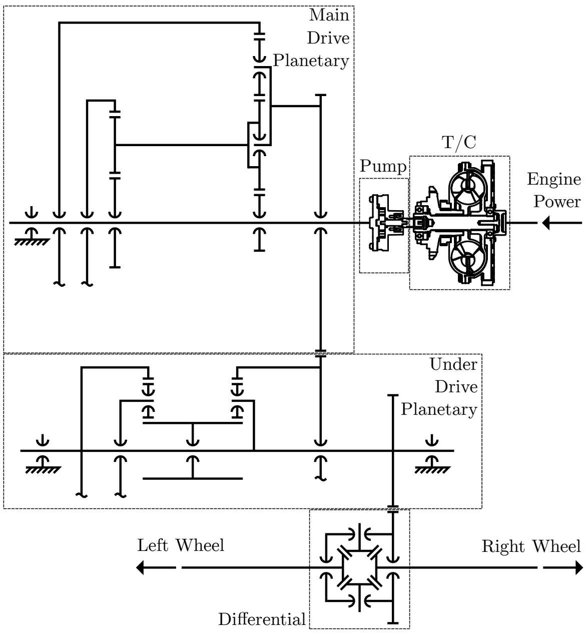


Figure 2. Full Aisin 55-50 Automatic Transmission.

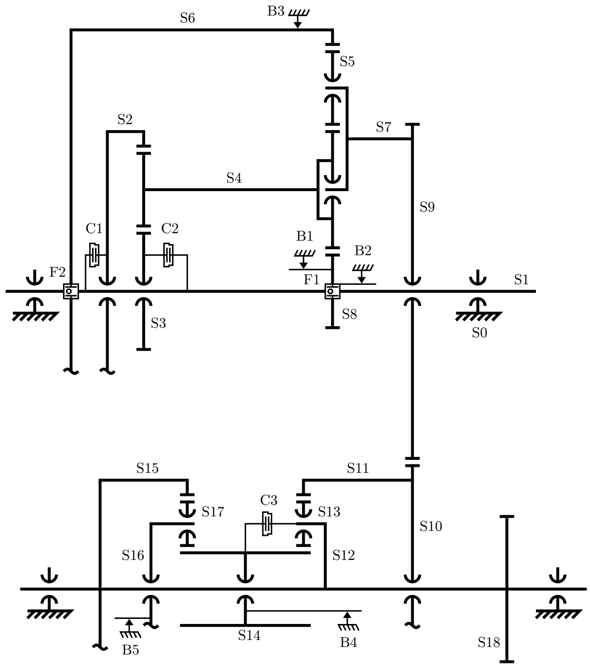


Figure 3. Gear trains and actuators of Aisin 55-50 Automatic Transmission

2.2. Equations of the gearbox

Since the entire gearbox is composed of rotating bodies (provided that the motion of the actuators is neglected), the virtual power principle corresponds to:

$$P_{v,tot} = \sum_{i=1}^{n_B} P_{v,i} = \sum_{i=1}^{n_B} (I_i \dot{\omega}_i - T_{ext/i})\omega_{v,i} = 0 \tag{4}$$

where the total virtual power $P_{v,tot}$ is the sum of the virtual power $P_{v,i}$ of each body. n_B is the number of bodies, I_i and $T_{ext/i}$ are the moment of inertia and the external torque applied on body i respectively while $\dot{\omega}_i$ and $\omega_{v,i}$ are the rotational acceleration and the virtual rotational speed of body i respectively.

Assuming that all the actuators are disengaged, the number of configuration parameters n_{cp} is determined by the difference between the number of bodies and the number of constraints. By choosing n_{cp} independent angular position among the n_B bodies, the speed ω_i of each body can be computed as a linear combination of the derivatives Ω_j of the configuration parameter:

$$\omega_i = \sum_{j=1}^{n_{cp}} \lambda_{i,j} \Omega_j \tag{5}$$

where

$$\lambda_{i,j} = \frac{\partial \omega_i}{\partial \Omega_j} \tag{6}$$

2.3. Speed ratios determination using del Castillo's methodology

Since the gearbox is composed of planetary gear trains, del Castillo [7] showed that the partial contributions $\lambda_{i,j}$ of the derivatives of the configuration parameters in the speed of each body can be determined using a systematic matrix form methodology:

$$\lambda_{i,j} = \underline{\mathbf{e}}_i^{\mathrm{T}} \begin{bmatrix} \mathbf{C} \\ \underline{\mathbf{e}}_l^{\mathrm{T}} \\ \underline{\mathbf{e}}_j^{\mathrm{T}} \end{bmatrix}^{-1} \underline{\mathbf{e}}_N \tag{7}$$

where $\underline{\mathbf{e}}_i$, $\underline{\mathbf{e}}_j$, $\underline{\mathbf{e}}_l$ and $\underline{\mathbf{e}}_N$ are vectors with n_B rows filled with zeros except for the output gear, the input gear, the locked gear of the planetary gear train and the last row respectively. Since this method remains valid only for planetary gear trains, del Castillo [7] demonstrated that there is a maximum of 2 degrees of freedom (2 configuration parameters) in one planetary gear train. Therefore, the n_{cp} configuration parameters are split between the main drive and the underdrive planetary gear trains as shown in Figure 2. Then, in Eq. (7), the input gear is associated with one of the n_{cp} configuration parameters corresponding to the planetary gear train considered, the output gear is the gear for which the partial speed contribution $\lambda_{i,j}$ must be computed and the locked gear corresponds to the other configuration parameter of the planetary gear train. The matrix $\mathbf{C}$ is built using the construction of each planetary gear in the gearbox and the number of teeth of each gear as explained for this gearbox in [4].

2.4. Actuator modeling

According to the vehicle motion, when a particular set of gears is engaged in the transmission, the rotational speed of the drive wheels and the engine are directly linked requiring only one degree of freedom for the transmission. Therefore, $n_{cp} - 1$ clutches and brakes are used to set the gearbox in a specific configuration (corresponding to a specific gear ratio) by reducing the number of configuration parameters from n_{cp} to 1.

Due to the complexity of the actuators' behavior, it was assumed that the clutches and brakes included in the gearbox can be modeled as torques applied on the gears involved by a particular actuator. For the clutch, by combining the elements i and j, the torques applied on each element are respectively [4]

$$T_i = -T_j = K\left(\omega_j - \omega_i + \tau(\dot{\omega}_j - \dot{\omega}_i)\right) \tag{8}$$

which ensures that the speed of both gears are the same through the $K\left(\omega_j - \omega_i\right)$ term. The $\tau\left(\dot{\omega}_j - \dot{\omega}_i\right)$ term is implemented in order to stabilize the integration (a kind of proportional derivative control). Obviously, the speed and acceleration of the body j are set to zero when the body i is linked to the gearbox chassis with a brake.

2.5. Transmission equations

Considering the gearbox equations, the engine modeling, the torque converter and the actuators, the equations of motion of the powertrain transmission are given by [4]:

$$\mathbf{M}\dot{\underline{\mathbf{\Omega}}} + \mathbf{D}\underline{\mathbf{\Omega}} = \{T_{\mathrm{inp}}\ 0\ \cdots\ 0\ T_{\mathrm{out}}\}^{\mathrm{T}} \tag{9}$$

where $\underline{\mathbf{\Omega}}$ is the vector regrouping the derivatives of the n_{cp} configuration parameters and the rotational speed of the engine shaft, $\mathbf{M}$ and $\mathbf{D}$ are the matrices associated to the inertial properties and the actuator model respectively. The matrix $\mathbf{M}$ also contains the terms dedicated to the integration stabilization. T_{inp} and T_{out} are the torque delivered by the engine and the resistive torque applied to transmission output shaft (before the differential system) respectively.

3. Vehicle definition using multibody method

Since a vehicle contains many mechanical parts, an analytic approach presents several limitations. Therefore, the multibody approach can be a very convenient methodology as the rigid bodies can be completely defined by their mass/inertial properties at the center of mass, while the homogeneous transformation matrix to describe the location of the center of mass (the origin of the local frame of each body) with respect to a main frame using the configuration parameters.

The vehicle (4 wheels and a chassis) can then be defined using the multibody approach with minimal coordinates, resulting in the automatic computation of the equations of motion. This work was performed using EasyDyn [8], an open source and in-house library based on the multibody methodology described in [9] and [10].

3.1. A simple representation of the differential using the multibody methodology

If the vehicle motion in the three spatial directions must be computed, it is necessary to dispatch the power, generated by the engine and transformed in an adapted form by the transmission, between the two drive wheels. The power dispatch is ensured by the differential system which contains an input shaft linked to the transmission output shaft, a carrier and planetary gears driven by the carrier. As illustrated in Figure 4, both planetary gears dispatch the power between the output gears for the two wheels. However, the rotation of these output gears is allowed in the carrier which provides the transmission with equivalent torque to both wheels but with a different rotational speed.

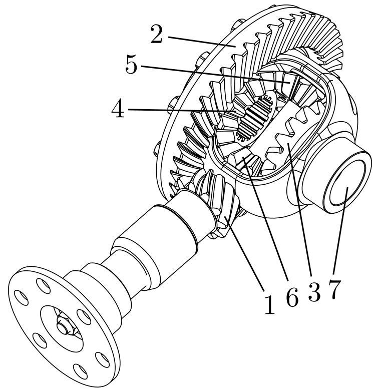


Figure 4. Representation of an open differential system. (1) Input gear, (2) Annular gear, (3) Left output gear, (4) Right output gear, (5) and (6) Planetary gears and (7) Carrier.

Following the physical workings of the differential system, a simplified multibody model can be established, shown in Figure 5. The model used considers that the rotational motion of the carrier is that of the transmission output shaft taking into account the gear ratio between the input shaft and the carrier. If one planetary gear is considered, its motion is defined by a new configuration parameter which is determined by the vehicle lateral motion during the integration process. The speed of the right wheel is then computed as the difference between the rotational speed of the carrier and the speed of the planetary gear. Conversely, the speed of the left wheel is then computed as the sum between the rotational speed of the carrier and the speed of the planetary gear

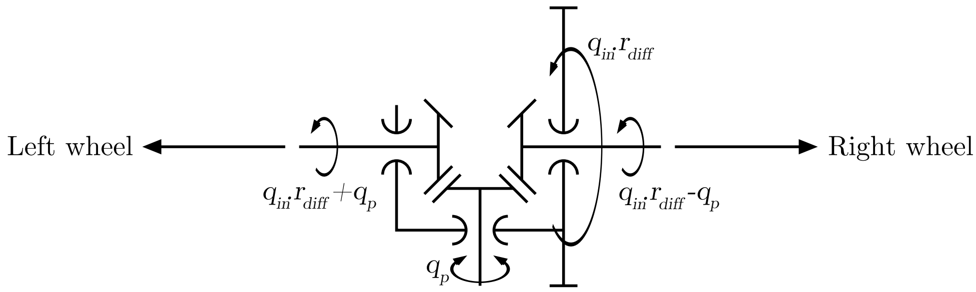


Figure 5. Kinematic scheme of a differential where q_{in} is the angular position of the transmission shaft, q_p is the angular position of the planetary gear and r_{diff} is the gear ratio of the differential.

3.2. Tire modeling

The traction force applied to the vehicle with respect to the torque applied on the wheel was established using a pneumatic tire model developed by Gim [11] which considers the tires to be an assembly of springs as illustrated in Figure 6. Following this model, a longitudinal force F_X, a lateral force F_Y, a vertical force F_Z and a self-aligning torque M_Z are applied to the tire depending on the specified tire parameters such as (illustrated in Figures 6 and 7):

- C_Z the radial stiffness which is the relation between the normal force applied on the tire F_Z and the radial deformation δ (also called penetration): $C_Z = \frac{F_Z}{\delta}$. This stiffness is often coupled with a damping ratio D_Z which represents the ability of the tire to absorb vibrations;
- C_L the longitudinal stiffness which is defined as $C_L = \left.\frac{dF_x}{ds}\right|_{s=0}$ where F_x is the longitudinal force and s the slip ratio;
- $C_l = \left.\frac{dF_y}{d\alpha}\right|_{\alpha=0}$ the lateral stiffness which is the relationship between the lateral force applied on the tire F_y and the slip angle α;
- $C_C = \left.\frac{dF_y}{d\gamma}\right|_{\gamma=0}$ the camber stiffness which is the relationship between the lateral force applied on the tire F_y and the camber angle γ;
- $s_L = \frac{v_{Px}}{|v_{ref}|}$ the longitudinal slip ratio;
- $s_l = \frac{v_{Py}}{|v_{ref}|}$ the lateral slip ratio;
- $s_C = |sin(\gamma)|$ the lateral slip ratio due to camber angle γ.

3.3. Vehicle/Transmission coupling

Considering the multibody modeling of the vehicle and the analytic modeling of the transmission, the equations of motion of the whole vehicle and transmission are given by:

$$\begin{array}{rcr} \mathbf{M}^*(\underline{\mathbf{q}}).\underline{\ddot{\mathbf{q}}} + \underline{\mathbf{h}}(\underline{\mathbf{q}}, \underline{\dot{\mathbf{q}}}) = \underline{\mathbf{g}}(\underline{\mathbf{q}}, \underline{\dot{\mathbf{q}}}, t) & \text{for} & \underline{\mathbf{q}} = \{q_1 \dots q_{n_{cp,v}}\} \\ \mathbf{M}\underline{\dot{\mathbf{\Omega}}} + \mathbf{D}\underline{\mathbf{\Omega}} = \{T_{\text{inp}}\ 0\ \cdots\ 0\ T_{\text{out}}\}^{\text{T}} & \text{for} & \underline{\mathbf{\Omega}} = \{\dot{q}_{n_{cp,v}+1} \dots \dot{q}_{n_{cp,v}+n_{cp,t}}\} \end{array} \quad (10)$$

where the first set of equations describes the automatically generated equations of motion of the vehicle part using EasyDyn with the $\mathbf{M}^*$ matrix regrouping the inertial terms, $\underline{\mathbf{h}}$ is the centrifugal and gyroscopic effects term and $\underline{\mathbf{g}}$ is the applied forces vector. These equations are defined for the $n_{cp,v}$ configuration parameters of the multibody vehicle part. Otherwise, the second set of equations (from the transmission model) is defined for the $n_{cp,t}$ degrees of freedom of the transmission (including the engine shaft).

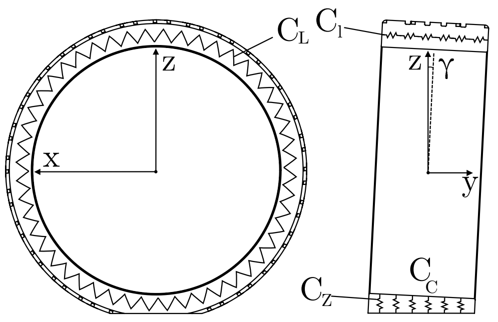


Figure 6. Tire equivalent spring modeling.

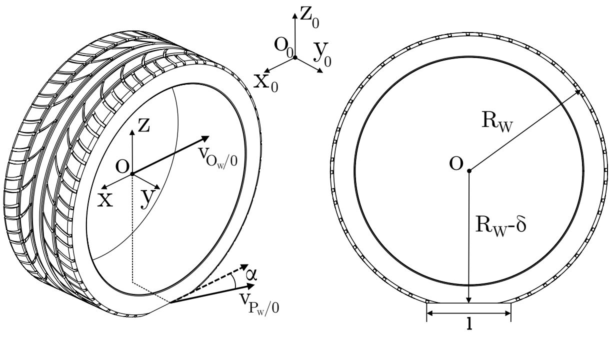


Figure 7. Tire motion definition using slip ratio and penetration.

In order to couple both systems, a penalty torque T_p (equivalent to the ones defined for the modeling of the actuators in the transmission — Eq. (8)) is added between the output shaft of the transmission and the input shaft of the differential. This pairing, which couples the output of the analytic modeling and the input of the multibody modeling, may be considered as a "virtual clutch" which is always engaged between two parts of the same transmission shaft. Figure 8 illustrates this methodology but for the sake of clarity, the represented vehicle is a rear wheel drive vehicle even though the simulated vehicle is a front wheel drive vehicle.

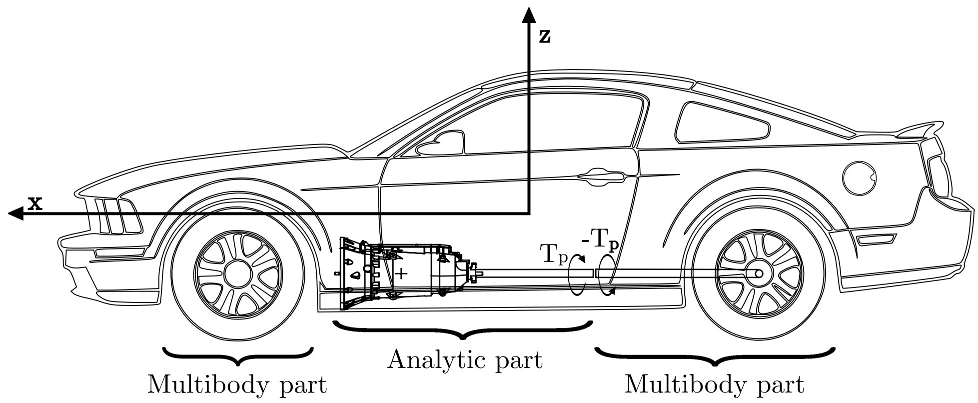


Figure 8. Analytic/Multibody vehicle composition.

4. Validation and additional numerical results

4.1. Acceleration performance and comparison with experimental results

The first step achieved using this hybrid modeling approach was the validation of the acceleration performances by comparing the numerical results with the experimental data for a forward motion at full charge (the accelerator pedal fully depressed). Figures 9 and 10 show the time history of the gearbox input speed (which is also the output speed of the torque converter) and the time history of the vehicle speed respectively. The comparison between the experimental and theoretical results demonstrates the efficiency of using the vehicle/transmission hybrid modeling approach.

By virtue of the multibody modeling of the vehicle part, the motion of the four wheels can be obtained. However, only the drive wheels were directly linked (due to the differential system) with the transmission. Indeed, the traction force applied on the vehicle chassis and the resistive

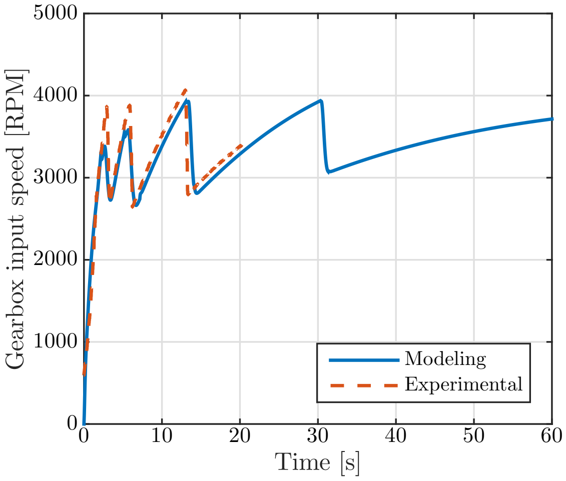


Figure 9. Time history of the gearbox input speed.

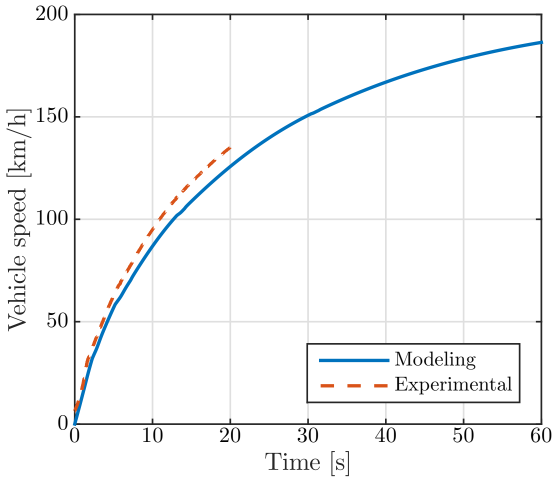


Figure 10. Time history of the vehicle speed.

torque applied to the transmission were deducted from the tire/road contact modeling. If the vehicle is supposed to move forward, the speed of the drive wheels is the same and the planetary gear of the differential does not rotate. Since 2 degrees of freedom (configuration parameters) were dedicated to the rotational motion of the driven wheels, their rotation speed are also computed using the tire modeling and the forward motion of the vehicle. Figure 11 shows the tangential speed of the drive wheels, the tangential speed of each driven wheel and the average forward speed of the vehicle for comparison. Due to the slip between the tire and the road, the speed of the vehicle is lower than the speed of the drive wheels but higher than the driven wheels (which have also the same rotational motion as long as the vehicle is moving forward).

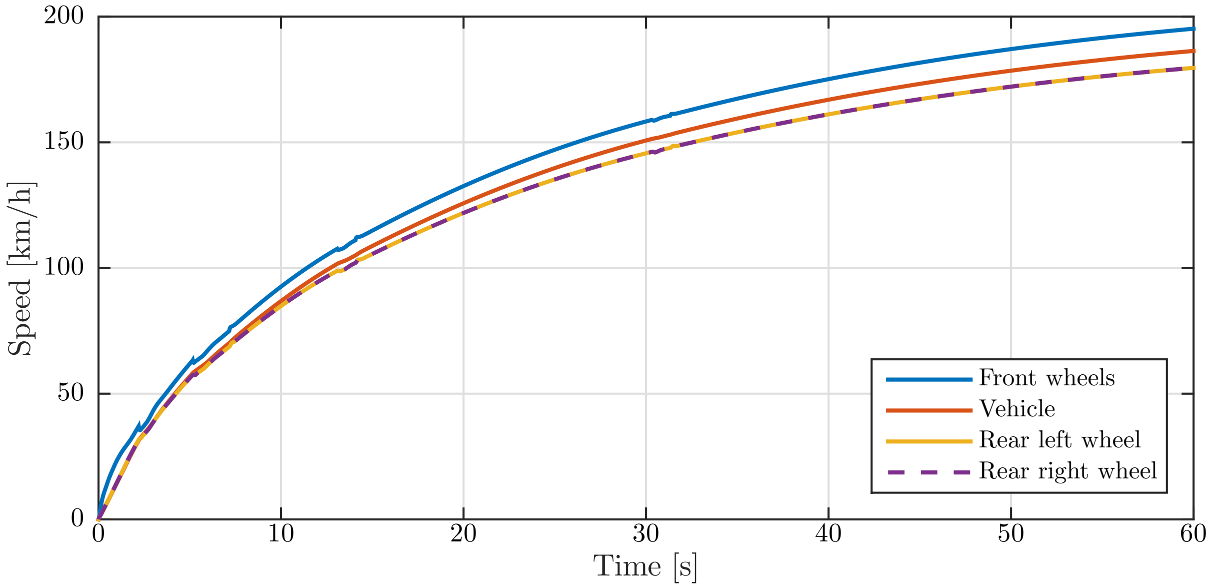
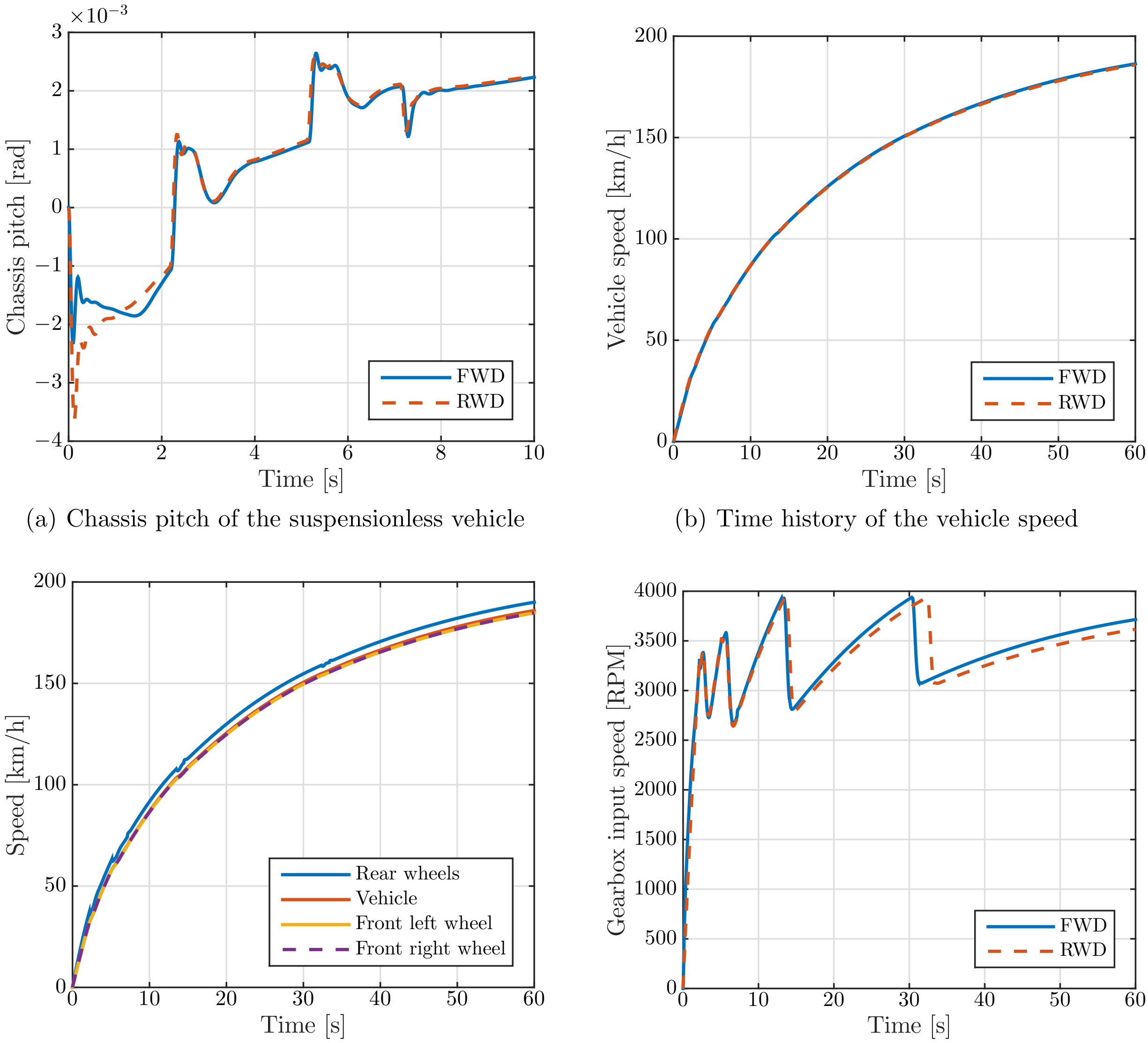


Figure 11. Time history of the speed of each wheel.

4.2. Front wheel drive and Rear wheel drive acceleration comparison

The use of multibody modeling offers many benefits which can easily be implemented. For example, it may be of interest to investigate the vehicle behavior if the power generated by the engine was dispatched to the rear wheels instead of the front wheels. This situation remains hypothetical because the Aisin 55-50 is a front wheel drive transmission only. The generation of the rear wheel drive vehicle is easily made in the definition of the homogeneous transformation matrices.



(a) Chassis pitch of the suspensionless vehicle

(b) Time history of the vehicle speed

(c) Time history of the wheel speeds

(d) Time history of the gearbox input speed

Figure 12. Rear wheel drive and front wheel drive vehicles comparison.

From the simulation, it can be said that:

- Figure 12(b) shows that the vehicle speed remains almost the same.
- In contrast with the front wheel drive vehicle, it can be seen in Figure 12(c) that the speed of the front wheels (driven wheels) is lower than the vehicle speed and the speed of the rear wheels (drive wheels) is higher than the vehicle due to slip. However, the speed of the

driven wheels appears to be closer to the vehicle's average speed than in the front wheel drive case.

- This speed modification could be explained by Figure 12(a) which shows that the vehicle leans forward (primarily because of the center of gravity location). This results in a higher penetration of the tire on the front axle leading to a higher rotation speed for the same vehicle speed.
- Figure 12(a) also shows that, during the starting phase the pitch decrease is much higher in the rear wheel drive case. Indeed, the torque applied on the vehicle (which decreases the pitch or increases the pitch in absolute value), due to high accelerations in the starting phase, involves higher vertical forces F_Z applied on the tires of the rear axle. Therefore, the longitudinal force F_X increases. Indeed F_X is directly proportional to F_Z (because of the tire modeling [11]).
- In contrast, after the starting phase, the accelerations become smaller due to the decrease in torque available at the engine. Therefore, the natural torque applied on the vehicle through the center of gravity location (due to the weight of the engine, the center of gravity is closer to the front wheel axle) becomes more important than the torque as a result of the accelerations. The pitch after the starting phase is then the same for both vehicle types.
- Due to this same pitch after the starting phase, the tire penetration is also the same for both models. However, the rear wheel radius related to the rear wheel drive vehicle is greater than the front wheel radius for the front wheel vehicle. For the same vehicle speed, the rotation speed of the drive wheels is smaller for the rear wheel drive vehicle than for the front wheel drive. Figure 12(d) shows that the rear wheel drive vehicle shifts the gears later for the two last speeds.

4.3. On-road vehicle behavior

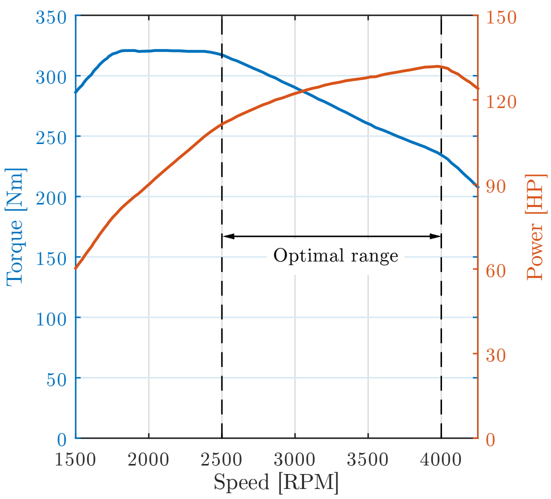


Figure 13. Torque and power curve at full charge for the actual engine used with the Aisin 55-50.

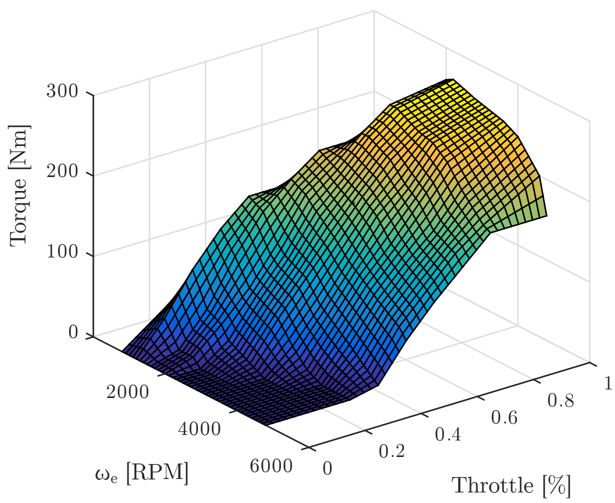
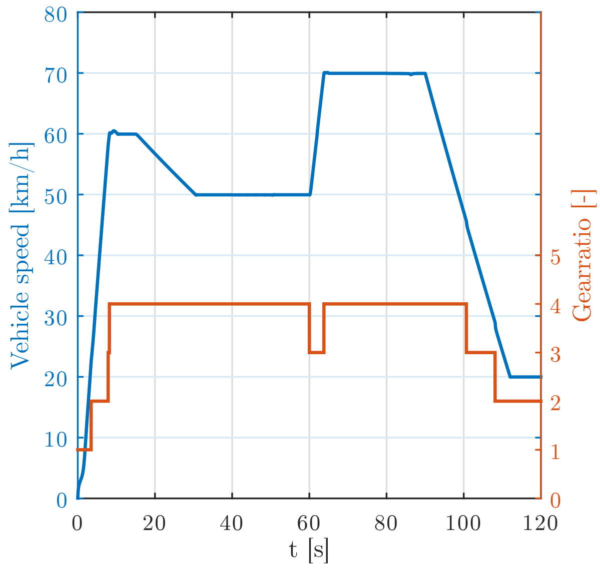
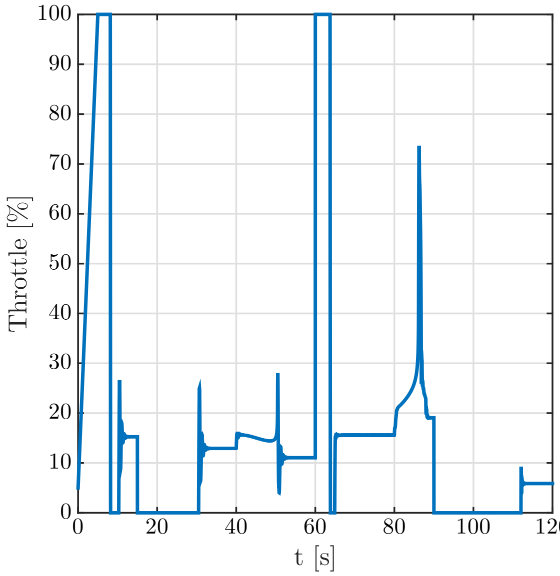


Figure 14. Simplified representation of another equivalent engine taking into account the throttle position.

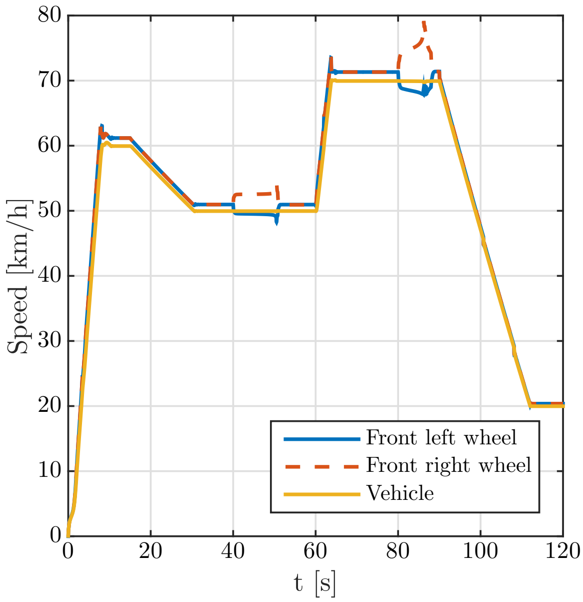
Through the multibody modeling of the vehicle, it is possible to implement an additional variable to the motion of the front wheels. The rotation angle of the front wheels around the vertical axis can be determined from the radius of the trajectory and the Ackermann formula



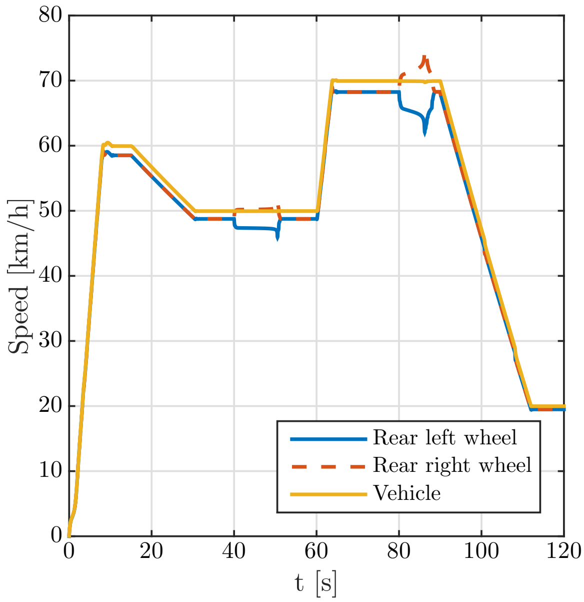
(a) Time history of the vehicle speed and gearratio



(b) Time history of the controlled throttle position



(c) Time history of the speed of the front wheels



(d) Time history of the speed of the rear wheels

Figure 15. Behavior of the vehicle on a track with two turns.

[12] (which expresses the dependency between the angle of the front wheels during a turning maneuver). Using a controller to compute the motion of the front wheels with respect to an imposed trajectory, it is possible to move the vehicle according to the lateral direction. Furthermore, it is possible to substitute the engine originally used with the Aisin transmission with another equivalent engine [13] that can take into account the throttle position. By adding this new engine representation and considering the shiftmaps of the transmission (describing when the controller must shift the gears with respect to the vehicle speed), the simulation of the

vehicle behavior can be achieved on a simplified road. Figure 13 shows the torque and power curve (at full charge) of the engine originally used with the transmission. Figure 14 illustrates a linear interpolation of another equivalent engine which takes into account the throttle position.

The imposed path involved three straight roads separated by two turns of 90 degrees. Different speeds involving gear shifts were imposed on the straight roads. Figure 15 shows the time history of the vehicle speed (and the gearbox behavior), the throttle position and the speed of each wheel. The presented results demonstrate the capability of the modeling to be used to calibrate the automatic gearbox.

5. Conclusions

By coupling an analytic representation of an automatic transmission with a multibody vehicle model, an efficient hybrid methodology dedicated to the vehicle and transmission simulation was developed.

After describing the methods used to model the vehicle and its transmission, validation of the model was undertaken through comparison with experimental results. The modularity of the entire modeling process was also demonstrated by comparing a front wheel drive vehicle with a rear wheel drive vehicle. Due to this modularity given by the multibody approach of the vehicle, it was possible to simulate the vehicle's behavior on a road path by taking into account the lateral motion.

The use of a systematic approach to describe the transmission (such as the del Castillo's methodology for planetary gear trains) allows the developed vehicle model to be used with other transmission types. Finally, the minimal coordinates multibody approach used to model the vehicle part (which does not include the transmission) provides the opportunity to complete the chassis with additional elements in order to simulate their behavior for a particular vehicle motion.

References

[1] Pan C H and Moskwa J J 1995 Dynamic modeling and simulation of the ford aod automobile transmission Tech. rep. SAE Technical Paper
[2] Otter M, Schlegel C and Elmqvist H 1997 *Proceedings of ESS* pp 19–23
[3] Bachinger M, Stolz M and Horn M 2015 *Mechatronics* **32** 67–78
[4] Kouroussis G, Dehombreux P and Verlinden O 2015 *Mechanism and Machine Theory* **83** 109–24
[5] Dempsey M and Roberts N 2012 *Proceedings 9th Modelica Conference* vol 981097 (Munich (Germany))
[6] Lin J and Parker R 2000 *Journal of Sound and Vibration* **233** 921–8
[7] DelCastillo J 2002 *Mechanism and Machine Theory* **37** 197–214
[8] Verlinden O, Ben Fekih L and Kouroussis G 2013 *Theoretical & Applied Mechanics Letters* **3** 013012
[9] Anantharaman M and Hiller M 1991 *International Journal for Numerical Methods in Engineering* **32** 1531–42
[10] Hiller M and Kecskeméthy A 1994 *Computer-Aided Analysis of Rigid and Flexible Mechanical Systems* (*NATO ASI Series* vol 268) ed Seabra Pereira M and Ambrósio J (Springer Netherlands) pp 61–100
[11] Gim G 1988 *Vehicle Dynamic Simulation with a Comprehensive Model for Pneumatic Tires* Ph.D. thesis University of Arizona
[12] Gillespie T 1992 *Fundamentals of Vehicle Dynamics* (Society of Automotive Engineers (SAE))
[13] Lechner G and Naunheimer H 1999 *Automotive Transmissions: Fundamentals, Selection, Design and Application* (Springer Berlin Heidelberg)