

Gravity localisation in a 6-dimensional brane world*

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Abstract

We study a 6-dimensional Einstein-Born-Infeld-Higgs model. In the limit of infinite Born-Infeld coupling, this model reduces to an Einstein-Abelian-Higgs model, in which gravity localising solutions were shown to exist. In this proceeding, we discuss further properties of the gravity localising solutions as well as of the solutions in the limit of vanishing cosmological constant.

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I. INTRODUCTION

Recently, brane world models with large (non-) compact extra dimensions have gained a lot of interest [1, 2, 3, 4, 5]. These models assume that we live on a 3-brane embedded in a higher dimensional manifold. The standard model fields and particles are confined to the brane, while gravity, which is a property of space-time itself, lives in the full dimensions. The idea that matter is confined to a lower dimensional manifold is not a new idea. The localisation of fermions on a domain wall has been discussed in [6]. Recently, it was newly motivated by results from string theory. In type I string theory so-called Dp-branes exist on which open strings, which represent matter fields, end. Gravitational fields, which are represented by closed strings, live in the full dimensions. However, as is well known, Newton's law in 4 dimensions is well tested down to 0.2 mm. Thus appropriate brane world models should localise gravity “well enough” to the 3-brane.

In [7], the localisation of gravity in more than one extra dimension has been discussed. This was achieved by studying higher dimensional topological defects such as Nielsen-Olesen strings [8, 9] and magnetic monopoles [10] in 6 and 7 space-time dimensions, respectively. It was found [7, 8] that gravity-localising (so-called “warped”) solutions are possible if certain relations between the defect's tensions hold.

Originally introduced to remove singularities associated with point-like charges in electrodynamics [11], the generalisation of the Born-Infeld (BI) action to non-abelian gauge fields has gained a lot of interest in topics related to string theory [12, 13]. It became apparent that when studying low energy effective actions of string theory the part of the Lagrangian containing the abelian Maxwell field strength tensor and its non-abelian counterpart in Yang-Mills theories has to be replaced by a corresponding (resp. abelian and non-abelian) BI term. That's why it seems interesting to generalise the brane world scenario for 6-dimensional Nielsen-Olesen strings recently proposed by Giovannini, Meyer and Shaposhnikov (GMS) [9] to Born-Infeld actions. This was done in [14] and it was found that gravity-localising solutions don't exist in a 6-dimensional Einstein-Born-Infeld-Higgs model. Here, we review the results and extend the discussions done in [14].

II. THE MODEL

The action reads:

$$S = S_{gravity} + S_{brane} = - \int d^6x \sqrt{-g} \frac{1}{16\pi G_6} (R + 2\hat{\Lambda}_6) + \int d^6x \sqrt{-g} \left(\hat{\beta}^2 (1 - \mathcal{R}) + \frac{1}{2} D_M \phi D^M \phi^* - \frac{\lambda}{4} (\phi^* \phi - v^2)^2 \right) \quad (1)$$

where $\hat{\Lambda}_6$ is the bulk cosmological constant, G_6 is the fundamental gravity scale with $G_6 = 1/M_{pl(6)}^4$ and g the determinant of the 6-dimensional metric. Moreover, we have the covariant derivative $D_M = \nabla_M - ieA_M$ and the field strength $F_{MN} = \partial_M A_N - \partial_N A_M$ of the U(1) gauge potential A_M . v is the vacuum expectation value of the complex valued Higgs field ϕ and λ is the self-coupling constant of the Higgs field.

The Ansatz reads [8, 9, 14]:

$$ds^2 = M^2(r) (dx_1^2 - dx_2^2 - dx_3^2 - dx_4^2) - dr^2 - l^2(r) d\theta^2 \quad (2)$$

for the metric and

$$\phi(r, \theta) = v f(r) e^{in\theta} \quad , \quad A_\theta(r, \theta) = \frac{1}{e} (n - P(r)) \quad (3)$$

for the gauge and the Higgs field [15], where n is the vorticity of the string. The equations of motion can be computed easily (see [14] for details) and depend after the rescalings $x = \sqrt{\lambda}vr$, $L(x) = \sqrt{\lambda}vl(r)$ only on the coupling constants $\alpha = e^2/\lambda$, $\gamma^2 = 8\pi G_6 v^2$, $\Lambda = \hat{\Lambda}_6/(\lambda v^2)$, $\beta^2 = \hat{\beta}^2/(\lambda v^2)$.

III. NUMERICAL RESULTS

The system of equations has been solved numerically subject to the following boundary conditions:

$$f(0) = 0 \quad , \quad P(0) = n \quad , \quad M(0) = 1 \quad , \quad M'|_{x=0} = 0 \quad , \quad L(0) = 0 \quad , \quad L'|_{x=0} = 1 \quad (4)$$

at the origin and

$$f(\infty) = 1 \quad , \quad P(\infty) = 0 \quad (5)$$

at infinity.

We have studied the set of equations numerically. We find that several branches of solutions exist. The pattern of these branches is very similar than in the 4 dimensional case [16]. For small gravitational coupling, $\gamma < \gamma_c(\alpha, \Lambda)$, the gravitational deformations of the Nielsen-Olesen string solutions as well as so-called Melvin solutions exist for the same values of the coupling constants. They differ, however, in their asymptotic behaviour. The metric functions on the string branch behave like $M(x \rightarrow \infty) \rightarrow a$, $L(x \rightarrow \infty) \rightarrow bx + c$, where a , b , c are constants depending on γ , α and Λ . The metric functions on the Melvin branch have the following asymptotic behaviour: $M(x \rightarrow \infty) \rightarrow Ax^{2/5}$, $L(x \rightarrow \infty) \rightarrow Bx^{-3/5}$, where again A and B are constants depending on the coupling constants. For $\gamma > \gamma_c(\alpha, \Lambda)$ closed solutions with zeros of the metric functions exist. For the so-called inverted string branch, which represents the strong gravity analogue of the string branch, the metric function $L(x)$ possesses a zero, $L(x = x_0) = 0$, while $M(x)$ stays finite at this x_0 . Furthermore, a so-called Kasner branch exists. The solutions on this branch possess a zero of the metric function $M(x)$, $M(x = \tilde{x}_0) = 0$, while $L(x \rightarrow \tilde{x}_0) \rightarrow \infty$. In Fig. 1, we show the dependence of the critical value γ_c on α for $\Lambda = 0$ and $n = 1, 2$. As is clearly seen, γ_c increases with increasing α and tends to zero for $\alpha \rightarrow 0$. This likely stems from the fact that the mass of the vortex decreases with increasing α and thus the strong gravity limit is reached for higher values of the gravitational coupling. Furthermore, the critical value of γ for a fixed α decreases with the winding number n as can be seen when comparing the two curves for $n = 1$ and $n = 2$. Again, this can be explained by the increased mass of the vortex solution.

Interestingly, the pattern of solutions persists for $\Lambda < 0$. For the $\beta^2 = \infty$ limit, it was found in [8] that so-called “anti-warped” and gravity-localising (“warped”) solutions exist. However, no classification was done. In [14], it was demonstrated that the “anti-warped” solutions are the by the presence of the cosmological constant deformed string and Melvin solutions, while the warped solutions are the solutions on the limiting line between the Kasner and inverted string solutions. Of course, this is only possible if a fine-tuning between the coupling constants is done, i.e. warped solutions exist only for specific triplets $(\gamma_W, \alpha_W, \Lambda_W)$ of the coupling constants. In Fig.1, we give the value of γ_W in dependence on α for $\Lambda_W = -0.0035$ and $n = 1, n = 2$.

As can be seen from Fig.1 gravity-localising solutions for a fixed α_W , Λ_W exist for lower values of the gravitational coupling for the $n = 2$ solutions than for the $n = 1$ solutions.

It is interesting to study how the value γ_W for $\Lambda < 0$ is related to the value γ_c for $\Lambda = 0$

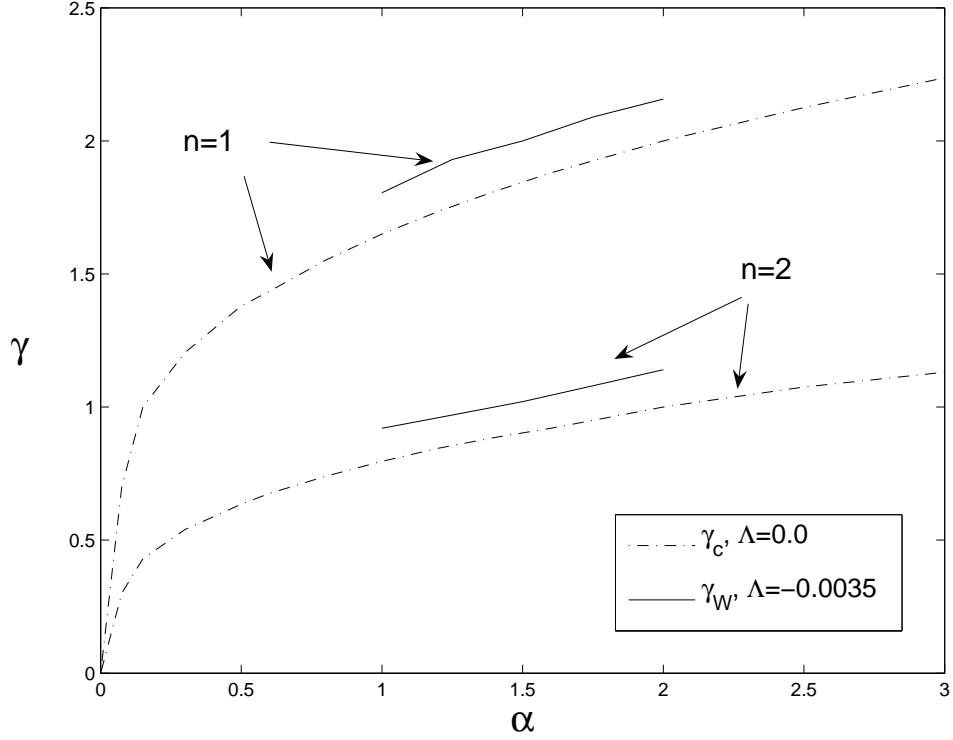


FIG. 1: The value γ_c for $\Lambda = 0$ as well as γ_W for $\Lambda_W = -0.0035$ are shown in dependence on α for $\beta^2 = \infty$ and $n = 1, n = 2$.

for a fixed value of α . We have studied this question in some detail and as can be seen from Fig.1, our results reveal that the curves are more or less parallel. This holds true for both $n = 1$ and $n = 2$. We thus find the following approximate relation:

$$\gamma_W(\alpha, \Lambda, n) \approx \gamma_c(\alpha, \Lambda = 0, n) + 3|\Lambda|^{1/2} \quad (6)$$

This completes the analysis done in [8, 14] in the sense that here we have determined the dependence of γ_c and γ_W on both α and n .

Let us remark that it was found [14] by an analytic argument that no gravity-localising solutions exist if $\beta^2 < \infty$. Of course, this result could be different if higher order corrections (e.g. higher derivative terms of the gauge field), which appear in the low energy effective action of string theory, are included.

IV. CONCLUSIONS

Brane world models have attracted a lot of attention recently. Since Newton's law is very well tested in 4 dimensions now, proper models should localise gravity well enough to the 3-brane. Randall-Sundrum models possess one extra codimension. For more than one extra dimension, alternative models have to be looked for. In [7, 8, 9, 10, 14], models with 2 and 3 codimensions have been studied by using topological defects in higher dimensions. Here, we have discussed further properties of the solutions in the 6-dimensional Einstein-Abelian-Higgs model. We have determined the α dependence of the critical gravitational coupling γ_c for $\Lambda = 0$. Further we have determined how the gravitational coupling γ_W , for which gravity-localising solutions exist, depends on α and n . We found an interesting relationship between γ_c and γ_W .

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- [1] G. Dvali and M. Shifman, Phys. Lett. **B396** (1997), 64; **407** (1997), 452.
 - [2] K. Akama, "Pregeometry", in Lecture Notes in Physics, 176, Gauge Theory and Gravitation, Proceedings, Nara, 1982, edited by K. Kikkawa, N. Nakanishi and H. Nariai, 267-271 (Springer Verlag, 1983) [hep-th/0001113]; I. Antoniadis, Phys. Lett. **B246** (1990), 377.
 - [3] N. Arkani-Hamed, S. Dimopoulos and G. Davli, Phys. Lett. **B429** (1998), 263; I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. Dvali, Phys. Lett. **B436** (1998), 257.
 - [4] L. Randall and R. Sundrum, Phys. Rev. Lett. **83** (1999), 3370.
 - [5] L. Randall and R. Sundrum, Phys. Rev. Lett. **83** (1999), 4690.
 - [6] V. A. Rubakov and M. E. Shaposhnikov, Phys. Lett. **125B** (1983), 136; **125B** (1983), 139.
 - [7] T. Gherghetta, E. Roessl and M. E. Shaposhnikov, Phys. Lett. **B491** (2000), 353.
 - [8] T. Gherghetta and M. E. Shaposhnikov, Phys. Rev. Lett. **85** (2000), 240.
 - [9] M. Giovannini, H. B. Meyer and M. E. Shaposhnikov, Nucl. Phys. **B691** (2001), 615.
 - [10] E. Roessl and M. E. Shaposhnikov, Phys. Rev. **D66** (2002), 084008.

- [11] M. Born and M. Infeld, Proc. R. Soc. London **A143** (1934), 410.
- [12] J. Polchinski, “TASI Lectures on D-branes”, hep-th/9611050.
- [13] A. A. Tseytlin, “Born-Infeld action, supersymmetry and string theory”, hep-th/9908105.
- [14] Y. Brihaye and B. Hartmann, Nucl. Phys. **B691** (2004), 79.
- [15] H. B. Nielsen and P. Olesen, Nucl. Phys. **B61** (1973), 45.
- [16] M. Christensen, A. L. Larsen and Y. Verbin, Phys. Rev. **D60** (1999), 125012; Y. Brihaye and M. Lubo, Phys. Rev. **D62** (2000), 085004.