# Mixed-symmetry fields in $(A)dS_{d+1}$

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Mostly based on [Thomas Basile, Xavier Bekaert and N.B. 1612.08166]

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# PLAN

1. Motivation

2. Bargmann-Wigner in Minkowski space

3. Bargmann-Wigner in $AdS_{d+1}$ space

4. UIR's of $SO(1, d+1)$ & fields in $dS_{d+1}$

## 1 Motivation for higher-spin fields : Group theory

- At dawn of QFT : Majorana (1932), Dirac (1936), Fierz-Pauli (1939), and most notably Wigner's 1939 classification of UIR's of Poincaré group $ISO(3,1)$.

- Relativistic, linear & covariant equations : Bargmann-Wigner (1948)

$\longrightarrow$ massless, helicity particles characterized by

- Mass $m = 0$ ; • helicity $s \in \{0, 1/2, 1, 3/2 \dots\}$

[ • Rem : In the $m = 0$ case, there are also the "continuous" or "infinite" spin UIR's $\rightsquigarrow \vec{\mu} \neq \vec{0}$ in $\mathbb{R}^{D-2}$. ]

## • Some comments about interactions

↳ Problems with :

- Minimal $u(1)$ coupling for $s \geq 3/2$ (1961)
- Minimal Lorentz coupling for $s \geq 5/2$ (1964)
- Infinite-component Majorana-like equations (1968)
  (tachyons)

↳ Together with the observation of high-spin hadronic resonances

Belief that consistent high-spin interactions require infinitely-many fields of unbounded spin.

Once the HS representations have been seen to exist in the sense of UIR's of spacetime isometry algebra, i.e. first quantization, then standard second quantization naturally requires a covariant Lagrangian.

Fierz-Pauli program :

Associate a quadratic, local and covariant Lagrangian to every UIR of maximally-symmetric spacetime-isometry algebra.

- Initiated by F.P. in 1939 for massive, spin-2 particle in $\mathbb{R}^{1,3}$. Then, notably [Chang (67), Schwinger (70), Singh-Hagen (74)]

- In 1978, Fronsdal and Fang gave Lagrangian for $m=0$ helicity-$s$ field around $\mathbb{R}^{1,3}$ and $(A)dS_4$ by taking the $m \to 0$ limit of Singh-Hagen's $\mathcal{L}$

<u>Questions</u>: Can we generalize this program to arbitrary spacetime dimensions $D > 4$? Interactions?

( <u>Rem</u> : $D = 2+1$ very interesting too. Not in this talk. )

- The BW program in $\mathbb{R}^{1,d}$ was achieved in late 80's [W. Siegel & B. Zwiebach] and in minimal form in [Labastida 89, X. Bekaert & N.B. 2001]

- The BW program in $AdS_{d+1}$ in the late nineties by R. Metsaev.

Before attacking the ambitious problem of introducing consistent interactions among the various fields, we first want to fill a gap: the Bargmann-Wigner program in $dS_{d+1}$: establish the dictionary in $dS_{d+1}$ between UIR's of $SO(1,d+1)$ and covariant linear wave equations in $dS_{d+1}$.

② Wigner's classification of UIR's of $ISO(1,d+1)$

↳ One-to-one with the $SO(1,d)$ orbits $\mathcal{O}_p$ of $p \in (\mathbb{R}^{1,d})^*$ together with UIR of little group $G_p \subseteq SO(1,d)$ stabilizing $p$.

1) $\forall g \in G_p \qquad g \cdot p = p$

2) Given repres. $R$ of $G_p$, induce a UIR $\mathcal{T}$ of ISO on the Hilbert space of functions on $\mathcal{O}_p$ valued in $R$

$$\mathcal{T}(\Lambda, a) \cdot \varphi(q) = \sqrt{\rho_{\Lambda^{-1}}(q)}\; e^{i(q,a)} R\left(g_q^{-1} \circ \Lambda \circ g_{\Lambda^{-1} q}\right) \cdot \varphi(\Lambda^{-1} q)$$

where $g_q \in SO(1,d)$ standard boost for $p$ : $g_q \cdot p = q$

s.t. $\quad g_q^{-1} \circ \Lambda \circ g_{\Lambda^{-1} q} : p \longrightarrow \Lambda^{-1} q \longrightarrow q \longrightarrow p$

The various orbits $\{\mathcal{O}\}$ correspond to $p$ being

1) Timelike $\rightsquigarrow$ Massive particle $p_\mu = (-m, 0, \dots, 0)$

$$p^2 = -m^2 \quad , \quad G_p \simeq SO(d) \quad \left[ E := p^0 \quad \& \quad \eta = \mathrm{diag}(-,+,\dots,+) \right]$$

2) Light-like $\rightsquigarrow$ Massless particle

$$p^2 = 0 \quad \& \quad p \neq 0 \qquad p_\mu = (-E, 0, \dots, 0, E)$$

In Light frame $x^\pm := \dfrac{x^d \pm x^0}{\sqrt{2}}$ , $p_\mu = (p_-, \overset{p_+}{0}, \overbrace{0, \dots, 0}^{p_i})$

Little group $G_p \cong ISO(d-1) \cong T_{d-1} \rtimes SO(d-1)$

$M_{i-}$ & $M_{-+}$ rejected $\Rightarrow \{ M_{i+} =: \pi_i \} \cup \{ M_{ij} \}$

$\hookrightarrow$ Take $\pi_i$ trivial $\rightsquigarrow$ helicity UIR's : $G_p \cong SO(d-1)$.

3) Spacelike $\rightsquigarrow$ tachyons : $G_p \cong SO(1, d-1)$

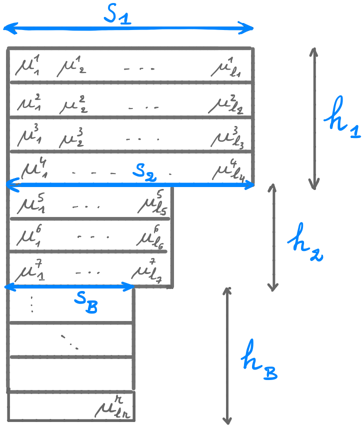
4) Nul $p = (0, \dots, 0)$ in $(\mathbb{R}^{1,d})^*$ : $G_0 \cong SO(1,d)$ .

As for covariant, linear wave equations

. Take $\varphi_{\mathbb{Y}}(x)$ valued in $\mathfrak{gl}(d+1)$ irrep $\rightsquigarrow \mathbb{Y}$

Young tableau

$\mathbb{Y}$ =




with $\boxed{c_1 + c_2 \leqslant d-1}$

$$\vec{s} = (\underbrace{\ell_1, \dots, \ell_4}^{s_1}, \underbrace{\ell_5, \dots, \ell_7}^{s_2}, \dots, \overset{s_B}{\ell_r})$$

- Symmetrizing the indices of any row with any index of a lower row gives zero identically.

- Antisymmetrizing the indices of a column with any index of a column at its right gives zero identically.

• Build the curvature

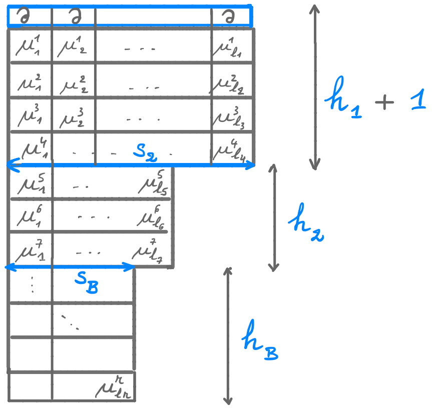
$$\boxed{K_{\overline{\mathbb{Y}}} := d^{(1)} \cdots d^{(s_1)} \varphi_{\mathbb{Y}}}$$

by acting on $\varphi_{\mathbb{Y}}$ with $s_1$ curls

and impose the wave equation

$$\boxed{\mathrm{Tr}\, K_{\overline{\mathbb{Y}}} \approx 0} \; .$$

$\overline{\mathbb{Y}} =$




From Bianchi identity $\quad d^{(i)} K_{\overline{\mathbb{Y}}} \equiv 0 \qquad \forall\, i \in \{1, \ldots, s_1\}$

Deduce that $\quad d_{(i)} K \approx 0 \quad \forall\, i \quad$ where $\quad \boxed{d_{(i)} := *_i\, d^{(i)} *_i}$ divergence.

Hence $\quad \{ d^{(i)}, d_{(i)} \} K_{\overline{\mathbb{Y}}} \equiv \Box K_{\overline{\mathbb{Y}}} \approx 0 \quad \Rightarrow K_{\overline{\mathbb{Y}}}$ massless field.

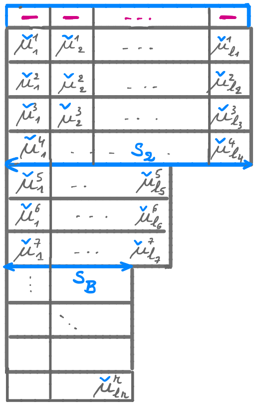
Fourier modes $\quad \tilde{K}_{\overline{\mathbb{Y}}}(p) \quad$ on $\quad p^2 = 0$

light cone : mass shell for light-like particles.

$$\boxed{\begin{array}{l} [d^{(i)}, d_{(j)}]_{\mathbb{Z}_2} = \delta^i_j\, \Box \\ [d^{(i)}, d^{(j)}]_{\mathbb{Z}_2} = 0 = [d_{(i)}, d_{(j)}]_{\mathbb{Z}_2} \end{array}}$$

## Bianchi Id.

$\hookrightarrow d^{(i)} K = 0$ : $\boxed{p_{[-} \tilde{K}_{\mu_1^1 \mu_1^2 \dots \mu_1^{p_B}]|\dots} \equiv 0}$ $\Leftrightarrow$ $\tilde{K}_{\underline{\mathbb{Y}}} \rightsquigarrow \mathbb{Y}$ of $gl(d, \mathbb{R})$ (*)

$\tilde{K} =$ 



$\check{\mu} \in \{+, 1, \dots, d-1\}$

$d_{(i)} K \approx 0$ Divergenceless : $p^+ \tilde{K}_{+\dots} \approx 0$

Means that $\tilde{K}$ valued in $\mathbb{Y}_{gl(d-1)}$

Tracelessness in $so(1,d) \longrightarrow$ Tracelessness $so(d-1)$

$\underline{\text{Ccl}}$ : $\tilde{K}_{\underline{\mathbb{Y}}}$ reduces on-shell to $\tilde{K}$ in UIR $R_{\mathbb{Y}}$ of $G_p$

$G_p \cong so(d-1)$ little group massless particle $\xrightarrow[\text{induce}]{}$ $T(\Lambda, a)$ UIR ▣

(*)

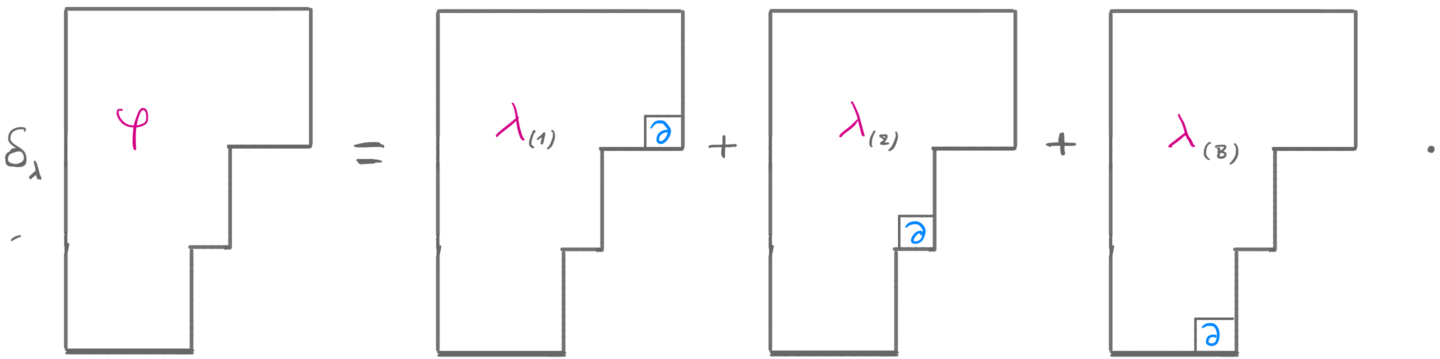
$K_{-\check{\mu}\check{\nu}|-\check{e}}$ $\quad 0 \equiv K_{[-\check{\mu}\check{\nu}|\check{e}]-} = 3\tilde{K}_{-[\check{\mu}\check{\nu}|\check{e}]-} - \cancel{\tilde{K}_{\check{\mu}\check{\nu}\check{e}|--}}^{\,0} \Leftrightarrow \tilde{K}_{-[\check{\mu}\check{\nu}|\check{e}]-} \equiv 0$

# Gauge invariance

$$K_{\bar{Y}} = d^{(1)} \dots d^{(s_1)} \varphi_Y$$

Wave equation $\mathrm{Tr}\, K_{\bar{Y}} \approx 0$ is PDE order $s_1$ for $\varphi$

Invariant under




On-shell, fixing gauge $\tilde{K}$ reduces to $\tilde{K} \approx (p_-)^{s_1} \varphi_{i_1 \dots j \dots}$

X.B.& N.B. Partial gauge fixing of $\mathrm{Tr}\, \tilde{K} = 0$ $SO(d-1)$.

to $\left(\Box - \sum_{i=1}^{s_1} d^{(i)} d_{(i)} + \frac{1}{2} \sum_{i,j=1}^{s_1} d^{(i)} d^{(j)} \mathrm{Tr}_{ij}\right) \varphi \approx 0$ : Labastida 89.

# ③ WAVE EQUATIONS in $(A)dS_{d+1}$

## Conventions and notation

Lie algebra $so\left(1+\frac{1+\sigma}{2},\, d+\frac{1-\sigma}{2}\right)$ with generators $M_{AB}=M_{AB}^{\dagger}$

$$(\eta^{(\sigma)}_{AB}) = \mathrm{diag}(-\sigma, \underbrace{-, \overbrace{+,\dots,+}^{(\delta_{ij})}}_{(\eta_{ab})}) \longrightarrow \begin{cases} \sigma = +1 & AdS_{d+1} \\ \sigma = -1 & dS_{d+1} \end{cases}$$

$A, B, \dots = 0_\sigma, 0, 1, \dots, d$

$a, b, \dots = 0, 1, \dots, d$

$$\eta_{ab} = \mathrm{diag}(\underset{0}{-}, \underset{1}{+}, \underset{\dots}{\dots}, \underset{d}{+}) \qquad so(1,d)$$

$$[M_{AB}, M_{CD}] = i\left(\eta^{(\sigma)}_{BC} M_{AD} - \eta^{(\sigma)}_{AC} M_{BD} - \eta^{(\sigma)}_{BD} M_{AC} + \eta^{(\sigma)}_{AD} M_{BC}\right)$$

• $P_a := \lambda \, M_{\sigma a}$ transvections (A)dS

$$\boxed{\begin{aligned} [M_{ab}, M_{cd}] &= i\,\eta_{bc} M_{ad} + \dots \\ [M_{ab}, P_c] &= 2i\,\eta_{c[b} P_{a]} \qquad [P_a, P_b] = i\sigma\lambda^2 M_{ab} \end{aligned}}$$

• Another useful decomposition of $M_{AB}$ :

$$D := i c_\sigma M_{\sigma 0} \,, \qquad P_i := M_{0i} + c_\sigma M_{\sigma i} \,, \qquad K_i := M_{0i} - c_\sigma M_{\sigma i}$$

where $c_\sigma = \begin{cases} i & \text{for } \sigma = +1 \\ 1 & \text{for } \sigma = -1 \end{cases}$ , s.t. $c_\sigma^2 = -\sigma$ .

$$\boxed{\begin{aligned} [M_{ij}, M_{kl}] &= i\,\delta_{jk} M_{il} + \dots & [K_i, P_j] &= 2\left(i M_{ij} + \delta_{ij} D\right) \\ [M_{ij}, P_k] &= 2i\,\delta_{k[j} P_{i]} & [M_{ij}, K_k] &= 2i\,\delta_{k[j} K_{i]} \\ [D, P_i] &= P_i & [D, K_i] &= -K_i \end{aligned}}$$

Quadratic Casimir $C_2\left[so\left(1+\frac{1+\sigma}{2}, d+\frac{1-\sigma}{2}\right)\right] = \frac{1}{2} M^{AB} M_{AB}$

Using $\quad M_{0i} = \frac{1}{2}(P_i + K_i), \qquad M_{0_\sigma i} = \frac{1}{2C_\sigma}(P_i - K_i)$

$$\frac{1}{2} M^{AB} M_{AB} = D(D-d) - P^i K_i + C_2[so(d)]$$

$\Rightarrow$ On a lowest-weight state $|\Delta, \vec{s}\rangle$ annihilated by ladder op. $K_i$,

$$\boxed{C_2\left[so\left(1+\frac{1+\sigma}{2}, d+\frac{1-\sigma}{2}\right)\right] = \Delta(\Delta-d) + \sum_{\ell=1}^{r} s_\ell(s_\ell + d - 2\ell)}$$

$$(D-\Delta)\,|\Delta, \vec{s}\rangle = 0, \qquad K_i\,|\Delta, \vec{s}\rangle = 0$$

In the $so(1,d)$-covariant basis where $P_a := \lambda M_{0a}$,

represent $\boxed{P_a = -i \nabla_a}$ as a diff. operator

$$\Rightarrow \quad C_2 = \frac{1}{2} M^{AB} M_{AB} \equiv C_2[so(1,d)] - \sigma\, \eta^{ab} M_{0a} M_{0b}$$

$$= C_2[so(1,d)] - \frac{\sigma}{\lambda^2} P^a P_a$$

$$\Rightarrow \quad \boxed{-\frac{\sigma}{\lambda^2} P^2 = \frac{\sigma}{\lambda^2} \nabla^a \nabla_a = \frac{1}{2} M^{AB} M_{AB} - \frac{1}{2} M^{ab} M_{ab}}$$

$\Rightarrow$ Relation between wave equation (linear, relativistic) and (abstract) UIR of $so\left(1+\frac{1+\sigma}{2}, d+\frac{1-\sigma}{2}\right)$.

$$\boxed{(\Box - \lambda^2 \sigma m^2)\varphi = 0}$$

Asking gauge invariance of wave equation

$$(\Box - \sigma\lambda^2 m^2)\,\varphi_{\mathbb{Y}} = 0 \;, \quad \mathrm{Tr}\,\varphi_{\mathbb{Y}} = 0 = \nabla\cdot\varphi_{\mathbb{Y}} \quad \text{on all indices}$$

under $\quad \delta_\lambda \varphi_{\mathbb{Y}} = \sum_{I=1}^{B} \nabla^{(I)} \lambda_{(I)}$

gives [Metsaev '95] a set of possibilities for fixed $I$

$$m_s^2 \in \left\{ (s_I - p_I - 1)(s_I - p_I + d - 1) - \sum_{k=1}^{r} \ell_k \right\}_{I=1,\dots,B}$$

and $\quad p_I := \sum_{J=1}^{I} h_J$

together with similar conditions on the gauge para. $\lambda_{(I)}$

and the gauge-for-gauge parameters $\{\lambda^i_{(I)}\}_{i=2,\dots,p_I}$

# Group theoretical description in $AdS_{d+1}$

Generalized Verma module

$$so(2) \oplus so(d) \subset so(2,d)$$

$$\mathcal{V} = \left\{ P_{i_1} \dots P_{i_n} \, | e_0, \vec{s} >_{j \dots k \dots} \right\}_{n = 0, 1, \dots}$$

Recall $\quad C_2[so(2,d)] = e_0(e_0 - d) + C_2[so(d)]$

with
$$\begin{cases} e_0 > s_1 - h_1 + d - 1 & \text{Massive unitary field} \\ e_0 = e_0^I := s_I - p_I + d - 1 & \text{Massless (gauge) fields} \\ e_0 \neq e_0^I \;\; \& \;\; e_0 < e_0^1 & \text{Massive non-unitary} \end{cases}$$

Observe $\quad m^2 \in \left\{ e_0^I (e_0^I - d) - \sum_{k=1}^{n} \ell_k \right\}_{I = 1, \dots, B}$

In accordance with $\quad \dfrac{\sigma}{\lambda^2} \Box = \dfrac{1}{2} M^{AB} M_{AB} - \dfrac{1}{2} M^{ab} M_{ab}$

# Gauge invariance of Fierz-Pauli-type wave equation

reflected by

Gauge field

Irr. module

$$\mathcal{D}(e_0^I, \mathbb{Y}) \cong \frac{\mathcal{V}(e_0^I, \mathbb{Y})}{\mathcal{D}(\underbrace{e_0^I + 1,}_{\delta\varphi = \nabla \lambda_{(I)}} \mathbb{Y}_{(I)})}$$

Generalized Verma m.

minimal energy of module

Gauge param. module, itself a quotient in general (gauge for gauge)

| $AdS_{d+1}$ | $dS_{d+1}$ |
| --- | --- |
| Vacuum $so(2) \oplus so(d)$ module | Vacuum $so(1,1) \oplus so(d)$ module |
| $\mathbb{V}(e_0, \mathbb{Y})$ | $\mathbb{V}(\Delta_c, \mathbb{Y})$ |
| • Casimir | • Casimir |
| $C_2 = (e_0 - d) + C_2[so(d)]$ | $C_2 = \Delta_c(\Delta_c - d) + C_2[so(d)]$ |
| • Critical mass | • Critical mass |
| $m^2 = e_0(e_0 - d) + \sum_{k=1}^{r} \ell_k$ | $m^2 = \Delta_c(\Delta_c - d) + \sum_{k=1}^{r} \ell_k$ |
| • massless for $e_0 = e_0^I$ | • massless for $\Delta_c = ?$ |
| unitarity known $(L_i^-)^\dagger = L_i^+$ | unitarity ? |

④ $\boxed{\text{UIR's of } so(1,d+1)}$

- Principal series : $\Delta_c = \frac{d}{2} + i\nu$, $\mathbb{Y}$ & $\nu \in \mathbb{R}$ arbitrary

- Complementary series : $p < \Delta_c < d-p$ , $p \in \{0, 1, \dots, r-1\}$

$P = P_B$

$\ell_k = 0$ for $k = p+1, \dots, r$ .

- Exceptional series : $\Delta_c = d-p$ ( or $\Delta_c = p$ ) , $p \in \{1, \dots, r-1\}$

$\ell_k = 0$ for $k = p+1, \dots, r$ . (no scalar)

- $(d = 2r+1)$ Discrete series : $\Delta_c = \frac{d}{2} + k$ , $k \in \frac{\mathbb{N}}{2}$

maximal height $0 < k \leq \ell_r$

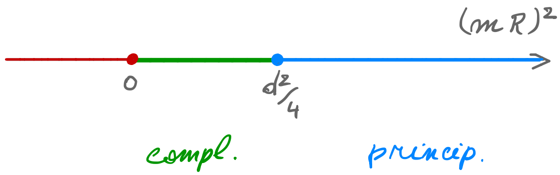
$so(1, 2r+2) \Rightarrow \text{rank } so(d) = \text{rank } so(1,d)$

# Dictionary

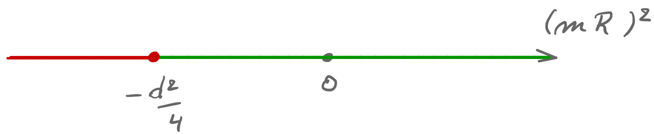
Computing de $so(d+2)$ characters of Generalized Verma modules [using Bernstein - Gel'fand - Gel'fand resolution] and comparing with characters of $SO(1,d+1)$ UIR's from the math litterature, we obtained the dictionary

- Principal & complementary : Massive fields, e.g.

$dS_{d+1}$




$AdS_{d+1}$




$R^2 m^2 = e_0(e_0-d)$ $\qquad e_0 = \frac{d-2}{2} \Rightarrow$ singleton

$e_0 \geq s-p+d-1$ $\qquad m^2_{Rac} = -\frac{1}{4}(d^2-4)$

$e_0 \geq d-1$ & $e_0 = \frac{d-2}{2}$

• Exceptional series : (partially) massless fields with less-than-maximal height

Unitarity: only the last block must be activated

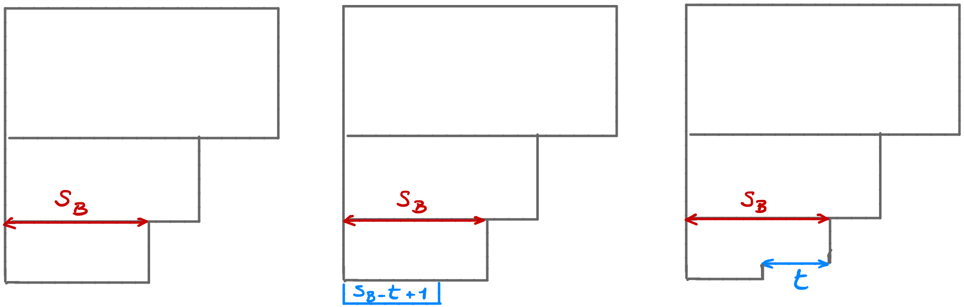
$$\Delta_c = s_B - p + d - t$$

$p \equiv p_B$

contrary to the first one in AdS.

Key • : The weights $(\Delta_c, \mathbb{Y})$ labelling the UIR ⇝ Curvature and not $\varphi$ potential

• Discrete series : massless field $\varphi$ with maximal height




$$\Delta_c = \ell_r - r + d - t$$

$p_B \equiv p = r$

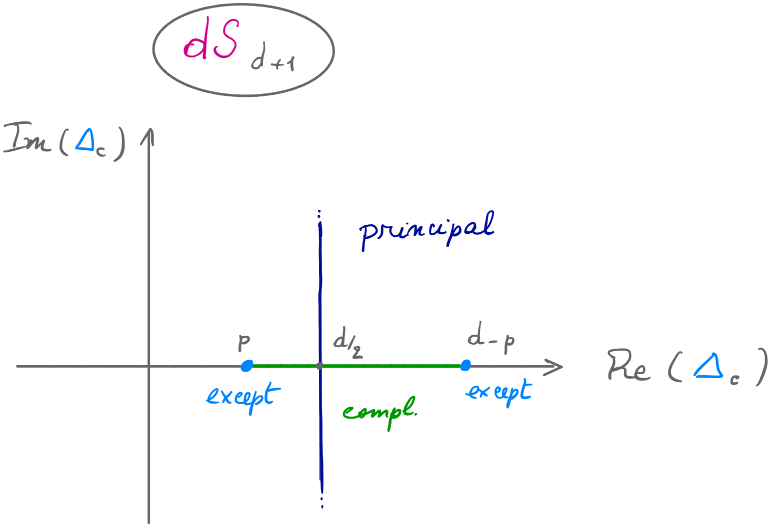
$s_B \equiv \ell_r$

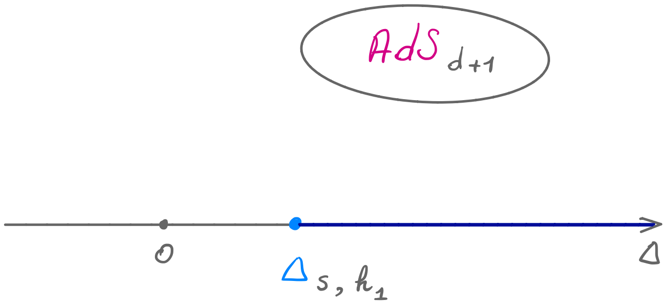
$\varphi$ potential $\quad K$ curvature $\quad \lambda$ gauge parameter

Massless cases : $t = 1$ ; PM : $1 < t \leq s_B$ .

# Conclusion

Unitary fields.



Note: In the scalar case $s = 0$, the primary Weyl tensor

$\phi(x)$ obeys $(\Box + 2\lambda^2)\,\phi(x) = 0$ in $AdS_4$, where

$$\bar{M}^2_{(0,0)} = -2 = C_2[so(2,3)\,|\,\mathcal{D}(e_0, 0)] = -e_0(-e_0+3)$$

leaving 2 possibilities compatible with unitarity:

↳ $e_0 = 1$ (Dirichlet) or $2$ (Neuman) BC's.

. So, in the zoology of "massless" UIR's

⇝ (bosonic) fields propagating in $AdS_4$, we have

$\mathcal{D}(s+1, s)$ $s=0,1,2,\dots$ and $\mathcal{D}(2,0)$ : Fronsdal on-shell fields

# Dirac singletons and Flato-Fronsdal

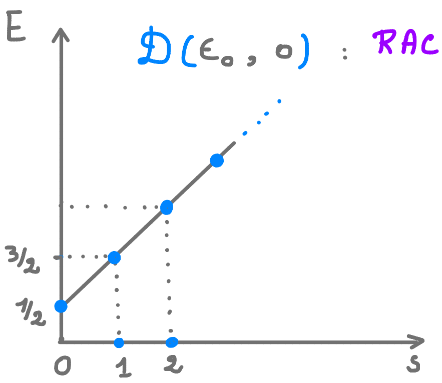
$\left[\epsilon_0 = \frac{d-2}{2}\right].$

Two remarkable $so(2,d)$-UIRs : $\mathcal{D}(\epsilon_0, 0)$ & $\mathcal{D}(\epsilon_0 + \frac{1}{2}, \frac{1}{2})$

Not propagating inside $AdS_{d+1}$ but at $\partial AdS_{d+1}$.

↳ Single line in compact weight space

$\mathcal{D}(\epsilon_0, 0)$ : RAC & $\mathcal{D}(\epsilon_0 + \frac{1}{2}, \frac{1}{2})$ : Di



## Flato - Fronsdal theorem ( $d=3$ )

- $\mathcal{D}(\tfrac{1}{2},0) \otimes \mathcal{D}(\tfrac{1}{2},0) \simeq \bigoplus_{s=0}^{\infty} \mathcal{D}(s+1,s)$

- $\mathcal{D}(1,\tfrac{1}{2}) \otimes \mathcal{D}(1,\tfrac{1}{2}) \simeq \mathcal{D}(2,0) \oplus \bigoplus_{s=1}^{\infty} \mathcal{D}(s+1,s)$

Consequence : Compositeness of massless particles in $AdS_4$

Rac : $\Box_3 \, \phi(x) = 0 \quad (*)$ with $\dim(\phi) = \frac{1}{2}$. $\left(\int d^3x \, \partial\phi \cdot \partial\phi\right)$

conformal scalar

- Symmetries of $(*)$ : $\dfrac{\mathcal{U}(so(2,d))}{\mathrm{Annih}(\mathrm{Rac})} \simeq \mathcal{A}$ associative algebra

$\downarrow [\cdot\,,\,\cdot]$

$hs(d+1)$

· Gauge symmetries : differential and algebraic

↳ Minimal set of fields & gauge parameters ⇝ Fronsdal.

$$-2\mathcal{L}(\varphi,\nabla\varphi) = \nabla_\nu \varphi_{\mu(s)} \nabla^\nu \varphi^{\mu(s)} - \frac{s(s-1)}{2} \nabla_\nu \varphi'_{\mu(s-2)} \nabla^\nu \varphi'^{\mu(s-2)}$$

$$+ s(s-1) \nabla_\nu \varphi'_{\mu(s-2)} \nabla_\rho \varphi^{\rho\nu\mu(s-2)} - s \nabla_\nu \varphi^\nu{}_{\mu(s-1)} \nabla_\rho \varphi^{\rho\mu(s-1)}$$

$$- \frac{s(s-1)(s-2)}{2} \nabla_\nu \varphi'^\nu{}_{\mu(s-3)} \nabla_\lambda \varphi'^{\lambda\mu(s-3)}$$

$$+ m_e^2 \varphi^{\mu(s)} \varphi_{\mu(s)} + m_e'^2 \varphi'^{\mu(s-2)} \varphi'_{\mu(s-2)} .$$

$$m_e = \lambda^2 \left( s^2 + (D-6)s - 2D + 6 \right)$$

$$\cdot\ \bar{R}_{\mu\nu\rho\sigma} = -\lambda^2 \left( \bar{g}_{\mu\rho} \bar{g}_{\nu\sigma} - \bar{g}_{\nu\rho} \bar{g}_{\mu\sigma} \right), \qquad \lambda^2 = \frac{-2\Lambda}{(D-1)(D-2)}$$

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 0$$

- Maxwell's theory : $A_\mu(x) := \varphi_\mu(x)$, $\delta_\epsilon A_\mu(x) = \partial_\mu \epsilon(x)$

  - $S[A_\mu] = -\frac{1}{4}\int d^4x \; F^{\mu\nu} F_{\mu\nu}$ , $F_{\mu\nu} := 2\,\partial_{[\mu} A_{\nu]}$

  - $\delta_\epsilon S[A_\mu] = 0 \iff \partial^\mu F_{\mu\nu} \equiv 0$ (Noether id.)

- Fierz-Pauli in metric-like notation :

  $h_{\mu_1\mu_2}(x) =: \varphi_{\mu(2)}(x)$, $\delta_\epsilon \varphi_{\mu(2)} = 2\,\partial_\mu \epsilon_\mu$ $\left(\delta_\epsilon h_{\mu\nu} = 2\,\partial_{(\mu}\epsilon_{\nu)}\right)$

  - $S_0[\varphi_{\mu(2)}] = -\frac{1}{2}\int d^4x \left[\partial^\nu \varphi^{\mu(2)}\,\partial_\nu \varphi_{\mu(2)} + \cdots\right]$

  - $\delta_\epsilon S_0[\varphi_{\mu(2)}] = 0 \iff \partial^\mu \overset{(1)}{G}_{\mu\nu}(x) \equiv 0$ , $\overset{(1)}{G}_{\mu\nu} := \overset{(1)}{R}_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}\overset{(1)}{R}$ .

- Fronsdal's formulation

- $\varphi_{\mu_1 \ldots \mu_s} = \varphi_{(\mu_1 \ldots \mu_s)} = \varphi_{\mu(s)}$ ,

  $\hookrightarrow$ Gauge transformation : $\delta_\epsilon \varphi_{\mu_1 \ldots \mu_s} = s\, \bar{\nabla}_{(\mu_1} \epsilon_{\mu_2 \ldots \mu_s)}$

Constr. : $\bar{g}^{\mu\nu} \bar{g}^{\rho\sigma} \varphi_{\mu\nu\rho\sigma\ldots} \equiv 0 \quad (s \geqslant 4)$ , $\quad \bar{g}^{\mu\nu} \epsilon_{\mu\nu\ldots} \equiv 0 \quad (s \geqslant 3)$

- $S^{Fr}[\varphi] = \int \mathcal{L}(\varphi, \bar{\nabla}\varphi)$ , $\qquad \dfrac{\delta S^{Fr}}{\delta \varphi_{\mu(s)}} =: G^{\mu(s)} \approx 0$

  $\nabla^{\mu_1} G_{\mu_1 \mu_2 \ldots \mu_s} \propto \bar{g}_{(\mu_2 \mu_3} \nabla^\alpha G'_{\mu_4 \ldots \mu_s)\alpha}$ Noether identity

# AdS / CFT & open problems

$\lambda \sim \left(\frac{R^2}{\alpha'}\right)^2$

$\lambda \to 0$

$\boxed{HS_4 / CFT_3}$ [Sezgin-Sundell, Klebanov-Polyakov]

| BC on $\varphi$ | type A | type B |
|---|---|---|
| $\Delta = 1$ | UV fixed-pt Free singlet. theory $CFT_3$ | Gross-Neveu model critical |
| $\Delta = 2$ | Critical O(N) model | Free Fermions $CFT_3$ |

$R \ll l_s$ , $N \to \infty$

$\updownarrow$

$\lambda \ll 1$ , $N \to \infty$

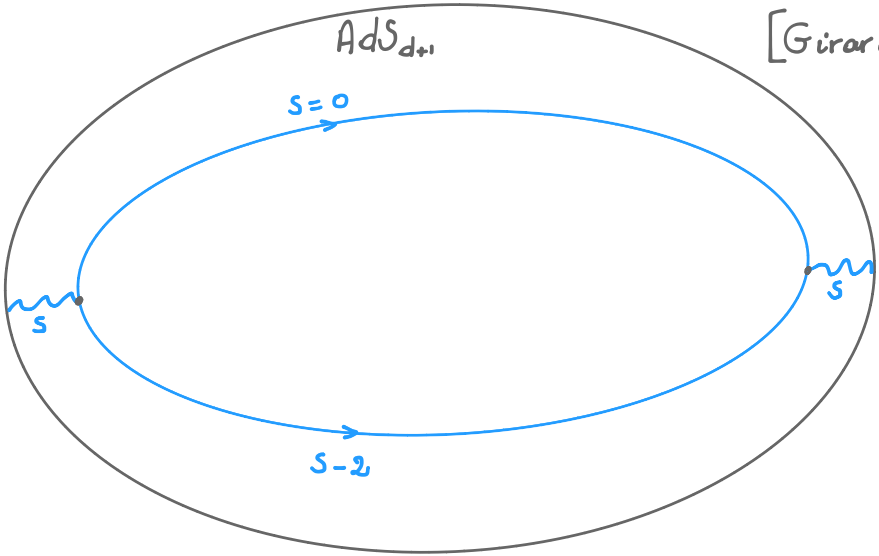
$$\frac{G}{R^2} \sim \frac{1}{N}$$

where $G$ = Newton's.

$\boxed{HS_3 / CFT_2}$ Prokushkin-Vasiliev $\longleftrightarrow$ Minimal model $CFT_2$

[Gaberdiel-Gopakumar]

When bulk scalar field in $\Delta = 2$ BC,




[Girardello - Porrati - Zaffaroni]

- Boundary CFT: $\partial^{\mu} J^{(s)}_{\mu \nu (s-1)} = \frac{1}{\sqrt{N}} \partial_{\nu} J^{(0)} \cdot J^{(s-2)}_{\nu (s-2)}$

- Bulk: Gives mass to $s > 2$ fields, perturbatively. Spin fields $s \leq 2$ protected.