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Generation of high frequency trains of chirped soliton-like pulses in inhomogeneous and cascaded active fiber configurations

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ABSTRACT

We present a theoretical formalism of description of soliton-like pulse generation in longitudinally inhomogeneous active optical fibers. Dynamics of the wave packet during the nonlinear stage of the modulation instability is described for fibers with distributed gain and group velocity dispersion profiles. Generation of pico- and sub-picosecond pulse trains operating in THz repetition range from quasi-continuous pump wave is simulated for fibers of different inhomogeneity and for cascade fiber configurations comprising homogeneous and inhomogeneous segments. Pulse train generation is shown to occur in resonant manner once the gain and GDV distributed fiber profiles are perfectly matched.

1. Introduction

Generation of THz ultra-short pulse trains at terahertz repetition rate and higher is among the current topical technological challenges. The induced modulation instability (MI) is an effective way to generate short optical pulses in optical fibers with a precise control of the pulse repetition rate [1–5]. In this process [3,4] a harmonic modulation of the input wave packet (WP) transforms into a train of ultrashort pulses separated by the modulation period. Commonly, dynamics of the generated pulses exhibit periodic behavior and the maximum pulse peak power achievable through the process could slightly exceed the initial pump power level. To form a pulse train of a maximal contrast the WP has to pass the fiber length that is about ~1.5 times of the dispersion length [3]. When passing longer fibers, the pulse structure collapses and finally returns to the original WP state.

In order to generate a stable pulse train with peak powers much higher than the initial pump power level by the means of MI, the effective incoherent optical amplification should be somehow incorporated into the fiber configuration. With homogeneous nonlinear active fibers the possibilities of incoherent amplification of solitons are quite limited. As the soliton initial energy increases e fold, independently of the way it is amplified, significant distortions of the soliton's shape and spectrum occur due to generation of non-solitonic components. As a result, the nonlinear WP loses its soliton properties becoming structurally unstable. For a long time, such behavior of the amplified soliton was commonly accepted as the only possible evolution. However, in [6–8], an efficient amplification of the optical soliton-like pulse (SLP) without modification of its shape has been demonstrated in situation when the initial SLP phase is a parabolic function of time and the gain increment is a hyperbolic function of a distance. Interaction of such frequency-modulated (FM) pulses becomes entirely elastic due to self-matching of the SLP phase and medium gain.

One of the main obstacles impeding an experimental realization of such an ''ideal'' SLP amplification is the need to implement the required gain profile along the fiber length. It has been shown, however, in [8–12] that for the effective SLP amplification one can use the fibers not only with a hyperbolic, but, practically, any chosen gain profile. The only condition for that is the distribution of the group velocity dispersion (GVD) along the fiber matching the given gain profile. The fibers with W-like cross section profile of the refractive index and anomalous GVD gradually decreasing along the fiber length seem to be a good candidate for this application. Such fibers could be designed and manufactured with the given GVD distribution along the fiber length ensuring negligible third-order dispersion parameter. Tuning of the GVD profile in these fibers is typically achieved through a controllable change of the fiber cross-section area. The current fiber drawing technology allows to control GVD slope within a significant range of values variation of the fiber diameter over the whole fiber length very small (typically <3%).

In the present work, we demonstrate generation of high-frequency trains of ultrashort pulses with the peak powers significantly exceeding the power of the initial pump-wave. The technique employs the

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nonlinear stage of MI (modulation amplitude is comparable with pump wave power) in active inhomogeneous optical fibers and fiber cascades enabling resonant generation of picosecond and sub-picosecond pulses once the initial WP parameters (power and chirp) and GDV distribution profile in the fiber are perfectly matched. In particular, the performed numerical simulations based on the Schrödinger equations explore the effect of the GVD distribution profile and active fiber gain spectrum width on formation of pico- and sub-picosecond laser pulse trains with THz repetition rate directly from the modulated CW wave.

It is worth noting that the possibility of formation, stable propagation and temporal compression of solitonic trains in inhomogeneous optical fiber configurations has been considered in [13–19]. Commonly, for formation of high repetitive rate pulse trains through modulation instability in an optical fiber a superposition of two monochromatic waves of nearly equal amplitudes and of two different frequencies are used as an input wave [15]. In contract to the previous works, while still employing inhomogeneous GVD fibers for high-frequency train generation, in this paper, we demonstrate resonant generation of frequency-modulated pico- and sub-picosecond SLP directly from a quasi-continuous initial wave. In particular, we highlight formation of a linear pulse chirp (parabolic phase) during the first stage of the pulse train development that makes possible their further effective soliton-like amplification up to pulse peak powers exceeding the power level of the initial wave by orders of magnitude. An important advance of the proposed method is the possibility of pico- and sub-pico-second periodic pulse train generation from the initially modulated wave with a very small modulation depth ($m \ll 10^{-3}$) that is orders of magnitude lower that the initial modulation depth ($m \sim 0.5-1$) used in early experiments mentioned above [14,15]. Potentially, it opens the way to produce ultrashort pulses through resonant MI arising from the spontaneous noise always present in the initial continues pump wave ($m \to 0$).

2. Dynamics of SLP amplification in inhomogeneous active fibers

Let us consider the dynamics of the optical WP propagating in an amplifying inhomogeneous optical fiber [20,21]. The electric wave field is given as

$$\mathbf{E}(t,r,z) = \frac{\mathbf{e}}{2}U(r,z)\left\{A(t,z)\exp\left[i\left(\omega_0 t - \int_0^z \beta'(\xi)d\xi\right)\right] + K.C.\right\}, \quad (1)$$

where $\mathbf{e}$ is the polarization unit vector, $U(r,z)$ is the radial field distribution along the fiber, ω_0 is the WP carrier frequency, β' is the real part of the propagation constant. The nonlinear Schrödinger equation [1–3] describing the temporal envelope amplitude $A(t,z)$ with the coefficients depending on the longitudinal coordinate could be presented as:

$$\frac{\partial A}{\partial z} - i\frac{D_2(z)}{2}\frac{\partial^2 A}{\partial \tau^2} + iR(z)|A|^2 A = \gamma(z)A. \quad (2)$$

Here, τ is the time in the running coordinate frame, $D_2(z) = (\partial^2\beta'(z)/\partial\omega^2)_0$ is GVD, $R(z)$ and $R(z)$ are the coefficient of Kerr nonlinearity and effective gain increment, respectively:

$$\gamma(z) = g(z) - (\partial S_m/\partial z)/2S_m - K(z), \quad (3)$$

where $g(z)$ is the gain increment determined by the optical fiber material, S_m is the effective fiber mode area [1,3,4] and $K(z)$ is the fiber loss coefficient. The energy of the pulse propagating along the fiber is determined by the material gain increment as:

$$W(z) = W_0\exp\left(2\int_0^z \gamma(\xi)d\xi\right), \quad (4)$$

where W_0 is the input pulse energy.

For a fiber with the constant anomalous GVD and nonlinearity, and the material gain increment distributed along the fiber length as

$$\gamma(z) = \gamma_0/(1-2\gamma_0 z) \quad (5)$$

the pulse with a hyperbolic secant profile amplified while $2\gamma_0 z < 1$ is a solution described by Eq. 2 [7,8]:

$$A(\tau,z) = A_0\frac{\text{sech}(\tau/\tau_s)}{1-2g_0 z}\exp\left(i\frac{\alpha_0\tau^2 - \Gamma z}{1-2g_0 z}\right). \quad (6)$$

Here, $\tau_s = \tau_0(1-2\gamma_0 z)$, $\Gamma = \gamma_0/2\alpha_0\tau_0^2$ and we assume that $D_2 R < 0$ and $2\Gamma = |D_2|/\tau_0^2 = R|A_0|^2$.

Nonlinear WPs described by Eq. (6) possess the properties of elastic interaction that are important for practical applications and referred to as "light" FM solitons [7,8].

The dispersion and non-linearity coefficients varying along the fiber length could be expressed in dimensionless variables: $d(z) = D_2(z)/D_0$ and $r(z) = R(z)/R_0$, where D_0 and R_0 are the values at the fiber input. With $\eta(z) = \int_0^z d(\xi)d\xi$ and $C(\tau,z) = \sqrt{r/d}\,A(\tau,z)$ Eq. (2) transforms into the form:

$$\frac{\partial C}{\partial \eta} - i\frac{D_0}{2}\frac{\partial^2 C}{\partial \tau^2} + iR_0|C|^2 C = \gamma_{ef}(\eta)C. \quad (7)$$

Thus, the problem of nonlinear pulse propagation along the optical fiber with the material parameters varying along the fiber length is reduced to the problem of pulse propagation along the fiber with constant dispersion D_0 and nonlinearity R_0 but with the variable effective gain $\gamma_{ef}(\eta)$:

$$\gamma_{ef}(\eta) = \frac{g(\eta) - K(\eta)}{d(\eta)} - \frac{1}{2}\frac{\partial}{\partial\eta}\ln\frac{\widetilde{S}_m(\eta)d(\eta)}{r(\eta)}, \quad (8)$$

where $\widetilde{S}_m = S_m(\eta)/S_m(0)$ is the normalized effective mode area.

Similar to Eqs. (2), (7) has a solution in the form of the amplified frequency-modulated (FM) soliton under conditions that $D_2(\eta)R(\eta) < 0$ and the effective gain increment (8) expressed as $\gamma_{ef}(\eta) = b_0/(1-2q\eta)$, where $b_0 = \gamma_{ef}(0)$. In this case, the solution of Eq. (7) is:

$$C(\tau,\eta) = C_0\frac{\text{sech}(\tau/\tau_s)}{1-2b_0\eta}\exp\left(i\frac{\alpha_0\tau^2 - \Gamma_0\eta}{1-2b_0\eta}\right), \quad (9)$$

where $\tau_s = \tau_0(1-2b_0\eta)$ and $\Gamma_0 = b_0/2\alpha_0\tau_0^2$, and the included parameters are linked as $\Gamma_0 = |D_0|/2\tau_0^2 = R_0|A_0|^2/2$.

The GVD profile supporting SLP formation could be expressed as

$$D_2(z) = D_0 f(z)\exp\left(-2b_0\int_0^z f(\xi)d\xi\right) \quad (10)$$

where $f(z) = F(z)\exp\left(2\int_0^z \gamma(\xi)d\xi\right)$ and $F(z) = R(z)S_m(0)/R_0 S_m(z)$. In the limiting case $\gamma(z) = 0$ and $F(z) = 1$ the dispersion profile reduces to $D_2(z) = D_0\exp(-2b_0 z)$, where $D_0 < 0$, and $\alpha_0 > 0$. The duration and chirp of the secant-hyperbolic FM soliton satisfy the relation $\tau_s(z)\alpha(z) = \tau_0\alpha_0$, where

$$\tau_s(z) = \tau_0\exp\left(-2b_0\int_0^z f(\xi)d\xi\right), \quad \alpha(z) = \alpha_0\exp\left(2b_0\int_0^z f(\xi)d\xi\right) \quad (11)$$

and $b_0 = \alpha_0|D_0|$. The energy of the pulse is described by Eq. (9) is $W_s = \tau_0|A_0|^2 = |D_0|/R_0\tau_0$.

It is worth noting that modern optical fiber technology allows manufacturing of fibers with negligible losses ($K(z) = 0$) and arbitrarily controllable GVD distribution along the fiber length (for example, single-mode fibers with W-type refractive index profile) [22,23,20,24]. For such fibers the effective mode area and nonlinearity coefficient can be considered almost unchangeable over the entire fiber length making our assumption $F(z) = 1$ used for delivering of Eq. (11) justifiable.

In the case of a constant gain increment $g(z) = g_0$ the function $f(z) = \exp(2g_0 z)$ and the expressions for the GVD providing an amplification of the FM pulse and its duration take the forms:

$$D_2(z) = -|D_0|\exp\left[-\frac{\alpha_0|D_0|}{g_0}\left(\exp(2g_0 z) - 1\right) + 2g_0 z\right], \quad (12)$$

$$\tau(z) = \tau_0\exp\left[-\frac{\alpha_0|D_0|}{g_0}\left(\exp(2g_0 z) - 1\right)\right]. \quad (13)$$

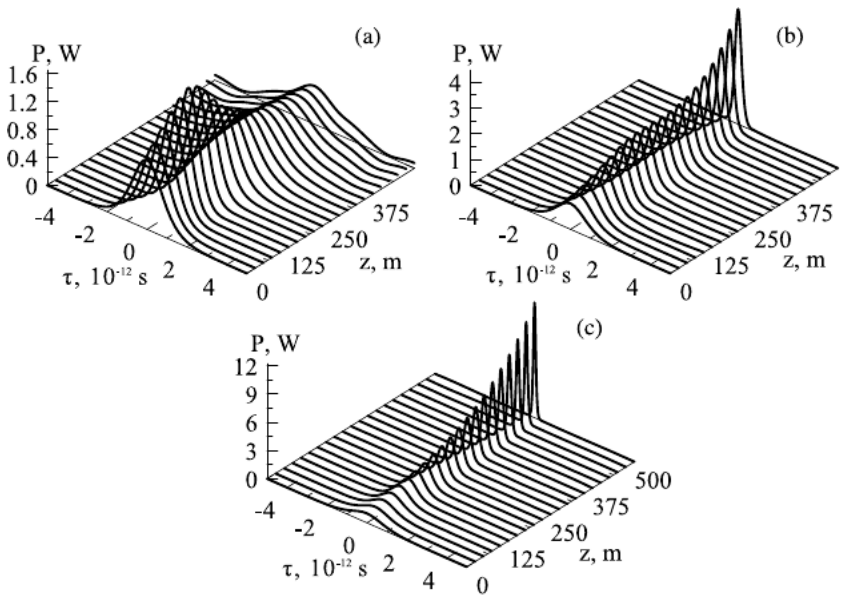

Fig. 1. Pulse evolution in the inhomogeneous fiber calculated for $R_0 = 10^{-2}$ W^{-1} m^{-1}, $D_0 = -10^{-26}$ s^2 m^{-1}, $b = (0, 3, 5) \cdot 10^{-3}$ m^{-1} (a, b, c), $\alpha_0 = 2.5 \cdot 10^{23}$ s^{-2}, $\tau_0 = 10^{-12}$ s, $P_0 = 1$ W.

In the low gain limit ($2g_0 z \ll 1$) with $\exp(2g_0 z) \approx 1 + 2g_0 z$ Eqs. (12), (13) are reduced to

$$D_2(z) \approx -\left|D_0\right| \exp(-2\alpha_0 \left|D_0\right| z), \qquad \tau(z) \approx \tau_0 \exp(-2\alpha_0 \left|D_0\right| z). \tag{14}$$

3. SLP evolution in inhomogeneous passive fibers

Fig. 1 demonstrates evolution of the FM pulse during its propagation in the passive optical fiber with the homogeneous nonlinearity $R_0 = 10^{-2}$ (W m)$^{-1}$ and exponentially decreasing anomalous GVD $D_2(z) = D_0 \exp(-bz)$, where $D_0 = -10^{-26}$s^2/m. The input pulse amplitude is set as

$$A(0, \tau) = A_0 \mathrm{sech}(\tau/\tau_0) \exp(i\alpha_0 \tau^2) \tag{15}$$

where $\alpha_0 = 2.5 \cdot 10^{23}$ s^{-2}, $\tau_0 = 10^{-12}$ s and $P_0 = \left|A_0\right|^2 = 1$ W. The GVD distribution parameter varies as $b = (0, 3, 5) \cdot 10^{-3}$ m^{-1} [Fig. 1(a, b, c)]. One can see that in a homogeneous fiber [Fig. 1(a)] the pulse evolution exhibits periodically repeating broadening and compression with a gradual temporal smearing of pulse causing a decrease of the peak power. In the inhomogeneous fiber, the pulse compresses more rapidly. The closer the initial pulse parameters to the parameters of the ideal FM soliton, the higher the degree of the compression is. The most effective pulse compression (down to sub-picosecond pulses) and the highest pulse amplitude are achieved when the initial chirp and distribution parameters satisfy the Eq. (14) $b = \alpha_0 \left|D_0\right| = 5 \cdot 10^{-3}$ m^{-1} [Fig. 1(a)]. It is worth noting that GVD plays the important "damping" role in the pulse dynamics: a small GVD value in conjugation with a large nonlinearity lead to a decay of a single WP caused by higher-order dispersion and nonlinear effects.

The described technique could be applied to increase the contrast of picosecond pulses in high-repetition rate pulse trains. The dynamics of the pulse train differs from the dynamics of a single pulse due to inter-pulse interaction. Fig. 2 shows the train evolution of FM sub-picosecond pulses described by Eq. (9) introduced with the period of $T = 5\tau_0$. One can see that in the case of homogeneous GVD, for the considered fiber length, a weak pulse compression is followed by broadening of the individual pulses and decreasing of their peak powers [Fig. 2(a)]. For the inhomogeneous GVD the pulses exhibit rapid compression and their amplitude monotonically increases with the fiber length. The optimal pulse compression is achieved for pulse trains presented in Fig. 2(c), when individual pulses are almost ideal FM solitons described by Eqs. (10)–(12).

In general, in order to describe the dynamics of a sub-picosecond pulses in the inhomogeneous fiber we have to take into account the dispersion of the third (and even higher) order that could significantly affect the pulse shape and cause pulse decay for short pulse durations involved. At the fiber input, the effect of third order dispersion can be quite low but starting from the certain length its effect becomes significant. Therefore, for the perfect amplification of the FM soliton, the condition $\left|D_3(z)\right| < \left|D_2(z)\right| / \Delta\omega(z)$ has to be held over the whole fiber length, where D_3 is the third-order dispersion parameter, and $\Delta\omega(z)$ is the WP spectral width. With a decrease of the pulse duration (and spectral broadening) and the decrease of the absolute GVD value along the fiber, the conditions for amplification of the FM soliton may not be satisfied if the fiber length is too long.

4. Generation of SLP trains in inhomogeneous passive fibers

As we mentioned initially, generation of pulse trains (even of low-power) with terahertz repetition rate is currently a topical but technically complicated scientific problem. In the previous sections, we have demonstrated formation and compression of ultrashort FM laser pulses in inhomogeneous fibers with THz repetition rate when the parameters of a single pulse are close to those of the ideal FM soliton described by Eqs. (10)–(12).

In this section, we demonstrate short pulse trains generation through the induced MI from a quasi-CW pump wave and with the peak powers significantly exceeding the pump power level. It is well known that in its nonlinear stage the MI breaks a modulated continuous wave into a train of soliton-like pulses possessing a linear chirp. Dynamics of MI in an inhomogeneous fiber is obtained by numerical simulations of Eq. (2) employing the split-step Fourier method [1]. We assume that a weakly modulated WP is introduced into the fiber:

$$A(0, \tau) = \sqrt{P_0} \left[1 + m \cos(\Omega_{\mathrm{mod}} \tau)\right], \tag{16}$$

where $m \ll 1$ is the modulation depth and Ω_{mod} is the modulation frequency corresponding to the MI gain spectrum peak $\Omega_{\mathrm{mod}} = \sqrt{2RP_0/\left|D_0\right|}$. Fig. 3 shows the evolution of the modulated quasi-continuous WP determined by Eq. (16) at the fiber input. Numerical simulations of Eq. (2) have been performed with the following parameters: $D_0 = -10^{-26}$ s^2/m, $P_0 = 1$ W, $b = (0, 3, 5, 7) \cdot 10^{-3}$ m^{-1} [Fig. 3(a, b, c, d)], $m = 0.01$. In the homogeneous optical fiber the process of pulse

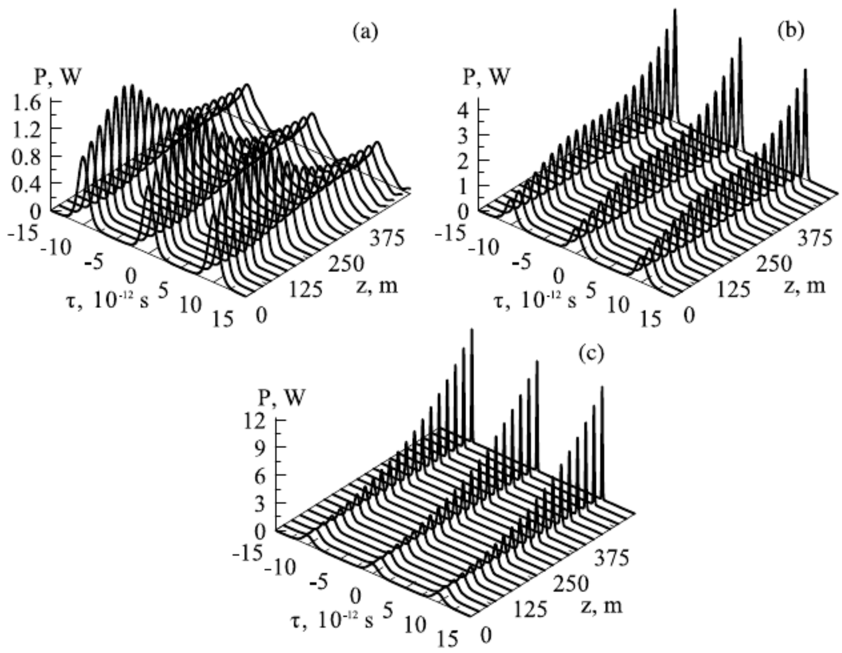

Fig. 2. Pulse train evolution in the inhomogeneous passive fiber. The pulse period is $T = 5\tau_0$. Other calculation parameters are the same as in Fig. 1.

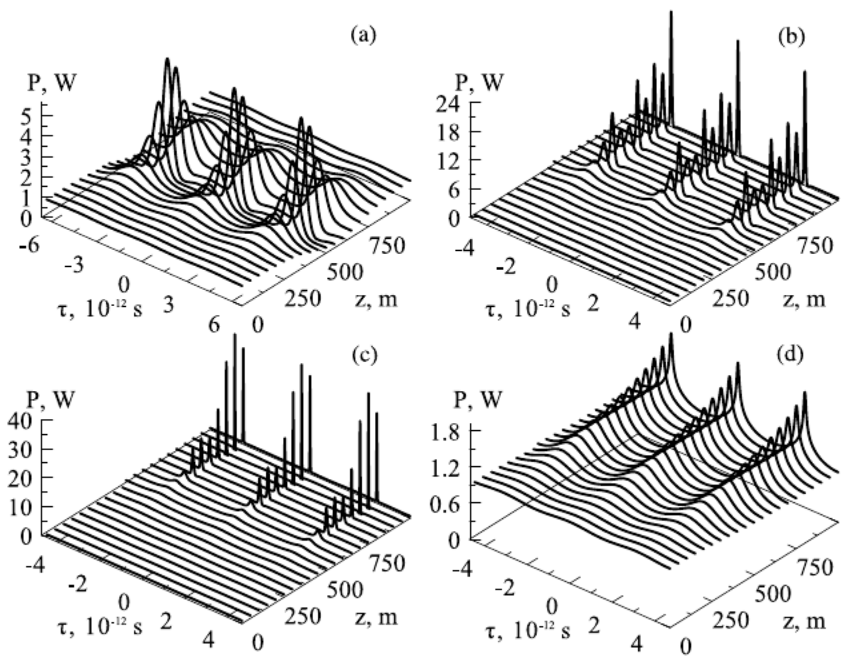

Fig. 3. Evolution of weakly modulated ($m = 0.01$) continues wave calculated for different b values: $b = (0, 3, 5, 7) \cdot 10^{-3}\,\mathrm{m}^{-1}$ (a, b, c, d), $P_0 = 1\,\mathrm{W}$.

compression during the nonlinear MI stage is replaced by the process of pulse broadening. Such a process is periodic and the corresponding structure is referred to as a breather — at the end of the cycle the generated pulses transform into the initial modulated continuous wave. In the inhomogeneous fibers the process of pulse train generation is no longer reversible due to the GDV decreasing along the fiber length. After spectral broadening, the generated train of pulses does not return to the initial state of the modulated continuous wave. However, the duration of the pulses reduces not monotonically, the pulse amplitude increases and period of pulsing decreases. An increase of the GVD distribution parameter b leads to much stronger compression of the pulses [Fig. 3(b, c)]. However, for too high rate of the GVD decrease breaking of the modulated wave into a train of ultra-short pulses has not sufficient time to occur within the given fiber length [Fig. 3(d)].

Fig. 4 shows temporal evolution of the pulse phase for different values of the GVD distribution parameter and the fiber length of $L = 1000\,\mathrm{m}$ (the other parameters are the same as in Fig. 3). One can see that in an inhomogeneous fiber the frequency modulation of the generated pulses becomes linear providing the positive phase profile which is very close to the parabolic profile in the center of the generated pulse. This provides the means for quite straightforward soliton compression along the whole fiber length for the generated FM pulses. As it has been mentioned above, the increase of the GVD distribution slope accelerates pulse compression. However, at high GDV distribution parameters the

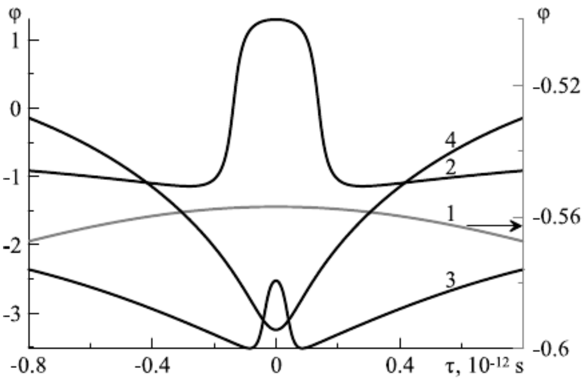

Fig. 4. Temporal phase profiles of the modulated continues wave calculated for different b values: $b = (0, 3, 5, 7) \cdot 10^{-3}\,\mathrm{m}^{-1}$ (curves 1–4), $P_0 = 1\,\mathrm{W}$, $L = 1000\,\mathrm{m}$.

GVD decreases too fast and breakage of the modulated wave into a train of ultrashort pulses does not have sufficient no time to occur in the given fiber length. Besides, at low GVD values ($|D_2| < 10^{-27}\,\mathrm{s}^2/\mathrm{m}$) the effect of higher order dispersion and nonlinearities becomes significant impairing stable generation of ultrashort pulses.

5. Generation of SLP trains in cascade fiber configurations

More stable and efficient generation of FM SLP (with shorter durations and higher peak powers) can be achieved using cascade fiber configurations. As an example, one could consequently combine two passive fibers — a homogeneous fiber with a relatively large (in absolute values) anomalous dispersion and an inhomogeneous fiber with a rapidly decreasing anomalous dispersion. In this configuration, the first fiber forms a SLP with a linear chirp and the second provides its efficient compression. Fig. 5 presents an evolution of a weakly modulated ($m = 0.01$) WP in the homogeneous [Fig. 5(a)] and inhomogeneous [Fig. 5(b)] optical fibers and in a two-cascade fiber configuration [Fig. 5(c)] simulated for the input power of $P_0 = 1\,\mathrm{W}$, modulation frequency $\Omega_{\mathrm{mod}} = \sqrt{2RP_0/|D_0|}$; Ω_{m} and GVD distribution over the fiber length as

$$D_2(z) = \begin{cases} D_0, & z < z_0, \\ D_0 \exp\left[-b(z - z_0)\right], & z \geq z_0, \end{cases} \tag{17}$$

where $z_0 = 500\,\mathrm{m}$ is the length of the homogeneous fiber section with GVD of $D_0 = -10^{-26}\,\mathrm{s}^2/\mathrm{m}$, $b = (0, 5) \cdot 10^{-3}\,\mathrm{m}^{-1}$ [Fig. 5(a, b)]. One can see that such a fiber cascade that combines the fiber with a constant and relatively large (in absolute values) anomalous dispersion and the second fiber with the inhomogeneous GVD distribution, provides stable generation of pulses with the peak power exceeding the peak power achievable individually in the fibers of the identical length.

The section of the homogeneous fiber plays an important role in the cascaded generation of a chirped pulse train. Fig. 6 shows the time dependence of the single pulse phase at the input of the inhomogeneous fiber section [Fig. 6(a)] and the distribution of the maximum pulse power over the fiber length [Fig. 6(b)] calculated for different lengths of the homogeneous fiber section. Comparison of the results allows us to conclude that formation of the breather with the maximal peak power (i.e. with the highest pulse/background level contrast) and pulse phase formation in the first fiber are both important for the effective FM pulse train generation in the cascaded configuration. On other hand, generation of short FM breathers in inhomogeneous fibers is much less efficient for those breathers whose peak powers just slightly exceed (less than 10%) the power of the initial pump wave. Thus, the compression efficiency for the train of pulses that passes the first fiber of the short length (300 m — the black curve) is considerably lower than that of the pulses with a higher peak power, although these pulses have nearly linear chirp.

The use of two-cascades fiber configuration consisting of the active homogeneous fiber segment and a segment of inhomogeneous passive fiber with a required GVD profile, allows to generate single pulses of much higher peak powers. Moreover, the active fiber section with a broadband gain spectral width enables generation of regular high-frequency pulse trains even with a weakly modulated CW ($m = 10^{-4}$) that is close to a spontaneous noise level attributed to the initial CW. Fig. 7 presents the evolution of the MI in two-cascades fiber configuration comprising homogeneous active fiber [Fig. 7(a)], and inhomogeneous passive fiber [Fig. 7(b, c, d)]. One can see that at low modulation depth m of the initial CW a weak MI development in the

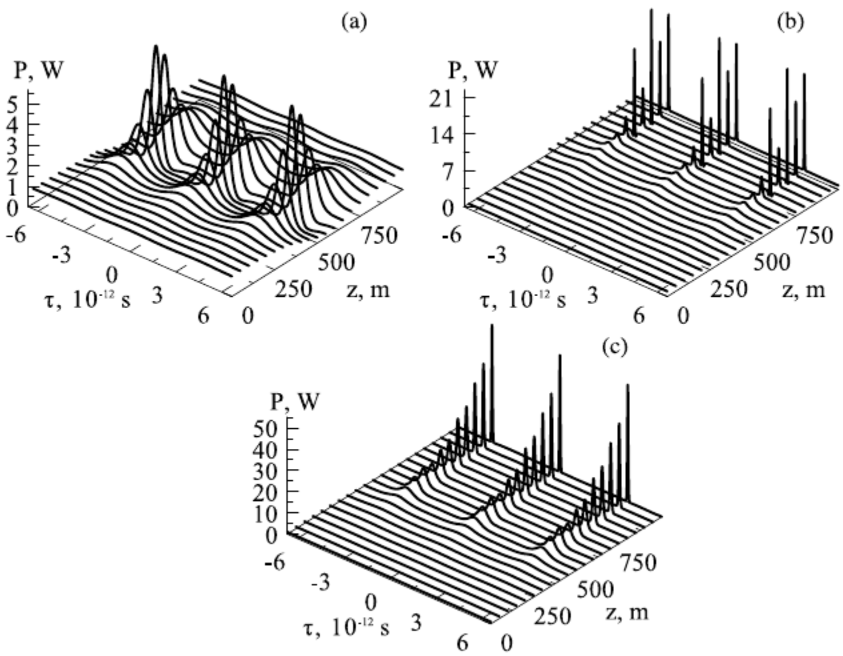

Fig. 5. Evolution of a weakly modulated ($m = 0.01$) continues wave calculated in the homogeneous (a) and inhomogeneous (b) passive optical fibers and in a passive two-cascades fiber configuration (c) calculated for the input power of $P_0 = 1$ W, modulation frequency $\Omega_{\mathrm{mod}} = \sqrt{2RP_0/|D_0|}$, $z_0 = 500$ m and $b = 5 \cdot 10^{-3}$ m^{-1}.

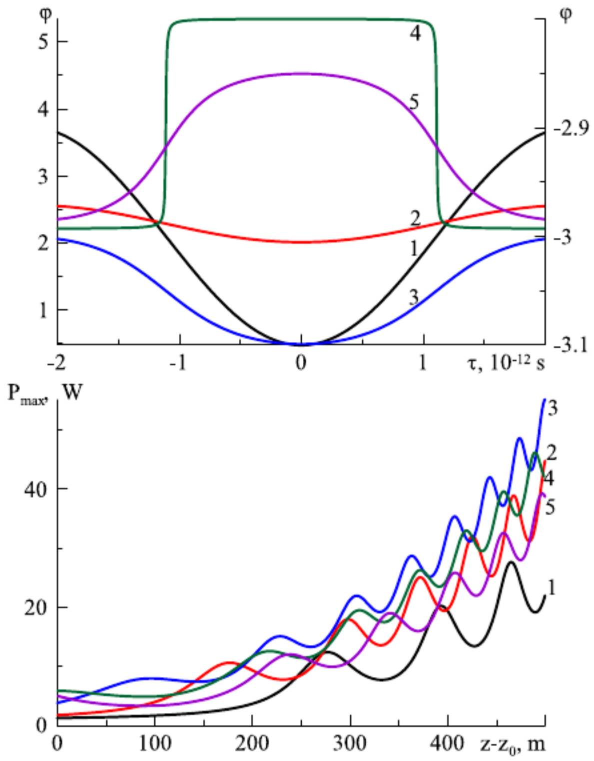

Fig. 6. (a) Time dependence of the pulse phase at the input of the inhomogeneous fiber section and (b) the distribution of the maximum pulse power over the inhomogeneous fiber length (b) calculated for different lengths of the homogeneous fiber section: $z_0 = (3; 4; 5; 5.65; 6) \cdot 10^2$ m — curves (1; 2; 3; 4; 5).

homogeneous active fiber causes generation of pulse trains of a low contrast with the peak amplitudes just slightly exceeding the average power level. However, passing less than 100 m of the inhomogeneous GVD fiber attached to the active homogeneous fiber this pulse train transforms into a high-contrast ultrashort pulse train with the THz repetition rate [Fig. 7(b, c, d)].

The efficiency of ultrashort pulse generation depends on relations between the parameters of the active fiber amplifier and the inhomogeneous passive fiber. Practically, for any input CW power we can find an active fiber with appropriate parameters (length, gain increment, GVD) and a passive fiber with a proper decreasing GVD profile to provide generation of a high-frequency pulse train with the preset peak power and repetition rate. Fig. 8 shows the effect of the active fiber gain line width on the dynamics of MI in the cascaded fiber configuration. Fig. 8(a, c, e) and (b, d, f) correspond to propagation in a homogeneous active fiber and inhomogeneous passive segments, respectively. One can see that with the optimal GVD profile parameter $b = 2.5 \cdot 10^{-2}$ m^{-1} of the passive fiber segment the dynamics of the ultra fast pulse train changes significantly as the gain line width of the homogeneous active fiber varies as $\Delta\omega_L = (0.1, 1, 10) \cdot 10^{12}$ s^{-1} [Fig. 8(b, d, f)]. Typically, the gain linewidth should be much wider than the inverse duration of the generated SLP (and pulse train repetitive rate as well). When the dispersion of the inhomogeneous segment and the gain increment of the active fiber are linked through Eq. (14), the MI demonstrates explosively fast growth as soon as the active fiber segment provides amplification within the spectral range wider than $\Delta\omega_L = 10^{12}\,c^{-1}$. In this context, the use of the Raman fiber amplifiers providing the gain line width of $\Delta\omega_L \sim 10^{13}\,c^{-1}$ [25] are very promising for the efficient generation of femtosecond SLPs with a repetition frequency over 1 THz that is cannot be achieved with the active fiber configuration.

6. Conclusion

The performed analysis demonstrates that in the fiber with decreasing anomalous GDV the MI dynamics strongly depends on the slopes, dispersion and gain increment change along the fiber length. It is shown that the induced MI can be used to generate high-frequency trains of pico- and sub-picosecond optical pulses at THz repetition rates. The generated pulses acquire the phase temporal profile close to parabolic. In this case, one can talk about formation of so-called frequency-modulated solitons, which in contrast to the "classical" fundamental

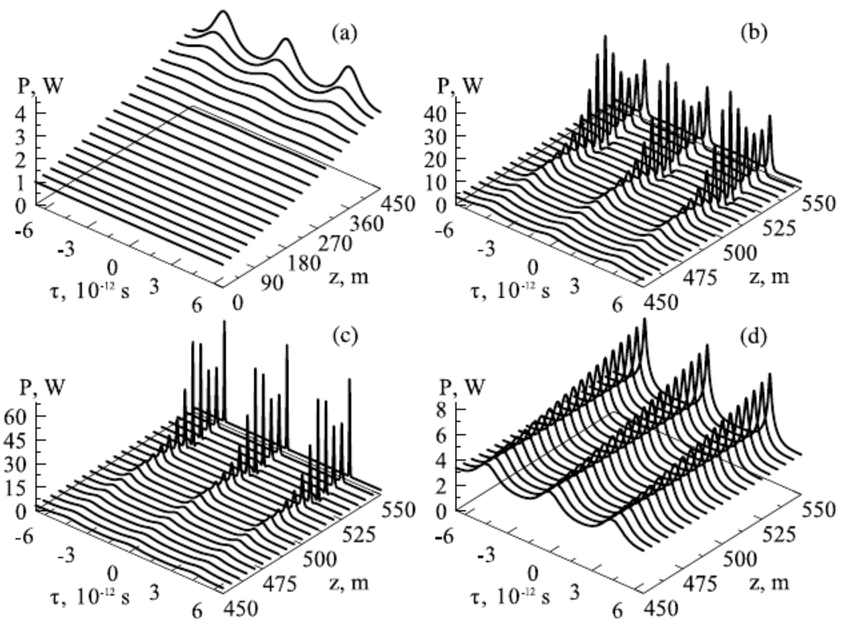

Fig. 7. Evolution of a weakly modulated ($m = 10^{-4}$) WP in a two-cascades fiber configuration comprising homogeneous active (a) and inhomogeneous passive (b–d) optical fibers simulated for the input power of $P_0 = 1$ W, modulation frequency $\Omega_{mod} = \sqrt{2RP_0/|D_0|}$, $R = 10^{-2}$ W^{-1} m^{-1}, $z_0 = 450$ m. Active and passive fiber parameters are $D = d_{20} - ig_0/\Delta\omega_L^2$, $g_0 = 1.5 \cdot 10^{-3}$ m^{-1}, $\Delta\omega_L = 10^{13}$ s^{-1} and $D = d_{20} \cdot \exp\left(-b\left(z - z_0\right)\right)$, $b = (1,25; 2,5; 5) \cdot 10^{-2}$ m^{-1} (b; c; d), respectively, $d_{20} = -10^{-26}$ s^2 m^{-1}.

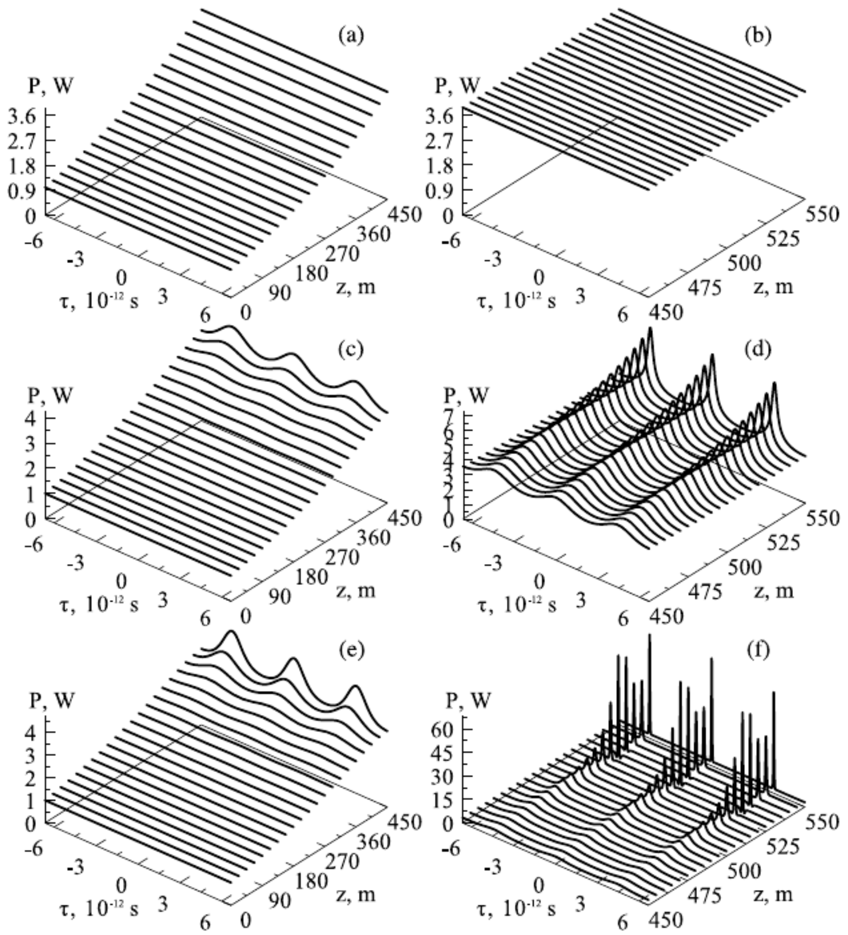

Fig. 8. Evolution of a weakly modulated ($m = 10^{-4}$) WP in a two-cascades fiber configuration comprising homogeneous active (a, c, e) and in homogeneous passive (b, d, f) optical fibers simulated for $b = 2.5 \cdot 10^{-2}\,\mathrm{m}^{-1}$ (b; c; d) and $\Delta\omega_L = (0.1; 1; 10) \cdot 10^{12}$ (a, b; c, d; e, f). Other parameters are the same as in Fig. 7.

solitons, have no restrictions for further increase of their energies and peak powers. We have shown that the efficiency of the FM soliton train generation can be significantly enhanced in a cascaded fibers configuration combining a homogeneous fiber with a relatively large (in absolute values) anomalous dispersion and a inhomogeneous fiber with a rapidly decreasing anomalous dispersion. As a result, the generation of femtosecond pulse trains with ultra-high repetitive frequency higher than 1 THz can be obtained. Fast formation of pulse trains from weakly modulated CW ($m = 10^{-4}$) in the configuration comprising a broadband fiber amplifier and an inhomogeneous passive fiber segment suggests that such a system would also support the effective rise of spontaneous modulation instability directly from the noise (when $m \to 0$). The obvious advantage of the proposed technique is the possibility to generate pulse trains with a sup THz repetition rate (potentially, higher than 10 THz). Importantly, the contrast of the generated pulses (the ratio between the pulse peak and average powers) is orders of magnitude higher than the pulse contract previously obtained in [13–19].

It is worth noting that similar results can be obtained with the use a single broadband amplification $\Delta\omega_L > 10^{13}\,c^{-1}$ fiber with the dispersion and gain profiles matching the Eq. (12). However, it seems to be technologically more challenging. So, the cascaded fibers configuration comprising the broadband fiber amplifier and inhomogeneous passive fiber present a good alternative to the quite complicated single amplifier fiber solution.

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