Hybrid linear observer for an activated sludge process with alternate phases

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Abstract: Alternate phase systems are widely applied to biological wastewater treatment. However the monitoring of these systems is delicate due to the lack of instrumentation and the fact that observability conditions might impair the construction of classical observers. This paper explores the use of a recently proposed hybrid linear observer (Tanwani *et al*, 2013). This observer allows to blend partial information from each phase of the switched process in order to reconstruct the evolution of unobservable states, whereas neither the aerobic nor the anoxic phases taken alone meets the usual observability conditions. The observer is tested in simulation with both linear and nonlinear process models.

Keywords: state estimation, hybrid linear observer, switched systems, wastewater treatment

PROCESS DESCRIPTION

The process under consideration is an activated sludge process with alternate phases. Nonlinear and linear models were developed in (C.S Gomez-Quintero, 2000, 2002, 2004), and some comparative simulation results are shown in Fig.1.

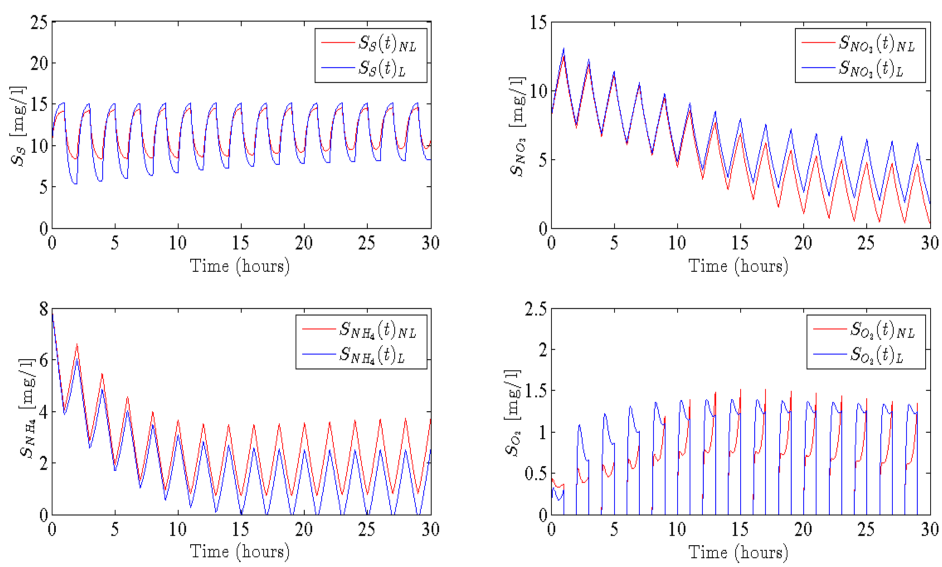

Figure 1. Comparison of linear and nonlinear model prediction over a period of 30 hours.

The usual instrumentation consists of sensors providing the dissolved oxygen concentration. Sensors for NH_4 and NO_3 concentrations are sometimes available in more advanced configurations but they are costly and difficult to maintain (Mulas *et al*, 2015). Alternatively, nitrogen compound concentrations are obtained through sampling and laboratory analysis.

In first approximation, it is interesting to represent the system as a switched linear system:

$$\dot{x} = A_q x(t) + B_q u(t), \qquad t \neq t_q \tag{1a}$$
$$x(t_q) = E_q x(t_q^-), \qquad q \geq 1 \tag{1b}$$
$$y(t) = C_q x(t), \qquad t \geq t_0 \tag{1c}$$

where $x(t) \in \mathbb{R}^n$ is the state, $y(t) \in \mathbb{R}^{d_y}$ the measured output, and $u(t) \in \mathbb{R}^{d_u}$ the input. The switching time $\{t_q\}$, $q \in \mathbb{N}$ is the time instant where the process moves from one phase to the next. In this formulation, a function exhibiting a discontinuity at time instant t_q is evaluated at t_q^- prior to the discontinuity, and at t_q just after the transition. $\tau_q := t_q - t_{q-1} = \tau = 1$h is the commutation period in our particular application example.

STATE ESTIMATION PROBLEM AND HYBRID LINEAR OBSERVER

Given a switched linear system and assuming both the persistence of the switching and its **determinability**, (Tanwani *et al*, 2013) introduced a hybrid linear observer based on the idea of **accumulating available information from individual subsystems**.

The minimal measurement configuration consists in measuring S_{O_2} in the aerobic phase and S_{NO_3} during the anoxic phase. Even though the system is not observable with this minimal sensor configuration, a hybrid observer design following the proposal of (Tanwani *et al*,2013) is possible. In fact, this observer is based on a weaker condition than the classical observability conditions: the determinability of the system which can be computed by means of the following theorem.
Let us denote by Q_q^m ($m \geq q$) the unobservable subspace for $[t_{q-1}, t_{m-1}^+)$. It can be shown that Q_q^m is computed recursively as follows:

$$\begin{aligned} \mathsf{Q}_q^q &= ker\, G_q, \\ \mathsf{Q}_q^k &= ker\, G_k \bigcap E_{k-1} e^{-A_{k-1}\tau_{k-1}} \mathsf{Q}_q^{k-1}, \quad q+1 \leq k \leq m \end{aligned} \tag{2}$$

G_q is the observability matrix defined by: $G_q := \begin{bmatrix} C_q & C_q A_q & \dots & C_q A_q^{n-1} \end{bmatrix}^T$
Theorem:
For a system (1) and given a switching signal $\sigma_{[t_0, t_{m-1})}$, the undeterminable subspace for $[t_0, t_{m-1})$ at t_{m-1} is given by Q_q^m of (2). Therefore, system (1) is $[t_0, t_{m-1})$ determinable if and only if :

$$\mathsf{Q}_1^m = \{0\}. \tag{3}$$

Measured variables		Determinability	Minimal commutation number
Aerobic Phase	Anoxic Phase		
$[S_{O_2}, S_S]$		×	∞
$[S_{O_2}, S_{NO_3}]$		✓	1
$[S_{O_2}, S_{NH_4}]$		×	∞
$[S_{O_2}]$	$[S_S]$	✓	2
$[S_{O_2}]$	$[S_{NO_3}]$	✓	2
$[S_{O_2}]$	$[S_{NH_4}]$	✓	2
$[S_{NO_3}]$	$[S_{NH_4}]$	×	∞

Table 1. Determinability of the linear model for two sensors configurations.

Table 1 shows some results of determinability analysis for two sensors configuration.

Hybrid Linear Observer Principle:
The hybrid linear observer dynamics are:

$$\dot{\hat{x}} = A_q\hat{x}(t) + B_q u(t), \qquad t \in [t_{q-1}, t_q], \quad t \neq \hat{t}_k \tag{4a}$$
$$\hat{x}(t_q) = \hat{x}(t_q^-), \qquad q \geq 1 \tag{4b}$$
$$\hat{x}(\hat{t}_k) = \hat{x}(\hat{t}_k^-) - \xi_k, \qquad k \geq 1 \tag{4c}$$

The correction vector ξ_k is computed at $\hat{t}_k = t_q + T_c$ where $T_c \leq \tau$ is introduced to take into account the computation time and/or eventual delays in measurements acquisition in real time application.

Lets N be the minimal number of switches to achieve system determinability. $N+1$ Luenberger observers are designed on the observable subspace of each of the $N+1$ previous phase. The information provided by those observers at the end of each phase are then combined at one instance for the computation of the correction vector ξ_k.
The observers gains are computed through classical pole-placement techniques such as to ensure a compromise between fast convergence and robustness to noise.

SIMULATION RESULTS

The designed observer is tested in simulation using either the same linear model as the process emulator or a more realistic nonlinear model. $T_c = 0.5h$.
The observer shows good performances when applied to a linear process emulator, as illustrated in Fig.2.

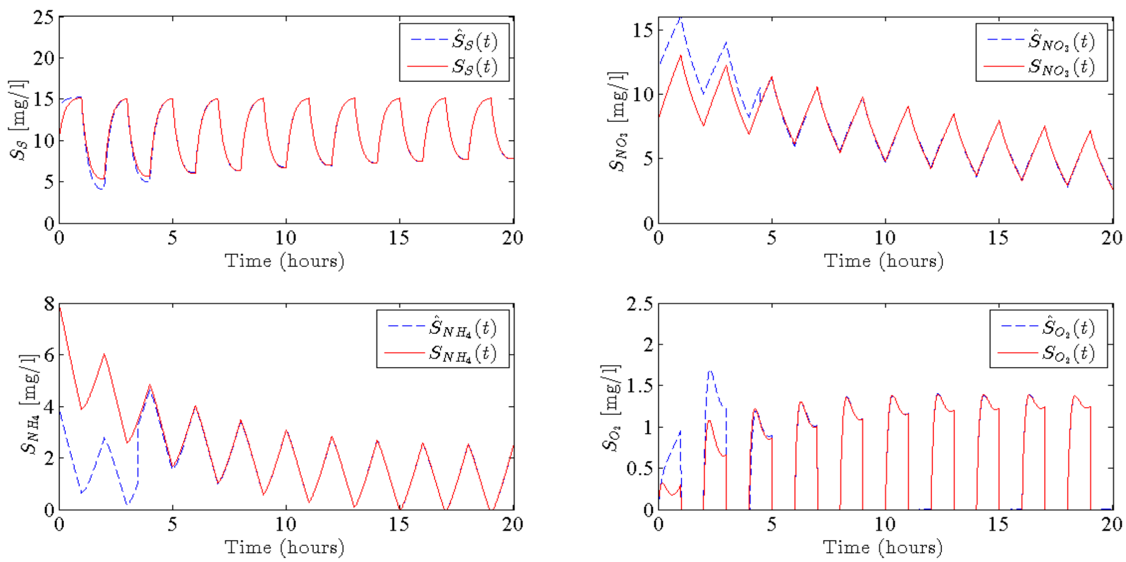

Figure 2. Hybrid observer tests with the same linear model over 20 hours: S_{O_2} is measured during the aerobic phase and S_{NO_3} during the anoxic phase.

Although the application of the observer to the nonlinear model seems less satisfactory at first sight (see Fig.3), the observer is still able to provide good estimates for the states of interest : S_{NH_4} and

S_{NO_3} .

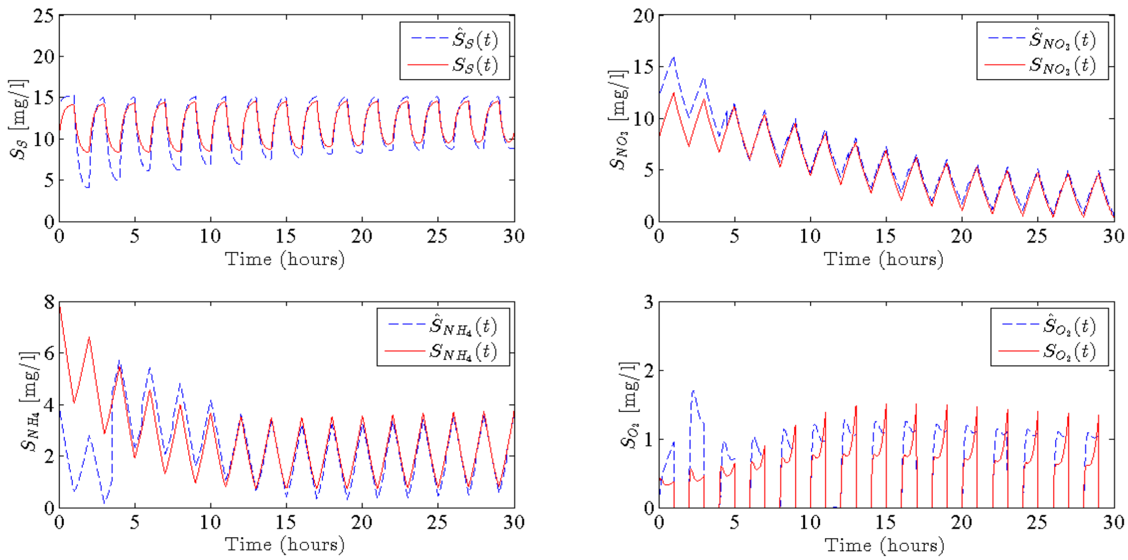

Figure 3. Hybrid observer tests with the nonlinear model over 30 hours: S_{O_2} is measured during the aerobic phase and S_{NO_3} during the anoxic phase.

FUTURE WORKS

The observer should be tested with actual experimental data.

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