Hairy Black Holes : Is Dark Matter a scalar field ?

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I. Introduction and motivations

Despite the unquestionable success of Einstein's General Relativity (GR) at the solar system scale, some **important phenomenons**, such as Dark Matter and Dark Energy (accounting together for almost 95% of the matter-energy content of the known universe), remains **beyond our understanding**. What is Dark Matter "made of" ? Where dœs Dark Energy "comes from" ? Over the last decades, there were numerous attempts to solve this problem. One of them is to **consider that the unrated phenomenons are due to unknown degrees of freedom** (that can be interpreted as new particles or as a new component in the description of gravity) and the most simple candidate for these degrees of freedom is a **scalar field** (generically named ϕ).

The **aim of this poster** is to present some general features of scalar-tensor theories of gravity and some results of my recent research related to black holes surrounded by scalar fields.

II. No (scalar) Hair Theorem

Surrounding a black hole with a scalar field is **a challenging problem**. In general, the simplest models are disfavoured, as illustrated by the following result due to Bekenstein :

Theorem 1 (Bekenstein) *Consider a stationary asymptotically flat black hole spacetime*

Hypothesis 1 : Consider a **minimally coupled** real scalar field :

$$S = \int_{\mathcal{M}} \left[F(g_{\mu\nu}, \partial_\alpha g_{\mu\nu}, \dots) + \nabla_\mu\phi\nabla^\mu\phi - V(\phi)\right]\sqrt{-g}\,\mathrm{d}^n x$$

Hypothesis 2 : The scalar field **share the space-time symmetries**.

Hypothesis 3 : (**Energetic condition**) $\phi V'(\phi) \geq 0 \;\; \forall\phi$, with $V'(\phi) = \mathrm{d}V/\mathrm{d}\phi$, & $\phi V'(\phi) = 0$ for some discrete values of ϕ, say ϕ_i.

The scalar field must be trivial *($\phi(x^\mu) = \phi_i, \forall x^\mu$) on the black hole exterior region.*

Consequently, in such theories, black holes will remain the same as in GR.

Trying to circumvent this result (or any generalisation) **motivate the study** of scalar fields **non-minimally coupled** to gravity and/or **violating some of the space-time symmetries**.

III. Horndeski

In the early 70's, the mathematician G.W. Horndeski classified the **most general theory** in 4 dimensions, including a single metric tensor with a Levi-Civita connexion and a single real scalar field, whose equations of motion are of second order. This theory is described by the following action :

$$S = \int_{\mathcal{M}} \mathscr{L}\sqrt{-g}\,\mathrm{d}^4x,$$

where

$$\begin{aligned}\mathscr{L} =& K(\phi,\rho) - G_3(\phi,\rho)\Box\phi + G_4(\phi,\rho)R + G_{4,\rho}(\phi,\rho)\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right] \\ &+ G_5(\phi,\rho)G_{\mu\nu}\nabla^\mu\phi\nabla^\nu\phi \\ &- \frac{1}{6}G_{5,\rho}(\phi,\rho)\left[(\Box\phi)^3 - 3\Box\phi\,(\nabla_\mu\nabla_\nu\phi)^2 + 2\,(\nabla_\mu\nabla_\nu\phi)^3\right],\end{aligned}$$

with

$$\rho = \nabla_\mu\phi\nabla^\mu\phi,$$

and where the functions $G_i(\phi,\rho)$ $(i \in \{3,4,5\})$ & $K(\phi,\rho)$ are **arbitrary** functions.

Overlooked for a while, Horndeski theory was rediscovered in the last decade and several generalizations (with complex scalar or vector field in, eventually, more than 4 dimensions) where constructed.

IV. From shift-symmetry to spontaneous scalarization

One interesting subclass of scalar-tensor theories is characterized by the following action :

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{1}{16\pi\mathcal{G}}R - \nabla_\mu\phi^*\nabla^\mu\phi + f(\phi)\mathcal{I}(g)\right],$$

where a **complex scalar field** is **non-minimally coupled to gravity via a geometrical invariant** $\mathcal{I}(g)$. In such a model, the space-time curvature will act as a source for the scalar field via the term $\mathcal{I}(g)$ and lead to non trivial scalar field configurations. This mechanism is known as **curvature induced scalarization**.

The pattern of the solutions depend on the invariant and on the term $f(\phi)$ defining the coupling.

The case of a scalar field coupled to the Gauss-Bonnet invariant $\mathcal{I}_{GB} = R^2 - 4R_{ab}R^{ab} + R_{abcd}R^{abcd}$ has been studied in the litterature for several choices of $f(\phi)$, revealing different kind of behaviours.

For our study, we have focussed on a coupling to $\mathcal{I}_{GB}$ of the form $f(\phi) = \gamma_1|\phi| + \gamma_2|\phi|^2$, where $\gamma_{1,2}$ are constants.

The case $\gamma_2 = 0$, in which the theory is invariant under a shift of the scalar field, is known as the "shift-symmetric" case. In this case, hairy black holes do exist for several values of $\gamma_1 \in [0, \gamma_{1,\max}]$.

The case $\gamma_1 = 0$ is known in the litterature as the "spontenously scalarized" case, since hairy black holes only exist for $0 < \gamma_{2,\min} \leq \gamma_2 \leq \gamma_{2,\max}$.

We have constructed black hole solutions in the "general case" ($\gamma_{1,2} \neq 0$) and shown how one can get hairy black holes in this case. In particular, for a given value of $\gamma_1 \neq 0$, solutions exist for $0 \leq \gamma_2 \leq \gamma_{2,\max}$ but different values of γ_1 reveal different patterns, as illustrated on figure 1.

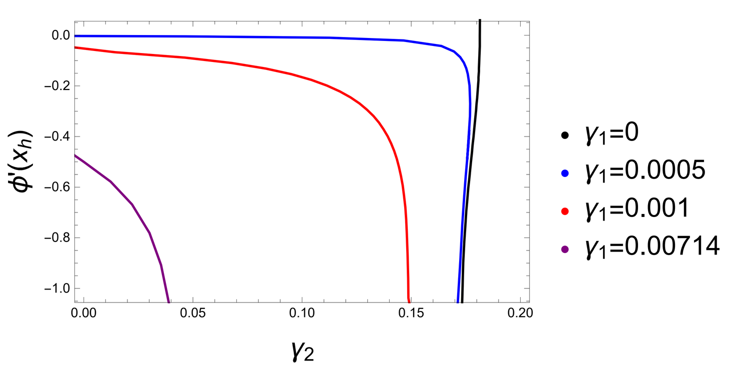

Figure 1: Derivative of the scalar field at the horizon as function of γ_2 for several values of γ_1.

On this figure, one can see how the range of accessible values of γ_2 is influenced by the presence of γ_1 and how the phenomenon of spontaneous scalarization (black line) appear in the limit $\gamma_1 \to 0$.

For really small values of γ_1 (see the blue curve), two different branchs of solutions appear. The first one, connected to the shift-symmetric solutions ($\gamma_2 = 0$) exist for $\gamma_2 \in [0, \gamma_{2,\max}]$ where it connect to a second branch existing only for $\gamma_2 \in [\gamma_{2,c}, \gamma_{2,\max}]$. In the limit $\gamma_1 \to 0$ the first branch (which exist only for small values of $\phi'(0)$) "disapear" while the second branch remains to give the spontaneously scalarized solutions.

V. Conclusions

The study of scalar tensor theories is a large subject due to the variety of possible interactions between the scalar field and the metric tensor which lead to very different kind of behaviours.

In the present work, we have shown the connections between two theories where black holes can be dressed by a scalar field thanks to spacetime curvature.

VI. Perspectives

We hope to return on these solutions in the near future to address the question of their stability.

Furthermore, we plan to extend our study to different kind of scalar-tensor interactions.

References

Y. Brihaye and L. Ducobu, "Hairy black holes: from shift symmetry to spontaneous scalarization," arXiv:1812.07438 [gr-qc].