# Frequently hypercyclic operators, and related notions

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In linear dynamics we study the behaviour of orbits $\{x, Tx, T^2x, \dots\}$ of vectors $x \in X$ under (continuous and linear) operators $T : X \to X$, where $X$ is typically a Banach- or Fréchet-space. An operator is called hypercyclic if it admits a dense orbit. In 2004, Bayart and Grivaux introduced the concept of a frequently hypercyclic operator: it admits a vector whose orbit not only meets any non-empty open set $U$ at least once (and hence infinitely often – like for hypercyclicity) but very often indeed in the sense that $\underline{\text{dens}}\{n \geq 0 : T^n x \in U\} > 0$. We will start by discussing the fundamental properties of frequently hypercyclic operators.

Now, the definition of Bayart and Grivaux involves the family $\mathcal{A}_{\text{ld}}$ of subsets of $\mathbb{N}_0$ of positive lower density. Recently, researchers have defined the general notion of an $\mathcal{A}$-hypercyclic operator by replacing the family $\mathcal{A}_{\text{ld}}$ by an arbitrary family $\mathcal{A}$ of subsets of $\mathbb{N}_0$. We will discuss some of the properties of this new notion. The results obtained help to understand why, in certain respects, frequent hypercyclicity behaves markedly differently from (ordinary) hypercyclicity.