Observability issues and unknown inputs in microalgae cultures

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1. Motivations

Microalgae cultures have a wide range of applications ranging from waste water treatment to biofuel production. Online measurements are mandotory for advanced control and monitoring purposes, however, in microalgae culture, it is impossible to measure online the internal quota (Q). Software sensors (observers) appear as an appealing solution: they blend partial information from available sensor into a mathematical model of the process in order to reconstruct online the unmeasured process states.

This poster shows the conditions under which even if the model appears theoretically observable, the observer will be unable to reconstruct the process states despite tuning. Furthermore, we study the performances of an Extended Kalman Filter and an Unknow Input Observer for Unknown Input estimation.

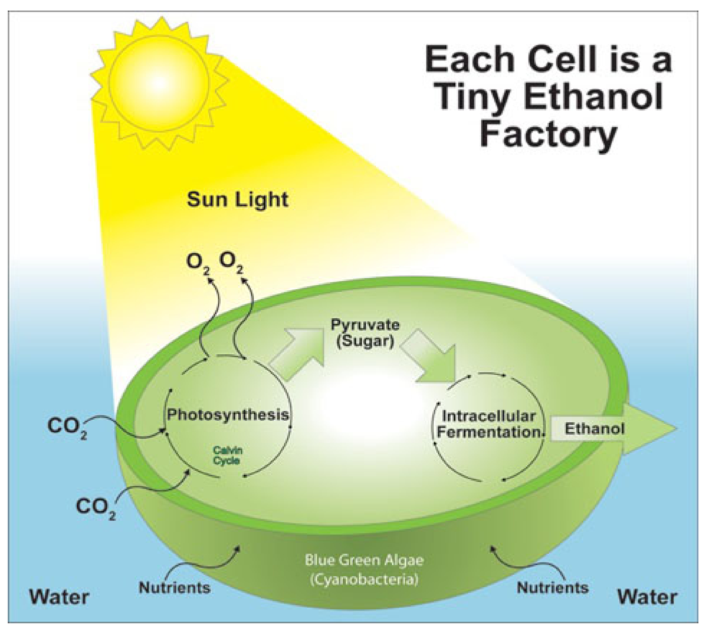
2. Process description

[1] discusses an extended Droop model [2] taking photo-acclimation and photo-inhibition into account:

$$\begin{cases} \dot{X} &= \mu X - DX - RX \\ \dot{S} &= -\rho X + D(S_{in} - S) \\ \dot{Q} &= \rho - \mu Q \\ \dot{I}^* &= \delta\mu(\bar{I} - I^*) \end{cases} \quad (1)$$

$$\begin{aligned} \mu(Q, I^*) &= \bar{\bar{\mu}}(Q, I^*)(1 - \frac{Q_0}{Q}) \\ \rho(S, Q) &= \rho_m \left(\frac{S}{K_S+S}\right)\left(1 - \frac{Q}{Q_1}\right) \end{aligned} \quad (2)$$

In these expressions, I^* is a conceptual variable representing the light to which the cells are photo-acclimated, D the dilution rate, $\rho(S,Q)$ the substrate uptake rate and $\mu(Q, I^*)$ the growth rate. Q_0 is the Minimal cell quota and Q_1 its upper bound. More information on parameters definition can be found in (Deschenes and Vande Wouwer, 2016).



3. Nonlinear Observability

To assess global observability, the model can be cast into a canonical observability form [3]:

$$\dot{\underline{x}} = \begin{bmatrix} \dot{\underline{x}}^1 \\ \vdots \\ \dot{\underline{x}}^i \\ \vdots \\ \dot{\underline{x}}^{q-1} \\ \dot{\underline{x}}^q \end{bmatrix} = \begin{bmatrix} f^1(\underline{x}^1, \underline{x}^2, \underline{u}) \\ \vdots \\ f^i(\underline{x}^1, \dots \underline{x}^{i+1}, \underline{u}) \\ \vdots \\ f^{q-1}(\underline{x}^1, \dots \underline{x}^q, \underline{u}) \\ f^q(\underline{x}^1, \dots \underline{x}^q, \underline{u}) \end{bmatrix}, \ \underline{y} = \underline{x}^1$$

Where: $\forall i \in \{1, \dots, q\}$, $\underline{x}^i \in \mathbb{R}^{n_i}$, $n_1 \geq n_2 \geq \dots \geq n_q$ and $\sum_{1 \leq i \leq q} n_i = n_x$. If:

$$\forall j \in \{1, \dots, n_1\}: \quad \frac{\partial h_j}{\partial x_j^1} \neq 0$$
$$\forall i \in \{1, \dots, q-1\}, \quad \forall (\underline{x}, \underline{u}) \in \mathbb{R}^{n_x} \times \mathbb{R}^{n_u}:$$
$$\text{rank} \frac{\partial f^i(\underline{x}, \underline{u})}{\partial \underline{x}^{i+1}} = n_{i+1}$$

$\Rightarrow$ the system is theoretically globally observable.

4. Observability Analysis

$y = X \longleftrightarrow H = [1\,0\,0\,0]$		$y = \begin{bmatrix} X \\ S \end{bmatrix} \longleftrightarrow H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$	
$\underline{x}^1 = X$	$\underline{f}^1 = \dot{X}$	$\underline{x}^1 = [X\ S]$	$\underline{f}^1 = [\dot{X}\ \dot{S}]$
$\underline{x}^2 = I^*$	$\underline{f}^2 = \dot{I}^*$	$\underline{x}^2 = I^*$	$\underline{f}^2 = \dot{I}^*$
$\underline{x}^3 = Q$	$\underline{f}^3 = \dot{Q}$	$\underline{x}^3 = Q$	$\underline{f}^3 = \dot{Q}$
$\underline{x}^4 = S$	$\underline{f}^4 = \dot{S}$		

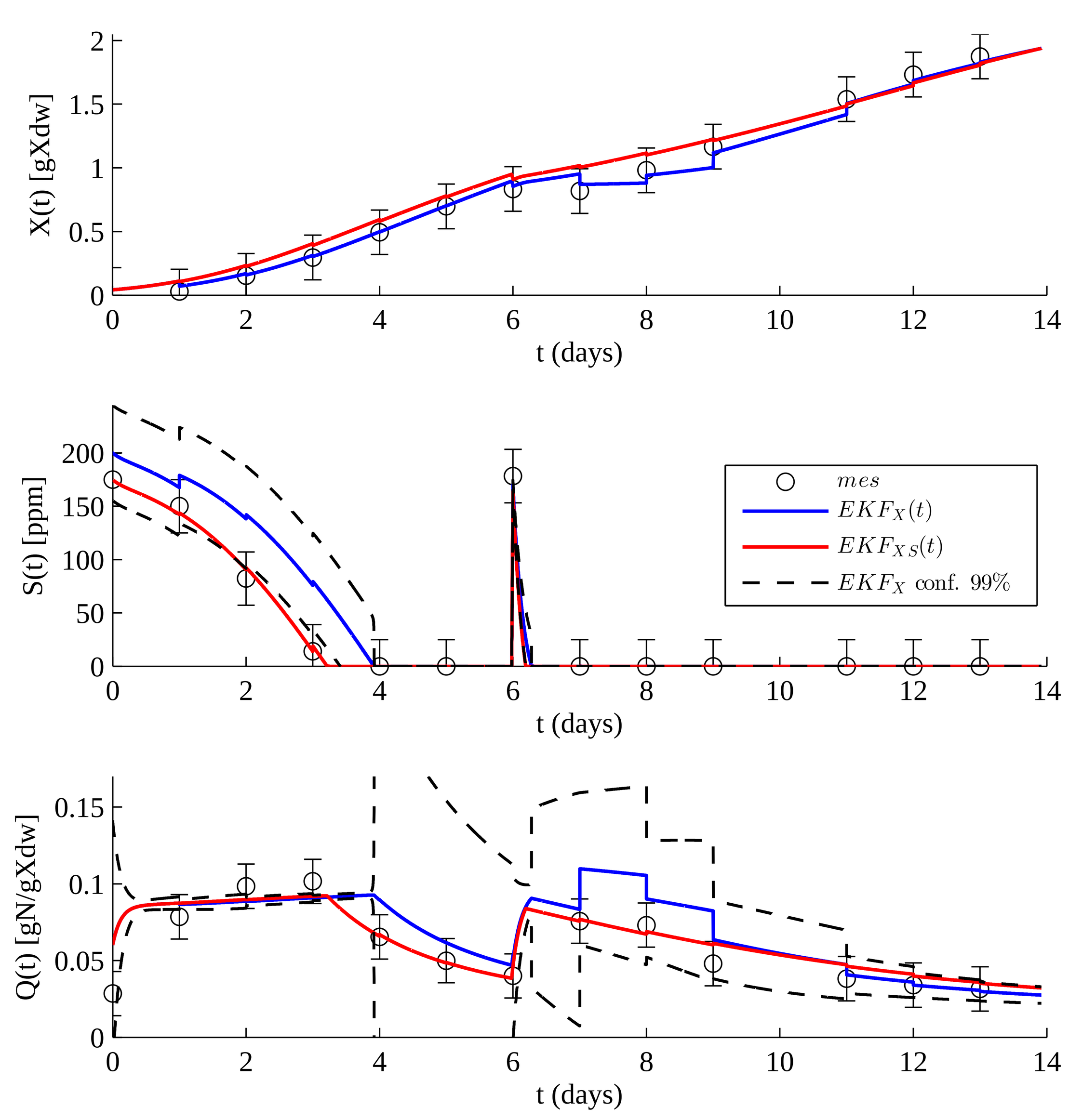
Table 1. Canonical observability form using only Biomass measurements and both Biomass and Substrate measurements

$y = X \longleftrightarrow H = [1\,0\,0\,0]$		$y = \begin{bmatrix} X \\ S \end{bmatrix} \longleftrightarrow H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$	
Observability Conditions	Fulfillment	Observability Conditions	Fulfillment
$\frac{\partial y}{\partial x_1^1} = \frac{\partial X}{\partial X} = 1 \neq 0$	If $X \neq 0$	$\begin{bmatrix} \frac{\partial y^T}{\partial x_1^1} \\ \frac{\partial y^T}{\partial x_2^1} \end{bmatrix} = \begin{bmatrix} \frac{\partial y^T}{\partial X} \\ \frac{\partial y^T}{\partial S} \end{bmatrix} = \mathbf{I}_2 \neq \mathbf{0}_2$	If $X \neq 0$ & $S \neq 0$
$\frac{\partial \underline{f}^1}{\partial \underline{x}^2} = \frac{\partial[\mu X - DX - RX]}{\partial I^*} = X\frac{\partial \mu(Q,I^*)}{\partial I^*} \neq 0$	If $X \neq 0$ & $Q \neq Q_0$	$\frac{\partial \underline{f}^{1T}}{\partial \underline{x}^2} = \begin{bmatrix} \frac{\partial[\mu(Q,I^*)X - DX - RX]}{\partial I^*} \\ \frac{\partial[-\rho(S,Q)X + D(S_{in} - S)]}{\partial I^*} \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \end{bmatrix}$	If $X \neq 0$
$\frac{\partial \underline{f}^2}{\partial \underline{x}^3} = \delta(\bar{I} - I^*)\frac{\partial \mu(Q,I^*)}{\partial Q} \neq 0$	If $I^* \neq \bar{I}$ & $Q \neq Q_0$	$\frac{\partial \underline{f}^2}{\partial \underline{x}^3} = \frac{[\partial \rho(Q,S) - \mu(Q,I^*)Q]}{\partial Q} \neq 0$	Always
$\frac{\partial \underline{f}^3}{\partial \underline{x}^4} = \frac{\partial[\rho - \mu Q]}{\partial S} = \frac{\partial \rho(S,Q)}{\partial S} \neq 0$	If $Q \neq Q_1$ & $S \neq 0$		

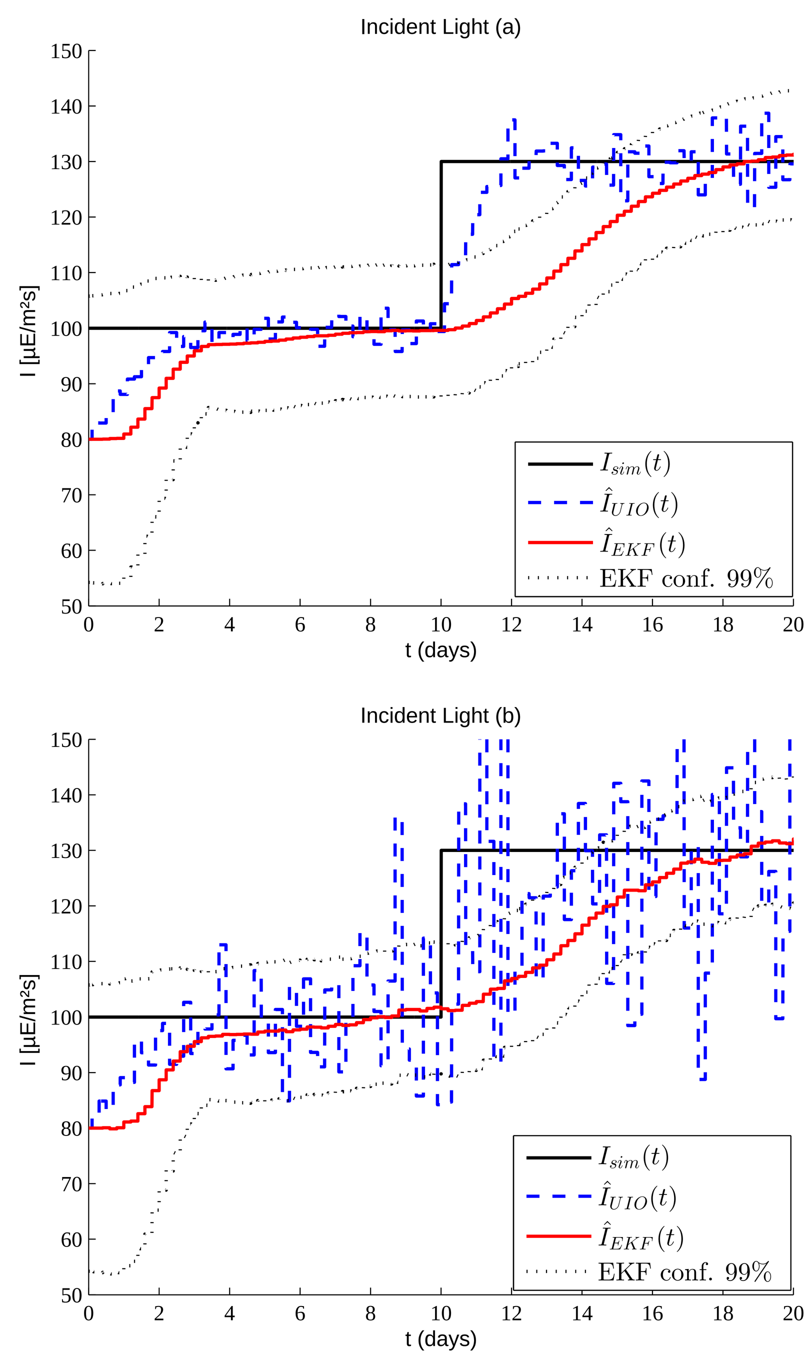
Table 2. Global Observability Analysis using only Biomass measurements and both Biomass and Substrate measurements - This table presents conditions under which loses of observability may occur.

5. Results

An EKF is tested with experimental data of scenesdemus obliquus [4] for the estimation of the internal quota Q first using only biomass measurements. A loss of observability occurs when: $Q \Rightarrow Q_1$ which affects the estimation of S. Moreover, when $S = 0$, the estimation of Q is affected. On the other hand, using both biomass and substrate measurements considerably improves the situation. Those results are in accordance with our prior observability analysis.



Unknown inputs can be estimated by extending the state vector and relying on an extended Kalman filter, or exploiting a dedicated unknown input observer as in [5]. The unknown input here is the incident light intensity. Simulation results (below) illustrate the convergence of both the augmented EKF and the UIO with low (a) and high (b) levels of noise. With this respect, the performance of the UIO deteriorates faster.



References

[1] Bernard O. and Rémond B. (2012) . *Validation of a simple model accounting for light and temperature effect on microalgal growth.* Bioresource Technology.

[2] Droop M. (1968). *Vitamin b12 and marine ecology. iv. the kinetics of uptake, growth and inhibition in monochrysis lutheri.* J. Mar. Biol. Assoc. UK, 48(3):689-733.

[3] Gauthier J.-P. and Kupka I. (1994). *Observability and observers for nonlinear systems.* SIAM Journal on Control and Optimization, 32(4):975-994.

[4] Deschênes J.-S. and Vande Wouwer A. (2016). *Parameter identification of a dynamic model of cultures of microalgae scenedesmus obliquus : an experimental study.* IFAC-PapersOnLine, 49(7).

[5] Rocha-Cozatl E. , Moreno J., and Vande Wouwer A. (July 2012). *Application of a continuous-discrete unknown input ovserver to estimation in phytoplanktonic cultures.* 2012 International Symposium on Advanced Control of Chemical Processes (IFAC ADCHEM).

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