Observability issues and unknown inputs in microalgae cultures

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Abstract: Microalgae cultures have a wide range of applications ranging from waste water treatment to biofuel production. For advanced control and monitoring purposes, it is required to develop software sensors reconstructing on-line the process state. However, this is a hard problem due to observability conditions and the presence of unknown inputs. In this paper, we provide an observability analysis and show the conditions under which, even if the observability conditions are satisfied, the sensitivity of the unmeasured states to the measured ones is weak and the observer convergence is affected. In addition, we consider the presence of unknown inputs and develop an extended Kalman filter and an unkonwn input observer to deal with this situation. Estimation is also illustrated with experimental data from cultures of Scenedesmus obliquus.

Keywords: state estimation, observability, unknown input estimation, extended kalman filter, microalgae.

MODEL DESCRIPTION

(Droop, 1968) introduced a mathematical model which uncouples biomass growth and nutrient uptake under constant light. This model was extended by (Bernard and R´emond, 2012) to account for photo-acclimation and photo-inhibition phenomena (1). The model parameters have recently been identified by (Deschenes and Vande Wouwer, 2016) for the cultures of Scenedesmus Obliquus in photo-bioreactors (PBR) where biomass (X), substrate (S) and internal quota(Q) were measured each day during 13 days through sampling and laboratory analysis. The model equations are the following:

$$\begin{cases} \dot{X} &= \mu X - DX - RX \\ \dot{S} &= -\rho X + D(S_{in} - S) \\ \dot{Q} &= \rho - \mu Q \\ \dot{I}^* &= \delta \mu (\bar{I} - I^*) \end{cases} \tag{1}$$

With:

$$\begin{aligned} \mu(Q,I^*) &= \bar{\bar{\mu}}(Q,I^*)(1-\frac{Q_0}{Q}) \\ \rho(S,Q) &= \rho_m\left(\frac{S}{K_S+S}\right)\left(1-\frac{Q}{Q_1}\right) \end{aligned} \tag{2}$$

In these expressions, I^* is a conceptual variable representing the light to which the cells are photo-acclimated, D the dilution rate, $\rho(S,Q)$ the substrate uptake rate and $\mu(Q,I^*)$ the growth rate. Q_0 is the Minimal cell quota and Q_1 its upper bound. More information on parameters definition can be found in (Deschenes and Vande Wouwer, 2016).

OBSERVABILITY ANALYSIS

To assess global observability, the model can be cast into a canonical observability form (Gauthier and Kupka, 1994, Zeitz, 1984)::

$$\underline{\dot{x}} = \begin{bmatrix} \underline{\dot{x}}^1 \\ \vdots \\ \underline{\dot{x}}^i \\ \vdots \\ \underline{\dot{x}}^{q-1} \\ \underline{\dot{x}}^q \end{bmatrix} = \begin{bmatrix} \underline{f}^1(\underline{x}^1, \underline{x}^2, \underline{u}) \\ \vdots \\ \underline{f}^i(\underline{x}^1, \dots \underline{x}^{i+1}, \underline{u}) \\ \vdots \\ \underline{f}^{q-1}(\underline{x}^1, \dots \underline{x}^q, \underline{u}) \\ \underline{f}^q(\underline{x}^1, \dots \underline{x}^q, \underline{u}) \end{bmatrix}, \underline{y} = \begin{bmatrix} h_1(x_1^1) \\ h_2(x_1^1, x_1^2) \\ \vdots \\ h_{n_1}(x_1^1, \dots, x_{n_1}^1) \end{bmatrix}$$

Where: $\underline{x}^T = [\underline{x}^1, \dots, \underline{x}^q]$, $\underline{f}^T = [\underline{f}^1, \dots, \underline{f}^q]$, $\underline{x}^{1,T} = [x_1^1, \dots, x_{n_1}^1]$, $\underline{h}^T = [h_1, \dots, h_{n_1}]$.
$\forall i \in \{1, \dots, q\}$, $\underline{x}^i \in \mathbb{R}^{n_i}$, $n_1 \geq n_2 \geq \dots \geq n_q$, $\sum_{1 \leq i \leq q} n_i = n_x$

A system is said globally observable if:

$$\forall j \in \{1, \dots, n_1\}: \quad \frac{\partial h_j}{\partial x_j^1} \neq 0$$
$$\forall i \in \{1, \dots, q-1\}, \quad \forall (\underline{x}, \underline{u}) \in \mathbb{R}^{n_x} \times \mathbb{R}^{n_u}: \quad \text{rank} \frac{\partial \underline{f}^i(\underline{x}, \underline{u})}{\partial \underline{x}^{i+1}} = n_{i+1}$$

The first conditions imply that the n_1 state variables can be inferred from the measurements. The second ensure that any differences in the state trajectory can be detected in the measurements thanks to a pyramidal influence of the state sub-vector $\underline{x}^{i+1}$ on the evolution equations $\underline{f}^i$.
Table 1 shows the results of this analysis, where H designates the output matrix in the linear measurement equation $y = Hx$.
When only biomass measurements are available (and biomass is nonzero), observability loss may occur when the substrate is depleted and/or the internal quota concentration is equal to its maximum value Q_1. On the other hand, combining biomass and substrate measurements can alleviate this condition.
Fig. 2 shows the sensitivity of the process states with respect to the substrate concentration: (a) X and I^* are not sensitive to S; (b) when the dilution rate is equal to 0 the sensitivity of Q is close to zero (loss of observability in batch operating mode).

EXPERIMENTAL RESULTS

State Estimation

Fig.1 illustrates the loss of observability when only biomass measurements are used for the estimation of the substrate and internal quota. As Q quickly reaches its maximum value, a loss of observability affects the estimation of S. Furthermore, a loss of observability appears on the Internal Quota estimation when the substrate is completely depleted. On the other hand, using both biomass and substrate measurements considerably improves the situation.

State and Unknown Input Estimation

Unknown incident light can also affect state estimation. Unknown inputs can be estimated by extending the state vector and relying on an extended Kalman filter, or exploiting a dedicated unknown input observer as in (Rocha-Cozatl et al., 2012). Fig 3 illustrates the convergence of both the augmented EKF and the UIO with low level of noise on measurements. In presence of higher level of noise, the UIO shows poor performance while the EKF remains robust.

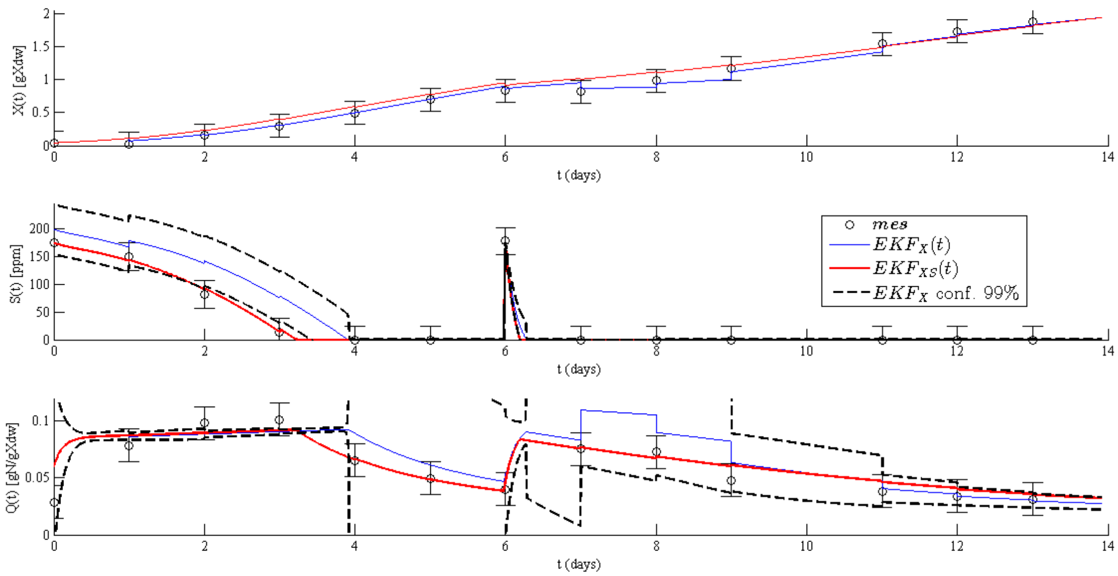

Figure 1. Internal quota estimation over 14 days with EKF with only Biomass measurements and both Biomass and Substrate measurements

$y = X \longleftrightarrow H = [1\,0\,0\,0]$		$y = \begin{bmatrix} X \\ S \end{bmatrix} \longleftrightarrow H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$	
$\underline{x}^1 = X$	$\underline{f}^1 = \dot{X}$	$\underline{x}^1 = [X\,S]$	$\underline{f}^1 = [\dot{X}\,\dot{S}]$
$\underline{x}^2 = I^*$	$\underline{f}^2 = \dot{I}^*$	$\underline{x}^2 = I^*$	$\underline{f}^2 = \dot{I}^*$
$\underline{x}^3 = Q$	$\underline{f}^3 = \dot{Q}$	$\underline{x}^3 = Q$	$\underline{f}^3 = \dot{Q}$
$\underline{x}^4 = S$	$\underline{f}^4 = \dot{S}$		

Table 1. Canonical observability form using only Biomass measurements and both Biomass and Substrate measurements

$y = X \longleftrightarrow H = [1\,0\,0\,0]$		$y = \begin{bmatrix} X \\ S \end{bmatrix} \longleftrightarrow H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$	
Observability Conditions	Fulfillment	Observability Conditions	Fulfillment
$\frac{\partial y}{\partial \underline{x}_1^1} = \frac{\partial X}{\partial X} = 1 \neq 0$	Always	$\begin{bmatrix} \frac{\partial y^T}{\partial \underline{x}_1^1} \\ \frac{\partial y^T}{\partial \underline{x}_2^1} \end{bmatrix} = \begin{bmatrix} \frac{\partial y^T}{\partial X} \\ \frac{\partial y^T}{\partial S} \end{bmatrix} = \mathbf{I}_2 \neq \mathbf{0}_2$	Always
$\frac{\partial \underline{f}^1}{\partial \underline{x}^2} = \frac{\partial[\mu X - DX - RX]}{\partial I^*} = X \frac{\partial \mu(Q,I^*)}{\partial I^*} \neq 0$	If $X \neq 0$ & $Q \neq Q_0$	$\frac{\partial \underline{f}^{1T}}{\partial \underline{x}^2} = \begin{bmatrix} \frac{\partial[\mu(Q,I^*)X - DX - RX]}{\partial I^*} \\ \frac{\partial[-\rho(S,Q)X + D(S_{in} - S)]}{\partial I^*} \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \end{bmatrix}$	If $X \neq 0$
$\frac{\partial \underline{f}^2}{\partial \underline{x}^3} = \delta(\bar{I} - I^*) \frac{\partial \mu(Q,I^*)}{\partial Q} \neq 0$	If $I^* \neq \bar{I}$ & $Q \neq Q_0$	$\frac{\partial \underline{f}^2}{\partial \underline{x}^3} = \frac{[\partial \rho(Q,S) - \mu(Q,I^*)Q]}{\partial Q} \neq 0$	Always
$\frac{\partial \underline{f}^3}{\partial \underline{x}^4} = \frac{\partial[\rho - \mu Q]}{\partial S} = \frac{\partial \rho(S,Q)}{\partial S} \neq 0$	If $Q \neq Q_1$ & $S \neq 0$		

Table 2. Global Observability Analysis using only Biomass measurements and both Biomass and Substrate measurements

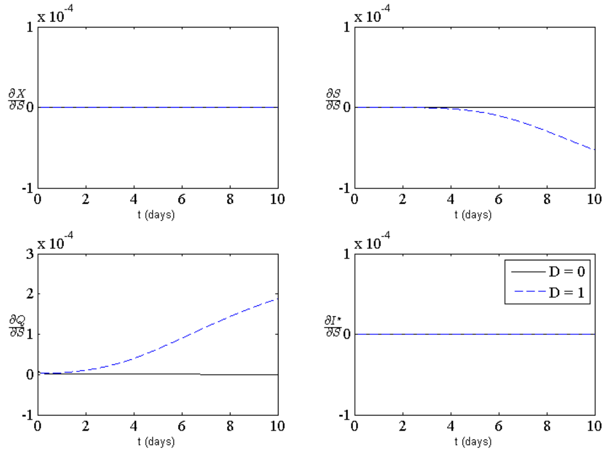

Figure 2. Sensitivity of the states with respect to the Substrate concentration with $D = 0$ and $D \neq 0$

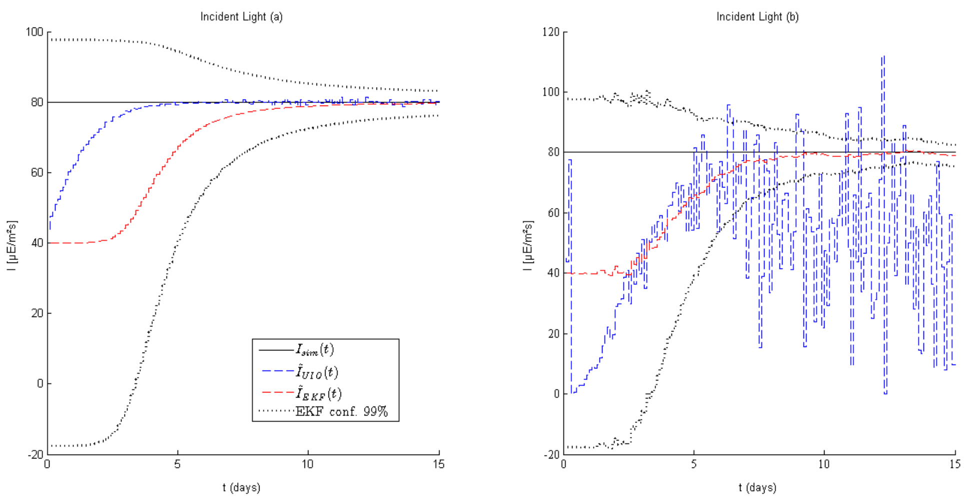

Figure 3. Incident light estimation with EKF and UIO (a) low noise level (b) higher noise level on measurements

REFERENCES

Bernard O. and R´emond B. (2012) . Validation of a simple model accounting for light and temperature effect on microalgal growth. Bioresource Technology.

Deschenes J.-S. and Vande Wouwer A. (2016). Parameter identification of a dynamic model of cultures of microalgae scenedesmus obliquus ?? an experimental study. IFAC-PapersOnLine, 49(7).

Droop M. (1968). Vitamin b12 and marine ecology. iv. the kinetics of uptake, growth and inhibition in monochrysis lutheri. J. Mar. Biol. Assoc. UK, 48(3):689?733.

Gauthier J.-P. and Kupka I. (1994). Observability and observers for nonlinear systems. SIAM Journal on Control and Optimization, 32(4):975?994.

Rocha-Cozatl E. , Moreno J., and Vande Wouwer A. (July 2012). Application of a continuous-discrete unknown input ovserver to estimation in phytoplanktonic cultures. 2012 International Symposium on Advanced Control of Chemical Processes (IFAC ADCHEM).

Zeitz M.(1984). Observability canonical (phase-variable) form for nonlinear time-variable systems. International Journal of System Science, 15(9):949-958.