Higher-spin extensions of Carrollian symmetries

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A.C., D. Francia, C. Heissenberg,
1703.01351, 1712.09591, 2011.04420

&

A.C., S. Pekar, arXiv:2110.07794

CARROLL WORKSHOP, Vienna, 18/2/2022

fnrs
LA LIBERTÉ DE CHERCHER

How mad are you willing to be?


Let's send c → 0
Great idea! May I add a spoon of higher spins?
???

Higher spins: another missed opportunity?

TEORIA RELATIVISTICA DI PARTICELLE CON MOMENTO INTRINSECO ARBITRARIO

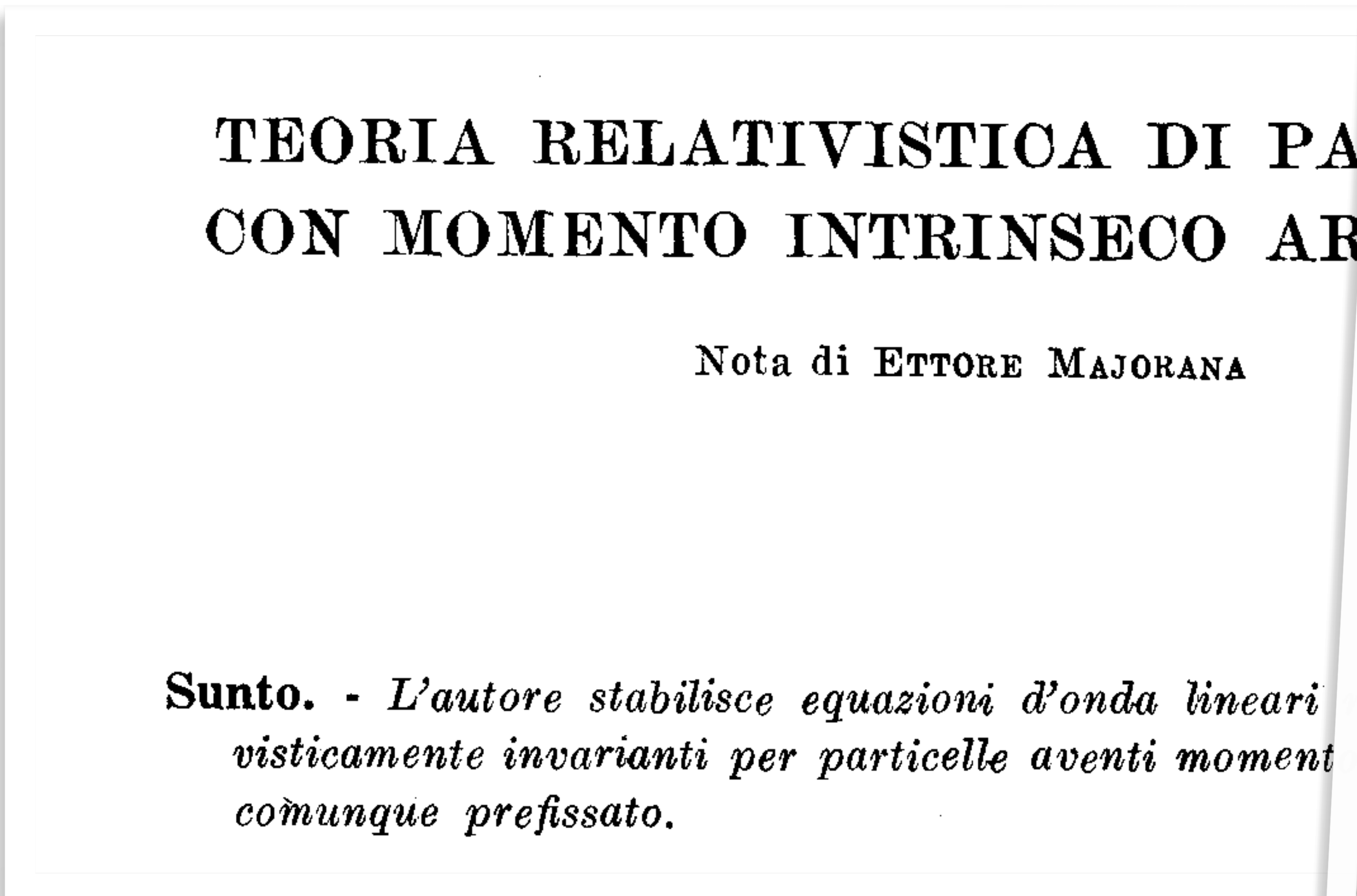
Nota di ETTORE MAJORANA

Sunto. - *L'autore stabilisce equazioni d'onda lineari nell'energia e relativisticamente invarianti per particelle aventi momento angolare intrinseco comunque prefissato.*

Summary. - *The author establishes wave equations that are linear in energy and relativistically invariant for particles with a fixed intrinsic angular momentum.*

E. Majorana, Nuovo Cimento 9 (1932) 335

Higher spins: another missed opportunity?



TEORIA RELATIVISTICA DI PA
CON MOMENTO INTRINSECO AR

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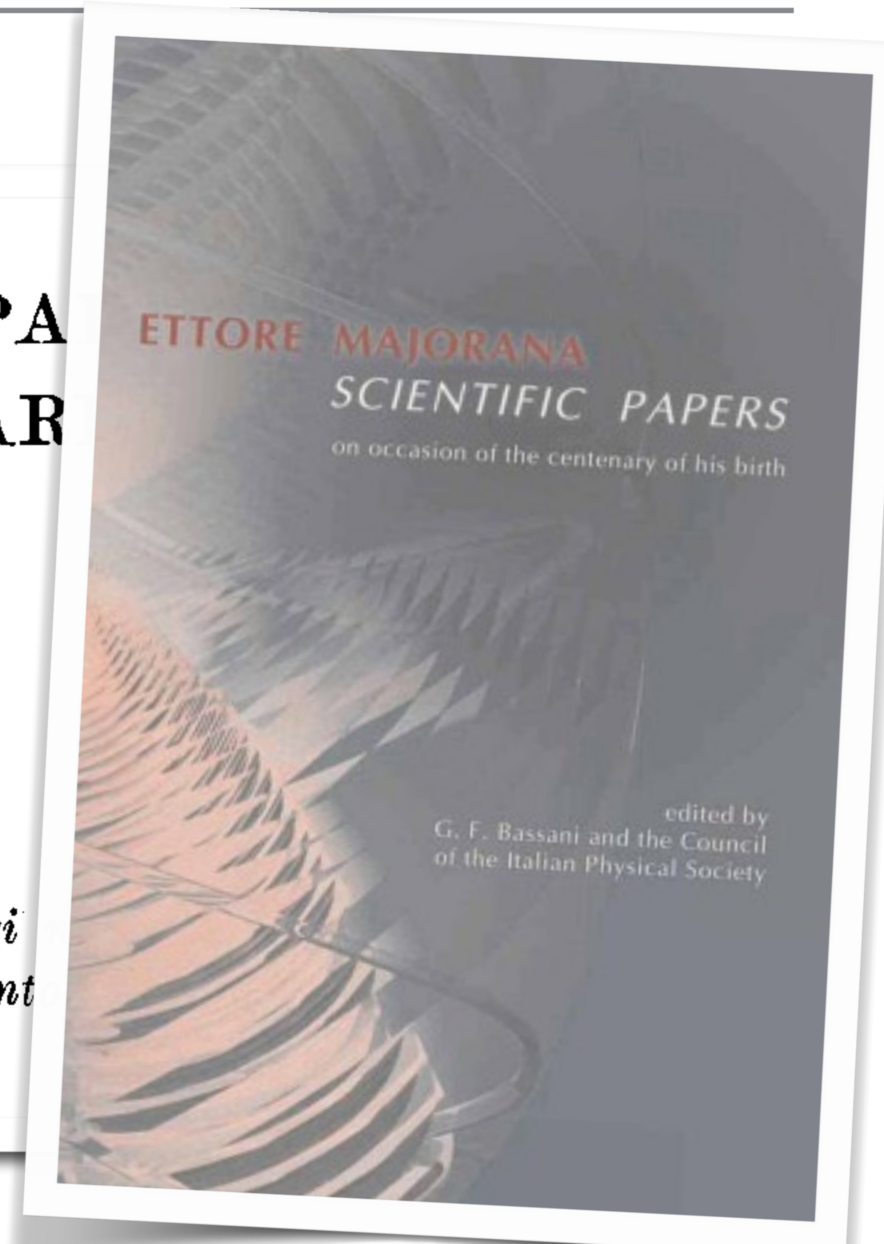

Summary. - *The author establishes wave equations that are linear in energy and relativistically invariant for particles with a fixed intrinsic angular momentum.*

E. Majorana, Nuovo Cimento 9 (1932) 335

(Massless) higher spins in a nutshell

- Irreps of the Poincaré group and free field theory: OK! — Wigner (1939); Fronsdal (1978)
- *1930's:* difficulties in coupling to electromagnetism — Fierz, Pauli (1939)
- *1960's:* extra problems & "no-go theorems"
  - Soft theorems for higher-spin particles ⇒ trivial S-matrix — Weinberg (1964); Coleman, Mandula (1967)
  - No minimal coupling ($\partial_\mu \to \nabla_\mu$) with gravity — Aragone, Deser (1971)
- *1980's:* cubic vertices in Minkowski & (A)dS — Bengtsson², Brink (1983); Berends, Burgers, van Dam (1984)
- *1990's:* Vasiliev's theory in (A)dS — Vasiliev (1990)
- *2000's:* AdS/CFT (holographic duals of weakly coupled CFT's) — Sezgin, Sundell (2002); Klebanov, Polyakov (2002) and many others…

Higher spins & (A)dS

- Why (massless) HS fields like (A)dS?
  - Classical HS interactions seem to require an infrared regulator: mass (String Theory) or cosmological constant (Vasiliev)

- Long range HS interactions imply:
  - in flat-space → trivial S-matrix

    Weinberg (1964)
  - in AdS → free CFT boundary correlators

    Sezgin, Sundell (2002); Klebanov, Polyakov (2002); Maldacena, Zhiboedov (2011) et al.

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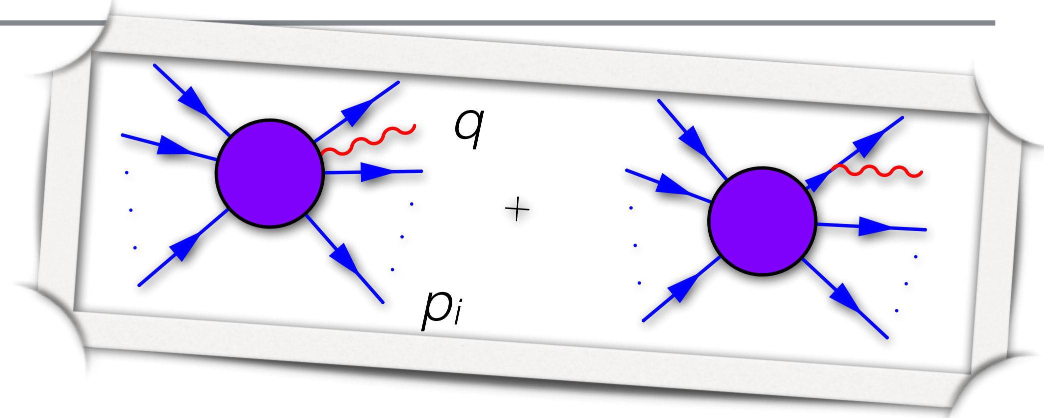
- May Minkowski still play a role?
  - Is String Theory a broken phase of a HS gauge theory?
  - Trivial S-matrix, but non-trivial interactions (*& asymptotic symmetries*)?

Skvortsov, Tran, Tsulaia (2018); A.C., Francia, Heissenberg (2017)

Weinberg soft theorems

(for any s and in any D)

Leading Weinberg's soft theorems



- Amplitude for *N–1* scalars and one soft particle

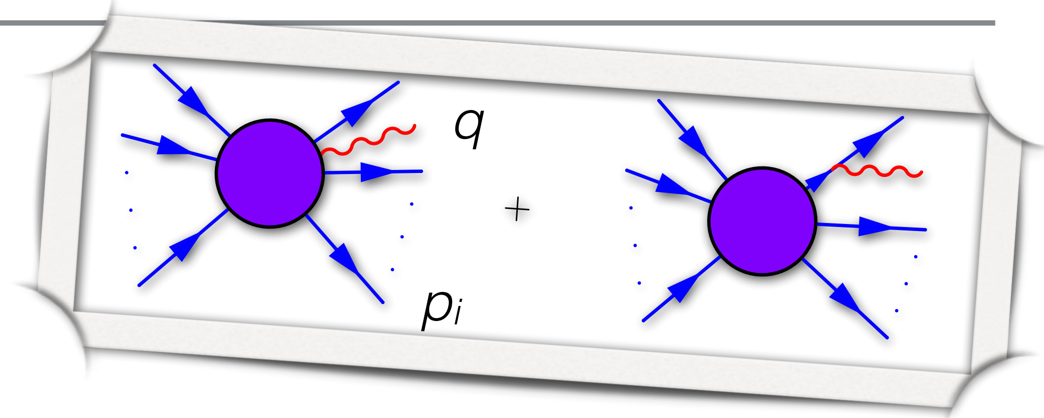
- In the limit $q \to 0$ the amplitude factorises:

Weinberg (1964)

$$\boxed{A_s(1,\ldots,N) \sim A(1,\ldots,N-1) \times \sum_{i=1}^{N-1} g_i^{(s)} \frac{p_i^{\mu_1} \cdots p_i^{\mu_s} \varphi_{\mu_1 \cdots \mu_s}(q)}{2p_i \cdot q}}$$

Leading Weinberg's soft theorems

- Amplitude for $N\!-\!1$ scalars and one soft particle



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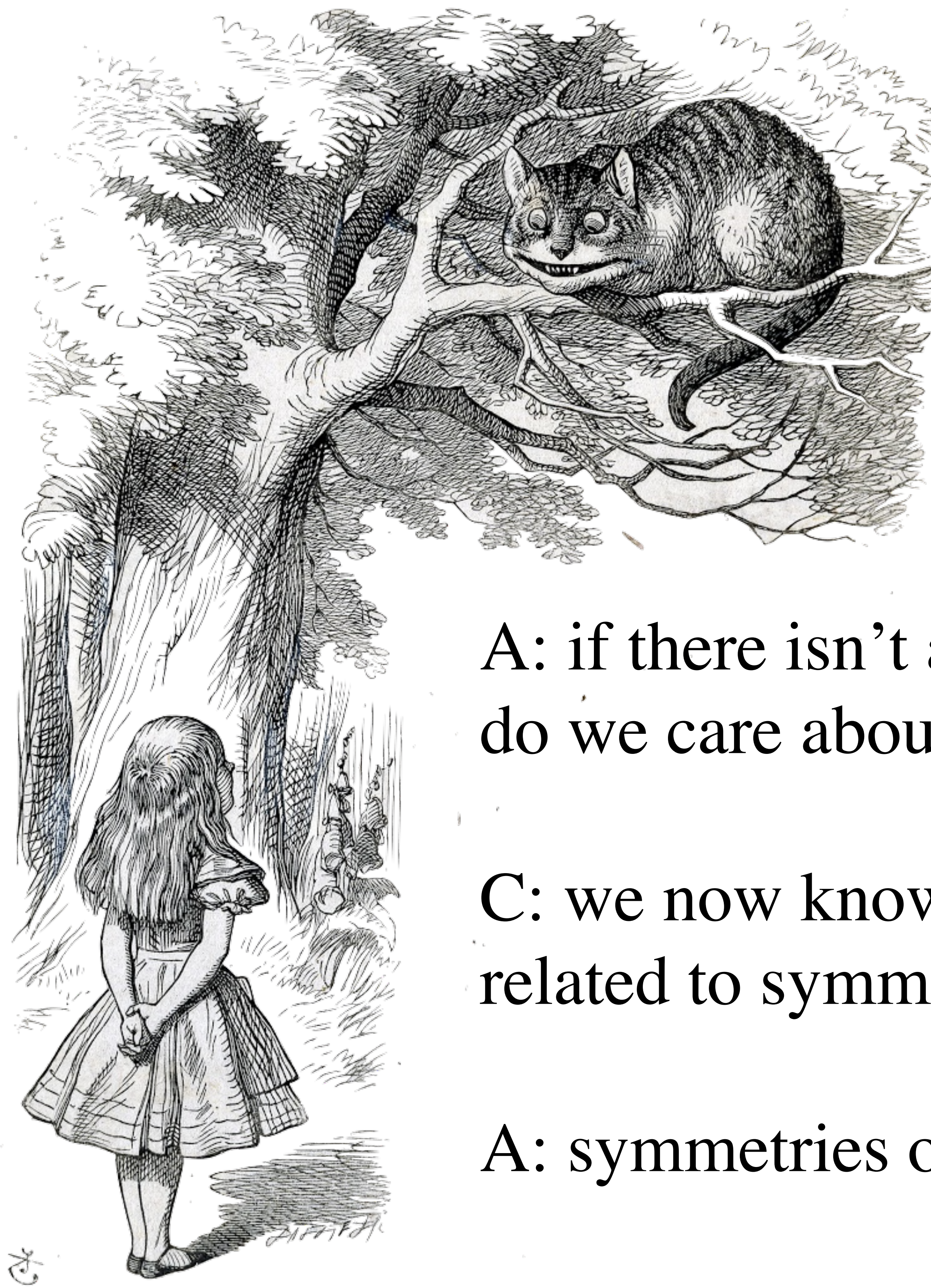
- Invariance under gauge transf.? e.g. $h_{\mu\nu}(q) \to h_{\mu\nu}(q) + i\, q_{(\mu}\Lambda_{\nu)}(q)$
  - QED: $\sum_{i=1}^{N-1} g_i^{(1)} = 0$ (*charge conservation*)
  - Gravity: $g_i^{(2)} = g_j^{(2)}\ \forall\, i,j \;\Leftrightarrow\; \sum_{i=1}^{N-1} p_i^{\mu} = 0$ (*equivalence principle*)
  - Higher spins? *Polynomial constraints in the momenta…*



A: if there isn’t any non-trivial S-matrix why do we care about higher spins in flat space?

C: we now know that soft theorems are related to symmetries!

A: symmetries of what if there isn’t anything?

C: wait and see Alice….

Higher-spin symmetries in flat space?

- Constructing an interacting field theory in flat space beyond cubic order is *subtle*
- Let's be pragmatic: let's begin by ignoring all subtleties for a while and let's try to **classify the symmetries** that may underlie any (possibly exotic) field theory and its possible "holographic dual"
- What can we use as a guiding principle?

The only thing that we know...
the free theory!

Fronsdal formulation of the dynamics

- Example 1: **Maxwell**
  - field equations: $\partial^\lambda F_{\lambda\mu} = 0 \quad \Rightarrow \quad \Box A_\mu - \partial_\mu \partial \cdot A = 0$
  - gauge symmetry: $\delta A_\mu = \partial_\mu \xi$

- Example 2: **linearised gravity**
  - field equations: $\Box h_{\mu\nu} - \partial_\mu \partial \cdot h_\nu - \partial_\nu \partial \cdot h_\mu + \partial_\mu \partial_\nu h_\lambda{}^\lambda = 0$
  - gauge symmetry: $\delta h_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$

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- **Particle of arbitrary spin *s***

Fronsdal (1978)

  - eom: $\mathcal{F}_{\mu_1 \cdots \mu_s} \equiv \Box \varphi_{\mu_1 \cdots \mu_s} - \partial_{(\mu_1} \partial \cdot \varphi_{\mu_2 \cdots \mu_s)} + \partial_{(\mu_1} \partial_{\mu_2} \varphi_{\mu_3 \cdots \mu_s)\lambda}{}^\lambda = 0$
  - gauge symmetry: $\delta \varphi_{\mu_1 \cdots \mu_s} = \partial_{(\mu_1} \xi_{\mu_2 \cdots \mu_s)}$

- Gauge invariance requires: $\xi_{\mu_1 \cdots \mu_{s-3}\lambda}{}^\lambda = 0$

What symmetries are we looking for?

- Two options:
  - **"rigid symmetries" of the vacuum:** $\bar{\nabla}_{(\mu_1}\epsilon_{\mu_2\cdots\mu_s)} = 0$ AC, Pekar (2021)
  - **asymptotic symmetries** AC, Francia, Heissenberg (2017)
- Why are the *isometries of the vacuum* interesting?
  - In gravity they are the basis of the Cartan formulation
  - Vasiliev's theory implements their gauging in (A)dS
  - Asymptotic symmetries are expected to include them as a subalgebra (or as a wedge algebra)

Part 1

Higher-spin isometries of the vacuum

aka

higher-spin algebras

A.C., S. Pekar, arXiv:2110.07794

∞-dim Lie algebras & higher spins

- The “Cartan" approach to higher-spin gauge theories:
  - *1987*: proposal for a **higher-spin algebra** in AdS_4 — Fradkin, Vasiliev
  - *1990*: procedure to implement its **gauging** → Vasiliev’s equations — Vasiliev
  - 2003: higher-spin algebras and interacting e.o.m. in AdS_D — Eastwood; Vasiliev

- Other recent (and less recent) developments
  - 3D HS algebras → Chern-Simons gauge theories (& matter couplings)

    Blencowe (1989); Porkushkin, Vasiliev (1999) & many others…
  - HS algebras for mixed symmetry and partially-massless fields

    Boulanger, Skvortsov (2011); Joung, Mkrtchyan (2016)

Higher-spin algebras

- Key ingredient in building HS theories and studying HS holography

- **What is a HS algebra?**
  - Poincaré & (A)dS algebras: isometries of the vacuum

HS “isometries” of the vacuum

- Fronsdal’s gauge transf.: $\delta\varphi_{\mu_1\cdots\mu_s} = \bar{\nabla}_{(\mu_1}\epsilon_{\mu_2\cdots\mu_s)} + \mathcal{O}(\varphi)$
- Vacuum-preserving symm.: $\bar{\nabla}_{(\mu_1}\epsilon_{\mu_2\cdots\mu_s)} = 0$
- Solution (in Minkowski): $\epsilon_{\mu_1\cdots\mu_{s-1}} = \sum_{k=0}^{s-1} M_{\mu_1\cdots\mu_{s-1}|\nu_1\cdots\nu_k} x^{\nu_1}\cdots x^{\nu_k}$

$$\epsilon_{\mu_1\cdots\mu_{s-3}\lambda}{}^{\lambda} = 0$$

Higher-spin algebras

- Key ingredient in building HS theories and studying HS holography
- **What is a HS algebra?** *Lie algebra on traceless Killing tensors*
  - Poincaré & (A)dS algebras: isometries of the vacuum

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- Fronsdal's gauge transf.: $\delta\varphi_{\mu_1\cdots\mu_s} = \bar{\nabla}_{(\mu_1}\epsilon_{\mu_2\cdots\mu_s)} + \mathcal{O}(\varphi)$
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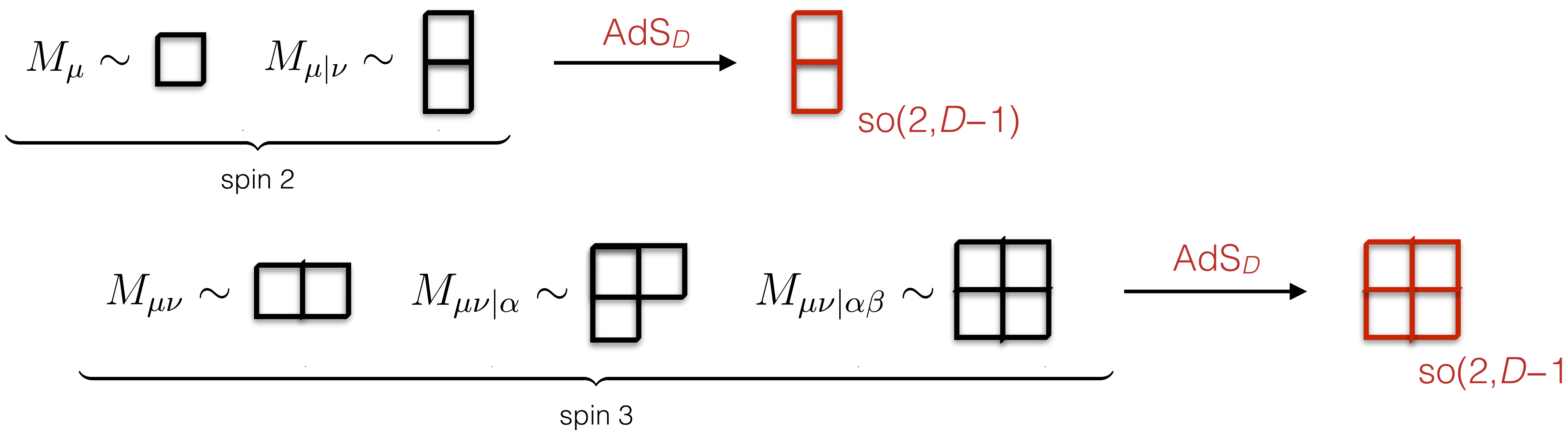
Higher-spin algebras

- Vector space of traceless Killing tensors:



Higher-spin algebras

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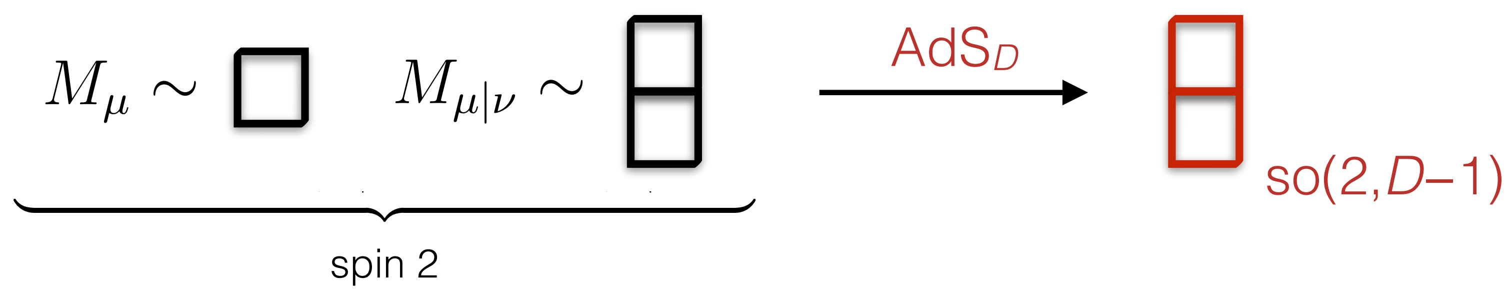
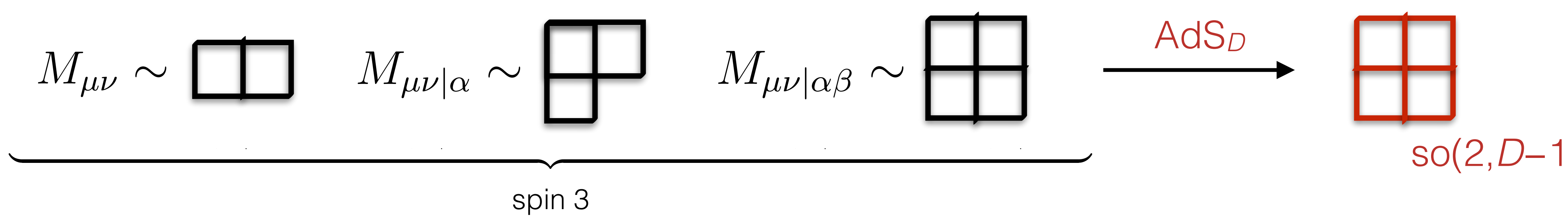
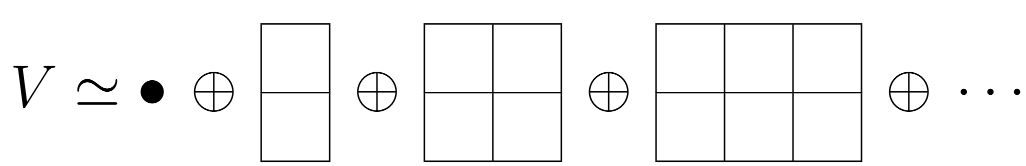




Eastwood-Vasiliev algebras in any D: non-Abelian Lie algebras on V including a so(2,D−1) subalgebra

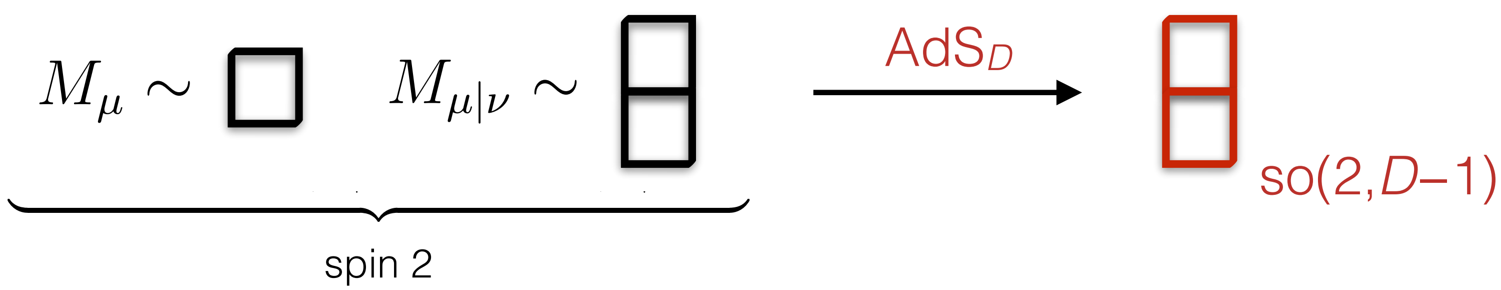


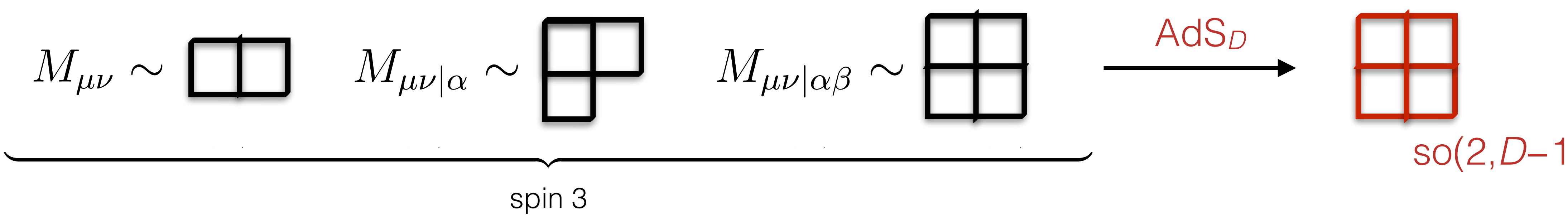
Fradkin, Vasiliev (1987); Eastwood (2002); Segal (2002); Vasiliev (2003)

Higher-spin algebras

so(2,D−1): isometries of AdS$_D$ &
<u>*conformal symmetries (in D−1)*</u>

- Vector space of traceless Killing te





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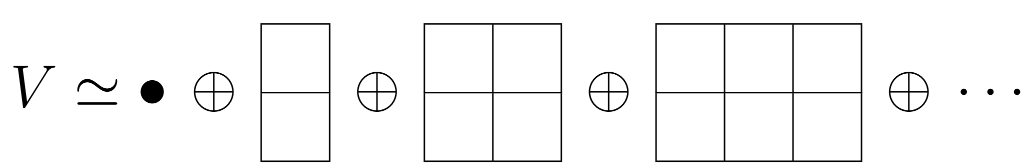


Fradkin, Vasiliev (1987); Eastwood (2002); Segal (2002); Vasiliev (2003)

Notable so(2,D–1) Inönü-Wigner contractions

AdS isometries

so(2,D–1)

conformal algebra

$\Lambda \to 0$

$c \to 0$

Minkowski isometries

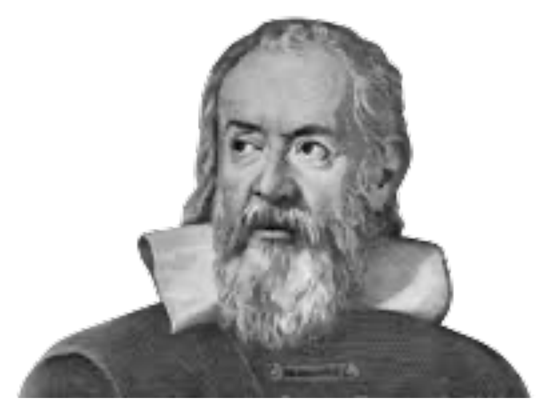
iso(1,D–1)

conformal Carroll algebra

$c \to \infty$

gca_{D-1}

conformal Galilei algebra



What about higher-spin algebras?

Goals & strategy/hypotheses

- **Goal:** classify Lie algebras defined on the vector space V (traceless Killing tensors) that

  1. contain a Poincaré subalgebra, **iso(1,*D*–1)**
  2. ~~contain a conformal Galilei subalgebra, **gca$_{D-1}$**~~ see the paper…

  …and discuss their properties

- **Strategy:** look for *coset algebras*, obtained by quotienting out an ideal from the universal enveloping algebra of iso(1,*D*–1) (or gca$_{D-1}$) (bonus: "good" Lorentz transf. for free) Eastwood (2002)

partial classification, still with interesting examples!

HS algebras in AdS$_D$

Conformal HS algebras in D–1 dimensions

Coset construction of HS algebras

- so(2,D–1) algebra: $[J_{AB}\,,\,J_{CD}] = \tilde\eta_{AC}\,J_{BD} - \tilde\eta_{BC}\,J_{AD} - \tilde\eta_{AD}\,J_{BC} + \tilde\eta_{BD}\,J_{AC}$

- Quadratic products of the generators

$$J_{A(B} \odot J_{C)D} - \text{traces} \sim \begin{array}{|c|c|}\hline & \\ \hline & \\ \hline \end{array} \qquad\qquad C_2 \equiv \frac{1}{2}\,J_{AB} \odot J^{BA} \sim \bullet$$

$$\mathcal{I}_{AB} \equiv J_{C(A} \odot J_{B)}{}^{C} - \frac{2}{D+1}\,\tilde\eta_{AB}\,C_2 \sim \begin{array}{|c|c|}\hline & \\ \hline \end{array} \qquad\qquad \mathcal{I}_{ABCD} \equiv J_{[AB} \odot J_{CD]} \sim \begin{array}{|c|}\hline \\ \hline \\ \hline \\ \hline \\ \hline \end{array}$$

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- Eastwood-Vasiliev algebras:

$$\boxed{\mathfrak{hs}_D = \frac{\mathcal{U}(\mathfrak{so}(2,D-1))}{\langle \mathcal{I}_{AB} \oplus \mathcal{I}_{ABCD} \rangle} \quad \Rightarrow \quad C_2 \sim -\frac{(D+1)(D-3)}{4}\,id}$$

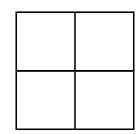
Coset construction of HS algebras

- so(2,D–

$$0 \sim \frac{3}{2}\,\mathcal{I}_{ABCD}J^{CD} - \mathcal{I}_{C[A}\,J_{B]}{}^{C} = \frac{1-D}{D+1}\left(C_2 + \frac{(D+1)(D-3)}{4}\,id\right) J_{AB}$$

$\tilde{\eta}_{BD}\,J_{AC}$

- Quadratic products of the generators

$$J_{A(B} \odot J_{C)D} - \text{traces} \sim$$ 

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Iazeolla, Sundel (2008)

scalar singleton module

Coset construction of HS algebras: summary

- Ideal to be factored out from $U(\mathrm{so}(2,D\!-\!1))$:

$$\mathcal{I}_{AB} \equiv J_{C(A} \odot J_{B)}{}^{C} - \frac{2}{D+1}\,\tilde{\eta}_{AB}\,C_2 \sim \begin{array}{|c|c|}\hline & \\ \hline \end{array}$$

$$\mathcal{I}_{ABCD} \equiv J_{[AB} \odot J_{CD]} \sim \begin{array}{|c|}\hline \\ \hline \\ \hline \\ \hline \\ \hline \end{array}$$

- Eastwood-Vasiliev algebras:

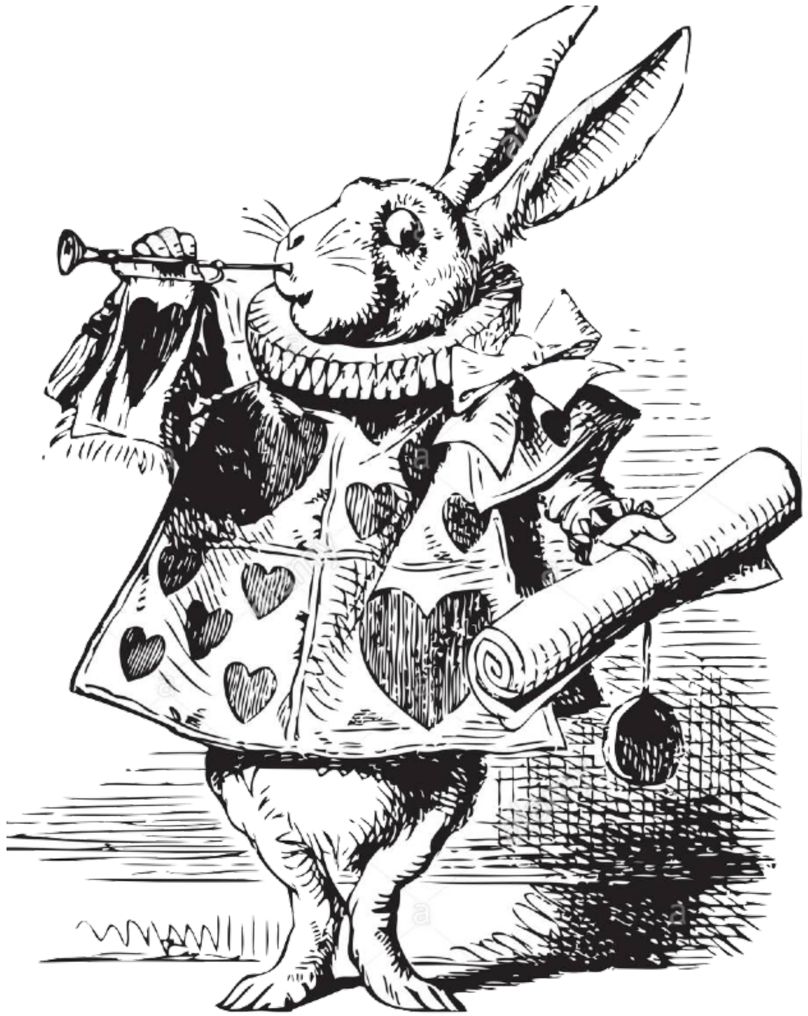
$$\mathfrak{hs}_D = \frac{\mathcal{U}(\mathfrak{so}(2,D-1))}{\langle \mathcal{I}_{AB} \oplus \mathcal{I}_{ABCD} \rangle} \quad \Rightarrow$$

$U(\mathrm{so}(2,D\!-\!1))$ evaluated on the scalar singleton module

- Isomorphic to the conformal symmetries of a free scalar in $D\!-\!1$ dim.

Carrollian conformal HS algebras

(in any dimensions)



From $U(\mathrm{so}(2,D-1))$ to $U(\mathrm{iso}(1,D-1))$

- Look at how the Carrollian contraction affects the so(2,D–1) ideal to *define* the iso(1,D–1) coset

$$[J_{AB}\,,\,J_{CD}] = \tilde{\eta}_{AC}\,J_{BD} - \tilde{\eta}_{AD}\,J_{BC} - \tilde{\eta}_{BC}\,J_{AD} + \tilde{\eta}_{BD}\,J_{AC}$$

$$\boxed{\mathcal{P}_a \equiv \epsilon\,J_{aD}\,, \qquad \mathcal{J}_{ab} \equiv J_{ab}}$$

$$[\mathcal{J}_{ab}\,,\,\mathcal{J}_{cd}] = \eta_{ac}\,\mathcal{J}_{bd} - \eta_{ad}\,\mathcal{J}_{bc} - \eta_{bc}\,\mathcal{J}_{ad} + \eta_{bd}\,\mathcal{J}_{ac}\,,$$

$$[\mathcal{J}_{ab}\,,\,\mathcal{P}_c] = \eta_{ac}\,\mathcal{P}_b - \eta_{bc}\,\mathcal{P}_a\,,$$

$$[\mathcal{P}_a\,,\,\mathcal{P}_b] = -\,\epsilon^2\,\mathcal{J}_{ab}\,,$$

- Next step: branching so(2,D–1) → so(1,D–1) of the ideal

From $U(so(2,D-1))$ to $U(iso(1,D-1))$

- Branching so(2,D−1) → so(1,D−1) of the ideal

$$\mathcal{Q}_{ab} \equiv \{\mathcal{P}_a, \mathcal{P}_b\} - \frac{2}{D}\,\eta_{ab}\mathcal{P}^2$$

$$\mathcal{S}_{ab} \equiv \{\mathcal{J}^c{}_a, \mathcal{J}_{bc}\} - \frac{4}{D}\,\eta_{ab}\mathcal{J}^2$$

$$\mathcal{I}_{AB} \sim 0 \;\;\Rightarrow\;\; \boxed{\begin{aligned} &\mathcal{J}^2 - \frac{D-1}{2}\,\epsilon^{-2}\,\mathcal{P}^2 \sim 0 \\ &\epsilon^{-1}\,\{\mathcal{P}^b,\,\mathcal{J}_{ba}\} \sim 0 \\ &\mathcal{S}_{ab} + \epsilon^{-2}\,\mathcal{Q}_{ab} \sim 0 \end{aligned}}$$

$$\mathcal{I}_{ABCD} \sim 0 \;\;\Rightarrow\;\; \boxed{\begin{aligned} &\epsilon^{-1}\,\{\mathcal{J}_{[ab}\,,\mathcal{P}_{c]}\} \sim 0 \\ &\{\mathcal{J}_{[ab}\,,\,\mathcal{J}_{cd]}\} \sim 0 \end{aligned}}$$

$$C_2 \equiv \mathcal{J}^2 + \epsilon^{-2}\,\mathcal{P}^2 \sim -\frac{(D+1)(D-3)}{4}id \;\;\Rightarrow\;\; \boxed{\begin{aligned} &\mathcal{J}^2 \sim \frac{D-1}{D+1}C_2 \sim -\frac{(D-1)(D-3)}{4}\,id \\ &\epsilon^{-2}\,\mathcal{P}^2 \sim \frac{2}{D+1}C_2 \sim -\frac{D-3}{2}\,id\,, \end{aligned}}$$

Coset construction from $U(\mathrm{iso}(1,D\!-\!1))$

- iso(1,D−1) ideal

we kept the leading terms in the $\epsilon \to 0$ limit

$$\mathcal{P}_a \mathcal{P}_b \sim 0$$

$$\mathcal{I}_a \equiv \{\mathcal{P}^b,\, \mathcal{J}_{ba}\} \sim 0$$

$$\mathcal{I}_{abc} \equiv \{\mathcal{J}_{[ab}\,,\mathcal{P}_{c]}\} \sim 0$$

$$\mathcal{I}_{abcd} \equiv \{\mathcal{J}_{[ab}\,,\, \mathcal{J}_{cd]}\} \sim 0$$

$$\mathcal{J}^2 + \frac{(D-1)(D-3)}{4}\, id \sim 0$$

- Leftover quadratic combinations, i.e. spin-3 generators:

$$\mathcal{S}_{ab} \equiv \{\mathcal{J}^c{}_{(a},\mathcal{J}_{b)c}\} - \mathrm{tr.} \quad \simeq$$

$$\mathcal{M}_{ab|c} \equiv \{\mathcal{J}_{a(c},\mathcal{J}_{d)b}\} - \mathrm{tr.} \quad \simeq$$

$$\mathcal{K}_{ab|cd} \equiv \{\mathcal{P}_{(a},\mathcal{J}_{b)c}\} - \mathrm{tr.} \quad \simeq$$

$$\mathfrak{ihs}_D \equiv \mathcal{U}(\mathfrak{iso}(1,D-1))/\langle \mathcal{I}_{\mathfrak{c}} \rangle$$

Coset construction from $U(\mathrm{iso}(1,D\!-\!1))$

- iso(1,D−1) ideal

we kept the leading terms in the $\epsilon \to 0$ limit

$$\mathcal{P}_a\mathcal{P}_b \sim 0$$

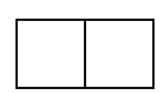
$$\mathcal{I}_a \equiv \{\mathcal{P}^b,\, \mathcal{J}_{ba}\} \sim 0$$

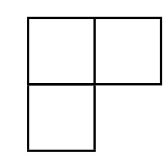
$$\mathcal{I}_{abc} \equiv \{\mathcal{J}_{[ab}\,,\mathcal{P}_{c]}\} \sim 0$$

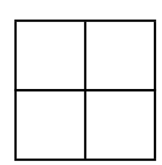
$$\mathcal{I}_{abcd} \equiv \{\mathcal{J}_{[ab}\,,\, \mathcal{J}_{cd]}\} \sim 0$$

$$\mathcal{J}^2 + \frac{(D-1)(D-3)}{4}\, id \sim 0$$

- Leftover quadratic combinations, i.e. spin-3 generators:

$$\mathcal{S}_{ab} \equiv \{\mathcal{J}^c{}_{(a},\mathcal{J}_{b)c}\} - \text{tr.} \simeq$$ 

$$\mathcal{M}_{ab|c} \equiv \{\mathcal{J}_{a(c},\mathcal{J}_{d)b}\} - \text{tr.} \simeq$$ 

$$\mathcal{K}_{ab|cd} \equiv \{\mathcal{P}_{(a},\mathcal{J}_{b)c}\} - \text{tr.} \simeq$$ 

$$\mathfrak{ihs}_D \equiv \mathcal{U}(\mathfrak{iso}(1,D-1))/\langle \mathcal{I}_{\mathfrak{c}} \rangle$$

Some commutators…

- All generators transform as Lorentz tensors

$$[\mathcal{J}_{ab}, \mathcal{S}_{cd}] = \eta_{ac}\mathcal{S}_{bd} + \eta_{ad}\mathcal{S}_{bc} - \eta_{bc}\mathcal{S}_{ad} - \eta_{bd}\mathcal{S}_{ac} \,,$$

$$[\mathcal{J}_{ab}, \mathcal{M}_{cd|e}] = 2\,\eta_{a(c}\mathcal{M}_{d)b|e} + \eta_{ae}\mathcal{M}_{cd|b} - 2\,\eta_{b(c}\mathcal{M}_{d)a|e} - \eta_{be}\mathcal{M}_{cd|a} \,,$$

$$[\mathcal{J}_{ab}, \mathcal{K}_{cd|ef}] = 2\left(\eta_{a(c}\mathcal{K}_{d)b|ef} + \eta_{a(e}\mathcal{K}_{f)b|cd} - \eta_{b(c}\mathcal{K}_{d)a|ef} - \eta_{b(e}\mathcal{K}_{f)a|cd}\right)$$

- Commutators with translations:

$$[\mathcal{P}_a, \mathcal{S}_{bc}] = -2\,\mathcal{M}_{bc|a} \,,$$

$$[\mathcal{P}_a, \mathcal{M}_{bc|d}] = 0 \,,$$

$$[\mathcal{P}_a, \mathcal{K}_{bc|de}] = -\,\eta_{ab}\,\mathcal{M}_{de|c} - \eta_{ac}\,\mathcal{M}_{de|b} - \eta_{ad}\,\mathcal{M}_{bc|e} - \eta_{ae}\,\mathcal{M}_{bc|d} - \frac{2}{D-2}\left(\eta_{d(b}\mathcal{M}_{c)e|a} + \eta_{e(b}\mathcal{M}_{c)d|a} - \eta_{bc}\mathcal{M}_{de|a} - \eta_{de}\mathcal{M}_{bc|a}\right)$$

Some commutators…

- All generators transform as Lorentz tensors

$$[\mathcal{J}_{ab}, \mathcal{S}_{cd}] = \eta_{ac}\mathcal{S}_{bd} + \eta_{ad}\mathcal{S}_{bc} - \eta_{bc}\mathcal{S}_{ad} - \eta_{bd}\mathcal{S}_{ac}\,,$$

$$[\mathcal{J}_{ab}, \mathcal{M}_{cd|e}] = 2\,\eta_{a(c}\mathcal{M}_{d)b|e} + \eta_{ae}\mathcal{M}_{cd|b} - 2\,\eta_{b(c}\mathcal{M}_{d)a|e} - \eta_{be}\mathcal{M}_{cd|a}\,,$$

$$[\mathcal{J}_{ab}, \mathcal{K}_{cd|ef}] = 2\left(\eta_{a(c}\,\mathcal{K}_{d)b|ef} + \eta_{a(e}\,\mathcal{K}_{f)b|cd} - \eta_{b(c}\,\mathcal{K}_{d)a|ef} - \eta_{b(e}\,\mathcal{K}_{f)a|cd}\right)$$

- Commutators with translations:

$$[\mathcal{P}_a, \mathcal{S}_{bc}] = -2\,\mathcal{M}_{bc|a}\,,$$

$$[\mathcal{P}_a, \mathcal{M}_{bc|d}] = 0\,,$$

$$[\mathcal{P}_a, \mathcal{K}_{bc|de}] = -\,\eta_{ab}\,\mathcal{M}_{de|c} - \eta_{ac}\,\mathcal{M}_{de|b} - \eta_{ad}\,\mathcal{M}_{bc|e} - \eta_{ae}\,\mathcal{M}_{bc|d} - \frac{2}{D-2}\left(\eta_{d(b}\mathcal{M}_{c)e|a} + \eta_{e(b}\mathcal{M}_{c)d|a} - \eta_{bc}\mathcal{M}_{de|a} - \eta_{de}\mathcal{M}_{bc|a}\right)$$

WARNING: the linearised curvatures do not reproduce those of Fradkin and Vasiliev

Structure of the algebra

- Higher-spin generators

$$\mathcal{Z}^{s,t} \equiv \begin{array}{|c|}\hline s-1 \\ \hline s-t-1 \\ \hline\end{array} \quad \text{with } t \in \{0,\dots,s-1\}$$

- t even: no P's
- t odd: one P

- Commutators with P

For D=4 see also
Fradkin, Vasiliev (1987)

$$\boxed{\begin{array}{ll} \left[\mathcal{P}, \mathcal{Z}^{(s,t)}\right] \propto \mathcal{Z}^{(s,t-1)} + \eta\, \mathcal{Z}^{(s,t+1)} & \text{for } t \text{ even} \\ \left[\mathcal{P}, \mathcal{Z}^{(s,t)}\right] = 0 & \text{for } t \text{ odd} \end{array}}$$

- $\mathfrak{ihs}_D$ as Inönü-Wigner contraction of $\mathfrak{hs}_D$

$$\left[\mathcal{Z}^{(s_1,t_1)}, \mathcal{Z}^{(s_2,t_2)}\right] \propto \sum_{s_3,t_3} \mathcal{Z}^{(s_3,t_3)} \quad \text{with} \quad \begin{array}{l} s_1+s_2-s_3 \mod 2 = 0 \\ t_1+t_2-t_3 \mod 2 = 0 \end{array}$$

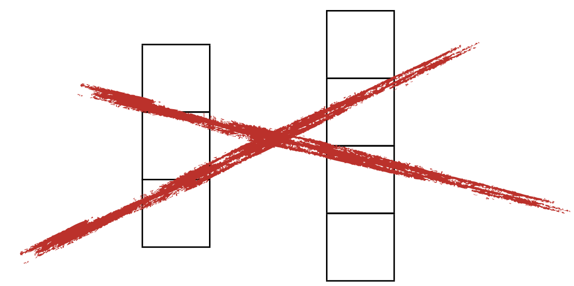
$$\Rightarrow \quad \boxed{\mathcal{Z}^{(s,t)} \to \epsilon^{-1}\, \mathcal{Z}^{(s,t)} \quad \text{for } t \text{ odd}}$$

Classification of consistent ideals

- Can one build other conformal Carrollian HS algebras from *U*(iso(1,*D*–1))?

- Portion of the ideal we need to quotient out:

$$\mathcal{I}_{ABCD} \sim 0 \;\; \Rightarrow \;\; \boxed{\begin{aligned} \epsilon^{-1}\,\{\mathcal{J}_{[ab}\,,\mathcal{P}_{c]}\} &\sim 0 \\ \{\mathcal{J}_{[ab}\,,\,\mathcal{J}_{cd]}\} &\sim 0 \end{aligned}}$$



- Candidate spin-3 generators:

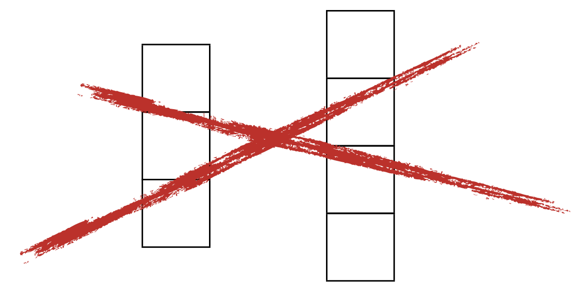
$$\{\mathcal{P}_\mu,\mathcal{P}_\nu\} - \text{tr.} \;\simeq\; \square\!\square \qquad \{\mathcal{J}^\rho{}_{(\mu},\mathcal{J}_{\nu)\rho}\} - \text{tr.} \;\simeq\; \square\!\square$$

Classification of consistent ideals

- Can one build other conformal Carrollian HS algebras from $U(\mathrm{iso}(1,D{-}1))$?

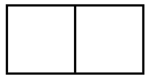
- Portion of the ideal we need to quotient out:

$$\mathcal{I}_{ABCD} \sim 0 \quad \Rightarrow \quad \boxed{\begin{array}{c} \epsilon^{-1}\,\{\mathcal{J}_{[ab}\,,\mathcal{P}_{c]}\} \sim 0 \\ \{\mathcal{J}_{[ab}\,,\,\mathcal{J}_{cd]}\} \sim 0 \end{array}}$$

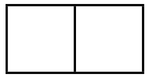


- Candidate spin-3 generators:

$$\{\mathcal{P}_\mu,\mathcal{P}_\nu\} - \mathrm{tr.} \;\simeq$$

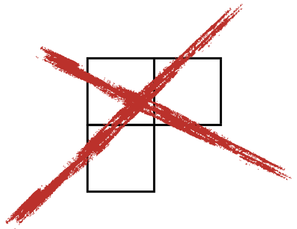


$$\{\mathcal{J}^\rho{}_{(\mu},\mathcal{J}_{\nu)\rho}\} - \mathrm{tr.} \;\simeq$$



- Can one use $P_\mu P_\nu$ as spin-3 generator?

$$[\mathcal{P}_\alpha\,,\,\cancel{\mathcal{J}^\rho{}_{(\mu}\mathcal{J}_{\nu)\rho} - \frac{2}{D}\eta_{\mu\nu}\,\mathcal{J}^2}] = \{\mathcal{J}_{\alpha(\mu},\mathcal{P}_{\nu)}\} + \cdots \quad \Rightarrow$$



Partial summary

- One can build *non-Abelian HS algebras* including iso(1,D–1) as a subalgebra (with the same spectrum as in AdS)
- “Good” Lorentz commutators guaranteed by UEA construction
- Atypical commutators with translations (counterpart of the absence of the “naive” minimal coupling?)
- The linearised **torsions** do not allow one to eliminate the HS auxiliary “spin-connections" à la Fradkin-Vasiliev

Can we recover these algebras asymptotically?

Part 2

Higher-spin asymptotic symmetries in flat space

A.C., D. Francia, C. Heissenberg,

1703.01351, 1712.09591, 2011.04420

Warming up: BMS symmetry

from Fierz-Pauli

The setup

- Action: $S = \int d^D x \, h^{\mu\nu} \left(R_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} R \right)$ (Fierz-Pauli)

$$R_{\mu\nu} = \Box h_{\mu\nu} - \partial_{(\mu} \partial \cdot h_{\nu)} + \partial_\mu \partial_n h_\lambda{}^\lambda$$

- Gauge symmetry: $\delta h_{\mu\nu} = \partial_{(\mu} \epsilon_{\nu)}$

- Minkowski in retarded Bondi coordinates:

$$ds^2 = -du^2 - 2dudr + r^2 \gamma_{ij} \, dx^i dx^j$$

- *Bondi "gauge":*

$$h_{rr} = h_{ru} = h_{ri} = 0 \quad \& \quad h_\lambda{}^\lambda = 0$$

Residual symmetries of the Bondi gauge

- u-independent linearised diffeos preserving the Bondi gauge:

$$\epsilon_\mu dx^\mu = T(\hat{\mathbf{x}})dr + \frac{1}{D-2}\left(\Delta + D - 2\right) T(\hat{\mathbf{x}})du + r\,\mathcal{D}_i T(\hat{\mathbf{x}})dx^i$$

arbitrary function & vector on the celestial sphere

- Full set of residual symmetries:

$$\epsilon_r = f$$

$$\epsilon_i = r^2 v_i + r\,\partial_i f \qquad \text{with} \quad f(u,\hat{\mathbf{x}}) = T(\hat{\mathbf{x}}) - \frac{u}{D-2}\mathcal{D}_i v^i(\hat{\mathbf{x}})$$

$$\epsilon_u = \epsilon_r + \frac{1}{r\,(D-2)}\mathcal{D}_i \epsilon^i$$

- ***We still have to set the boundary conditions on h_{uu}, h_{ui} and h_{ij}!***

Supertranslations & superrotations

- Variation of the component h_{ij}

$$\delta h_{ij} = r^2 \left\{ \mathcal{D}_{(i} v_{j)} - \frac{2}{D-2} \gamma_{ij} \mathcal{D} \cdot v \right\} + 2r \left\{ \mathcal{D}_i \mathcal{D}_j - \frac{1}{D-2} \gamma_{ij} \Delta \right\} f$$

- Typical falloff of radiation: $h_{ij} = \mathcal{O}(r^{3-\frac{D}{2}}) \xrightarrow[D=4]{} \mathcal{O}(r)$

Supertranslations & superrotations

- Variation of the component h_{ij}

$$\delta h_{ij} = r^2 \underbrace{\left\{ \mathcal{D}_{(i} v_{j)} - \frac{2}{D-2}\, \gamma_{ij} \mathcal{D} \cdot v \right\}}_{0} + 2r \left\{ \mathcal{D}_i \mathcal{D}_j - \frac{1}{D-2}\, \gamma_{ij} \Delta \right\} f$$

- Typical falloff of radiation: $h_{ij} = \mathcal{O}(r^{3-\frac{D}{2}}) \xrightarrow[D=4]{} \mathcal{O}(r)$

- Boundary conditions in *D=4*:

  • "natural option": $h_{ij} = \mathcal{O}(r) \quad \Rightarrow \quad \begin{cases} T(\hat{\mathbf{x}}) \;\; \text{arbitrary} \\ \mathcal{D}_{(i} v_{j)} - \frac{2}{D-2}\, \gamma_{ij} \mathcal{D} \cdot v = 0 \end{cases}$

Barnich, Troessaert (2010)

Supertranslations & superrotations

- Variation of the component h_{ij}

$$\delta h_{ij} = r^2 \left\{ \mathcal{D}_{(i} v_{j)} - \frac{2}{D-2}\, \gamma_{ij} \mathcal{D}\cdot v \right\} + 2r \left\{ \mathcal{D}_i \mathcal{D}_j - \frac{1}{D-2}\, \gamma_{ij} \Delta \right\} f$$

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    Barnich, Troessaert (2010)

  - other option: $h_{ij} = \mathcal{O}(r^2) \quad \Rightarrow \quad T(\hat{\mathbf{x}}) \ \ \& \ \ v_i(\hat{\mathbf{x}}) \ \text{ arbitrary}$

    Campiglia, Laddha (2014)

related to the subleading soft graviton theorem

Cachazo, Strominger (2014)

Supertranslations & superrotations in any *D*

- Variation of the component h_{ij}

$$\delta h_{ij} = r^2 \left\{ \mathcal{D}_{(i} v_{j)} - \frac{2}{D-2}\, \gamma_{ij} \mathcal{D} \cdot v \right\} + 2r \left\{ \mathcal{D}_i \mathcal{D}_j - \frac{1}{D-2}\, \gamma_{ij} \Delta \right\} f$$

- Typical falloff of radiation: $h_{ij} = \mathcal{O}(r^{3-\frac{D}{2}})$

Supertranslations & superrotations in any *D*

Hollands, Ishibashi (2005); Tanabe, Kinoshita, Shiromizu (2011); Hollands, Ishibashi, Wald (2017)

- Variation of the component h_{ij}

$$\delta h_{ij} = r^2 \underbrace{\left\{ \mathcal{D}_{(i} v_{j)} - \frac{2}{D-2}\, \gamma_{ij} \mathcal{D} \cdot v \right\}}_{0} + 2r \underbrace{\left\{ \mathcal{D}_i \mathcal{D}_j - \frac{1}{D-2}\, \gamma_{ij} \Delta \right\}}_{0} f$$

- Typical falloff of radiation: $h_{ij} = \mathcal{O}(r^{3-\frac{D}{2}})$

- Boundary conditions in *any D*:

  - "natural option": $h_{ij} = \mathcal{O}(r^{3-\frac{D}{2}}) \quad \Rightarrow$ Poincaré residual symmetry

Supertranslations & superrotations in any *D*

Kapec, Lysov, Pasterski, Strominger (2015)

- Variation of the component h_{ij}

$$\delta h_{ij} = r^2 \underbrace{\left\{ \mathcal{D}_{(i} v_{j)} - \frac{2}{D-2}\, \gamma_{ij} \mathcal{D} \cdot v \right\}}_{0} + 2r \left\{ \mathcal{D}_i \mathcal{D}_j - \frac{1}{D-2}\, \gamma_{ij} \Delta \right\} f$$

- Typical falloff of radiation: $h_{ij} = \mathcal{O}(r^{3-\frac{D}{2}})$

- Boundary conditions in *any D*:
  - “natural option”: $h_{ij} = \mathcal{O}(r^{3-\frac{D}{2}})$ $\Rightarrow$ Poincaré residual symmetry
  - option 2: $h_{ij} = \mathcal{O}(r)$ $\Rightarrow$ $T(\hat{\mathbf{x}})$ arbitrary + Lorentz

Supertranslations & superrotations in any D

- Variation of the component h_{ij}

Avery, Schwab (2015); Capone (2018); Colferai, Lionetti (2020); AC, Francia, Heissenberg (2020)

$$\delta h_{ij} = r^2 \left\{ \mathcal{D}_{(i} v_{j)} - \frac{2}{D-2}\,\gamma_{ij} \mathcal{D}\cdot v \right\} + 2r \left\{ \mathcal{D}_i \mathcal{D}_j - \frac{1}{D-2}\,\gamma_{ij}\Delta \right\} f$$

- Typical falloff of radiation: $h_{ij} = \mathcal{O}(r^{3-\frac{D}{2}})$

- Boundary conditions in *any D*:
  - "natural option": $h_{ij} = \mathcal{O}(r^{3-\frac{D}{2}})$ $\Rightarrow$ Poincaré residual symmetry
  - option 2: $h_{ij} = \mathcal{O}(r)$ $\Rightarrow$ $T(\hat{\mathbf{x}})$ arbitrary + Lorentz
  - option 3: $h_{ij} = \mathcal{O}(r^2)$ $\Rightarrow$ $T(\hat{\mathbf{x}})$ & $v_i(\hat{\mathbf{x}})$ arbitrary

Higher-spin supertranslations

&

Weinberg’s theorem

Higher-spin asymptotic symmetries: the setup

- Action: $$S = \int d^D x \, \varphi^{\mu_1 \cdots \mu_s} \left(\mathcal{F}_{\mu_1 \cdots \mu_s} - \frac{1}{2} \, \eta_{(\mu_1 \mu_2} \mathcal{F}_{\mu_3 \cdots \mu_s)\lambda}{}^{\lambda} \right)$$ Fronsdal (1978)

$$\mathcal{F}_{\mu_1 \cdots \mu_s} = \Box \varphi_{\mu_1 \cdots \mu_s} - \partial_{(\mu_1} \partial \cdot \varphi_{\mu_2 \cdots \mu_s)} + \partial_{(\mu_1} \partial_{\mu_2} \varphi'_{\mu_3 \cdots \mu_{s-2})\lambda}{}^{\lambda}$$

- Gauge symmetry: $\delta \varphi_{\mu_1 \cdots \mu_s} = \partial_{(\mu_1} \epsilon_{\mu_2 \cdots \mu_s)}$ (with traceless ϵ)

Higher-spin asymptotic symmetries: the setup

- Action: $$S = \int d^D x\, \varphi^{\mu_s} \left(\mathcal{F}_{\mu_s} - \frac{1}{2}\, \eta_{\mu\mu} \mathcal{F}'_{\mu_{s-2}} \right)$$ Fronsdal (1978)

$$\mathcal{F}_{\mu_s} = \Box \varphi_{\mu_s} - \partial_\mu \partial \cdot \varphi_{\mu_{s-1}} + \partial_\mu \partial_\mu \varphi'_{\mu_{s-2}}$$

- Gauge symmetry: $\delta\varphi_{\mu_s} = \partial_\mu \epsilon_{\mu_{s-1}}$ (with traceless ϵ)

Higher-spin asymptotic symmetries: the setup

- Action: $S = \int d^D x \, \varphi^{\mu_s} \left(\mathcal{F}_{\mu_s} - \frac{1}{2} \eta_{\mu\mu} \mathcal{F}'_{\mu_{s-2}} \right)$ Fronsdal (1978)

$$\mathcal{F}_{\mu_s} = \Box \varphi_{\mu_s} - \partial_\mu \partial \cdot \varphi_{\mu_{s-1}} + \partial_\mu \partial_\mu \varphi'_{\mu_{s-2}}$$

- Gauge symmetry: $\delta \varphi_{\mu_s} = \partial_\mu \epsilon_{\mu_{s-1}}$ (with traceless ϵ)

- Minkowski in retarded Bondi coordinates:

$$ds^2 = -du^2 - 2dudr + r^2 \gamma_{ij} \, dx^i dx^j$$

- ***Bondi-like "gauge"*** (or *part 1* of the boundary conditions)

$$\varphi_{r\mu_{s-1}} = 0 = \gamma^{ij} \varphi_{ij\mu_{s-2}}$$

AC, Francia, Heissenberg (2017 and 2020)

Boundary conds I: HS supertranslations

- Bondi-like gauge

AC, Francia, Heissenberg (2017 and 2020)

$$\varphi_{r\mu_{s-1}} = 0 = \gamma^{ij}\varphi_{ij\mu_{s-2}}$$

- Remaining field components? (part 2 of the boundary conditions)

	Falloffs	Asymptotic symmetries
D = 4	$\varphi_{u_{s-k}i_k} = \mathcal{O}(r^{k-1})$	*infinite dimensional*
Any D	$\varphi_{u_{s-k}i_k} = \mathcal{O}(r^{k+1-\frac{D}{2}})$	*only global Killing symmetries*
Any D	$\varphi_{u_{s-k}i_k} = \mathcal{O}(r^{k-1})$	*infinite dimensional*

u-independent asymptotic symmetries

- *u*-independent residual symmetries of the Bondi-like gauge:

$$\epsilon^{u_{s-k-1} i_k} \propto r^{-k}\, \mathcal{D}^i \cdots \mathcal{D}^i T(\hat{\mathbf{x}}) + \cdots$$

(depend on an arbitrary function on the celestial sphere)

- Spin-3 example:

$$\epsilon^{uu} = T(\hat{\mathbf{x}})\,, \qquad \epsilon^{ui} = -\frac{1}{r}\,\partial^i T(\hat{\mathbf{x}})\,, \qquad \epsilon^{ij} = \frac{1}{2r^2}\left[\mathcal{D}^i\mathcal{D}^j - \frac{1}{D}\,\gamma^{ij}(\Delta - 2)\right] T(\hat{\mathbf{x}})$$

- Compatible with $\varphi_{u_{s-k} i_k} = \mathcal{O}(r^{k-1})$ but not with radiation falloffs!

- OK, you got infinite-dimensional symmetries… but *what is the interpretation of the terms "above radiation"*?

u-independent asymptotic symmetries

- Obs 1: on shell the overleading terms must be *pure gauge*

$$\varphi_{u_{s-k} i_k} = r^{k-1} \frac{k(D+k-5)!}{s(D+s-5)!} (\mathcal{D}\cdot)^{s-k} C_{i_k}{}^{(1-s)}(\hat{\mathbf{x}}) + \mathcal{O}\left(r^{k+1-\frac{D}{2}}\right)$$

(so they are perfectly fine at least for $s = 1$)

- Obs 2: they do not contribute to surface charges for any s

$$\begin{aligned}
&(-1)^{s-1} \mathcal{Q}_T(u) \\
&= \lim_{r\to\infty} r^{D-3} \oint d\Omega_{D-2} \sum_{k=0}^{s-1} \frac{r^{-k}}{k!} T \left[(s-k-2)\, r\partial_r + (s-k-1)(D-k-2) \right] (\mathcal{D}\cdot)^k \varphi_{u_{s-k}} \\
&= \lim_{r\to\infty} r^{D-4} \underbrace{\left(\sum_{k=1}^{s-1} \alpha_k \right)}_{0} \oint d\Omega_{D-2}\, T\, (\mathcal{D}\cdot)^s C^{(1-s)} + \mathcal{O}(r^{\frac{D-4}{2}}),
\end{aligned}$$

Comments on surface charges

- The overleading terms do not contribute, but a divergent contribution from radiation is still present!

- A "prescription" curing this problem (and giving the "correct" Ward identities): AC, Francia, Heissenberg (2017 and 2020)

  - Assume that for $u < u_0$ the fields are stationary
  - Compute the (finite!) charge for $u < u_0$
  - Define $Q_T(u)$ as the evolution under the eom of $Q_T(-\infty)$

- Final result: $$\boxed{\mathcal{Q}_T(u) \propto \oint d\Omega_{D-2}\, T(\hat{\mathbf{x}})\, \mathcal{U}^{(0)}(u, \hat{\mathbf{x}})}$$

it should be possible to recover it from a more systematic renormalisation… See the works by Adrien, Romain and Laurent

(with $\varphi_{u\cdots u} = r^{3-D}\mathcal{U}^{(0)}(u, \hat{\mathbf{x}}) + \cdots$)

Recovering Weinberg's theorem

- "Standard" techniques to recover Weinberg's theorem apply
  - rewriting of the charge: $\mathcal{Q}_T\big|_{\mathscr{I}^+_-} = \mathcal{Q}_T\big|_{\mathscr{I}^+_+} - \int_{-\infty}^{+\infty} \frac{d\mathcal{Q}_T(u)}{du}\, du$,
  - $\mathcal{Q}_T\big|_{\mathscr{I}^+_-} = (-1)^s\,(D+s-4)\int_{-\infty}^{+\infty} du \oint d\Omega_{D-2}\, T(\hat{\mathbf{x}})\,\partial_u\,\mathcal{U}^{(0)}(u,\hat{\mathbf{x}})$,
  - eom: $\partial_u^{\frac{D-4}{2}}\,\mathcal{U}^{(0)} = \dfrac{\mathscr{D}\,(\mathcal{D}\cdot)^3\, C^{\left(\frac{D-8}{2}\right)}}{(D-1)(D-2)(D-3)}$
- *The charge can be rewritten in terms of radiation data in any D!*
- Obs 3: Weinberg's theorem follows by substitution in the Ward identity

$$\langle\mathrm{out}|\left(\mathcal{Q}_{\mathscr{I}^+_-}S - S\mathcal{Q}_{\mathscr{I}^-_+}\right)|\mathrm{in}\rangle = \sum_\ell g_\ell^{(3)} E_\ell^2\, T(\hat{\mathbf{x}}_\ell)\langle\mathrm{out}|S|\mathrm{in}\rangle$$

Awery, Schwab (2015)

requires antipodal matching

Higher-spin superrotations

&

???

Higher-spin superrotations

- Back to the *residual symmetries of the Bondi-like gauge* (spin 3)

$$\epsilon_{rr} = f \,,$$

$$\epsilon_{ri} = r^2 v_i + \frac{r}{2}\,\partial_i f \,,$$

$$\epsilon_{ij} = r^4 K_{ij} + r^3 \left(\mathcal{D}_{(i} v_{j)} - \frac{2}{D-1}\,\gamma_{ij} \mathcal{D}\cdot v \right) + \frac{r^2}{2} \left(\mathcal{D}_i \mathcal{D}_j - \frac{1}{D}\,\gamma_{ij}\,(\Delta - 2) \right) f$$

with

$$K_{ij} = K_{ij}(\hat{\mathbf{x}}) \,,$$

$$v_i = \rho_i(\hat{\mathbf{x}}) - \frac{u}{D}\,\mathcal{D}\cdot K_i(\hat{\mathbf{x}}) \,,$$

$$f = T(\hat{\mathbf{x}}) - \frac{2u}{D-1}\,\mathcal{D}\cdot\rho(\hat{\mathbf{x}}) + \frac{u^2}{D(D-1)}\,\mathcal{D}\cdot\mathcal{D}\cdot K(\hat{\mathbf{x}}) \,.$$

Higher-spin superrotations

- Back to the *residual symmetries of the Bondi-like gauge* (spin 3)

$$K_{ij} = K_{ij}(\hat{\mathbf{x}})\,,$$
$$v_i = \rho_i(\hat{\mathbf{x}}) - \frac{u}{D}\,\mathcal{D}\cdot K_i(\hat{\mathbf{x}})\,,$$
$$f = T(\hat{\mathbf{x}}) - \frac{2u}{D-1}\,\mathcal{D}\cdot\rho(\hat{\mathbf{x}}) + \frac{u^2}{D(D-1)}\,\mathcal{D}\cdot\mathcal{D}\cdot K(\hat{\mathbf{x}})\,.$$

- Induced variations of non-vanishing field components

$$\begin{aligned}\delta\varphi_{ijk} = {} & r^4\left\{\mathcal{D}_{(i}K_{jk)} - \frac{2}{D}\,\gamma_{(ij}D\cdot K_{k)}\right\}\\ & + r^3\left\{\mathcal{D}_{(i}\mathcal{D}_j\rho_{k)} - \frac{2}{D}\,\gamma_{(ij}\left[(\Delta + D - 3)\,\rho_{k)} + 2\,\mathcal{D}_{k)}D\cdot\rho\right]\right\}\\ & + \frac{r^2}{2}\left\{\mathcal{D}_{(i}\mathcal{D}_j\mathcal{D}_{k)}T - \frac{2}{D}\,\gamma_{(ij}\mathcal{D}_{k)}\,(3\,\Delta + 2(D-3))\,T\right\}\end{aligned}$$

Higher-spin superrotations

- Back to the *residual symmetries of the Bondi-like gauge* (spin 3)

$$K_{ij} = K_{ij}(\hat{\mathbf{x}})\,,$$
$$v_i = \rho_i(\hat{\mathbf{x}}) - \frac{u}{D}\,\mathcal{D}\cdot K_i(\hat{\mathbf{x}})\,,$$
$$f = T(\hat{\mathbf{x}}) - \frac{2u}{D-1}\,\mathcal{D}\cdot\rho(\hat{\mathbf{x}}) + \frac{u^2}{D(D-1)}\,\mathcal{D}\cdot\mathcal{D}\cdot K(\hat{\mathbf{x}})\,.$$

- Induced variations of non-vanishing field components

$$\delta\varphi_{ijk} = r^4\left\{\mathcal{D}_{(i}K_{jk)} - \frac{2}{D}\,\gamma_{(ij}D\cdot K_{k)}\right\} = 0$$
$$+\, r^3\left\{\mathcal{D}_{(i}\mathcal{D}_j\rho_{k)} - \frac{2}{D}\,\gamma_{(ij}\left[(\Delta + D - 3)\,\rho_{k)} + 2\,\mathcal{D}_{k)}D\cdot\rho\right]\right\} = 0$$
$$+\,\frac{r^2}{2}\left\{\mathcal{D}_{(i}\mathcal{D}_j\mathcal{D}_{k)}T - \frac{2}{D}\,\gamma_{(ij}\mathcal{D}_{k)}\,(3\,\Delta + 2(D-3))\,T\right\} = 0$$

only global Killing symmetries

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$$\varphi_{ijk} = \mathcal{O}(r^2)$$

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supertranslations
+ Lorentz (if D>4)

Higher-spin superrotations

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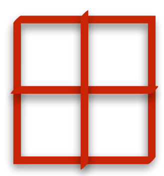
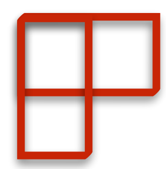
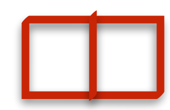
supertranslations
+ superrotations

Boundary conds II: higher-spin superrotations

- Summary: $\varphi_{u_{s-k}i_k} = \mathcal{O}(r^{s+k-2}) \;\Rightarrow\;$ *HS superrotations*

- Interpretation?
  - s = 2 $\quad \delta h_{ij} = r^2\left(\mathcal{D}_{(i}v_{j)} - \frac{2}{D-2}\gamma_{ij}\mathcal{D}\cdot v\right) + \mathcal{O}(r)$ — Campiglia, Laddha (2014)
  - s = 3 $\quad K_{ij} \sim$  $\quad \rho_i \sim$  $\quad T \sim$  — Global symmetries of a spin-3 field!

- Do they make sense?
  - *Overleading terms are still pure gauge*
  - *We recover all structures in the “rigid symmetries"*
  - Charges? More problematic…

cf., however,
Compère, Fiorucci, Ruzziconi (2018);
Freidel, Hopfmuller, Riello (2019);
Colferai, Lionetti (2020);

Summary & overview

- Boundary conditions allowing angle dependent asymptotic symmetries can be defined for any D and any s (part 2 of the talk)
- All contributions above radiation are (large) pure-gauge terms
- u-independent symmetries ⇒ (any-s) supertranslations
- Supertranslation Ward identities ⇒ Weinberg's soft theorems
- Even weaker falloffs ⇒ (any-s) superrotations

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- The global portion of these symmetries is in one-to-one correspondence with the "HS isometries" of the vacuum
- One can define a Lie bracket for "HS isometries" (part 1 of the talk)
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Right symmetry, but for the "wrong" setup?

A higher-spin extension of the Poincaré algebra may provide the natural extension of the $w_{1+\infty}$ symmetry in D=4… (see Laurent's talk)