Backstepping observer design for a tubular catalytic cracking reactor

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Abstract

Distributed parameter systems (DPSs) are a class of important processes in which process variables depend not only on time but also on spatial coordinates. The description of DPSs often takes the form of hyperbolic, parabolic or elliptic partial differential equations (PDEs). Parabolic PDEs represent the dynamics of industrial processes involving convection and diffusion effects. One of the most important examples of such class of systems is the chemical tubular reactor (CTR) with axial dispersion. In order to capture the effects of reactions, diffusion and convection in the reactor, the reactor model may take the form of a set of coupled parabolic PDEs.

This work aims at designing boundary observers for systems described by sets of nonlinear parabolic PDEs. By linearizing the nonlinear equations about the steady state profile of the system, a set of linear parabolic PDEs with spatially varying coefficients is produced. Then, the state estimation problem is transformed into a well-posed boundary stabilization problem for the dynamics of the state estimation error which is approached using the backstepping method.

The backstepping method for PDEs, as it is known today, was first introduced in the seminal work of Smyshlyaev and Krstic (Smyshlyaev and Krstic, 2004). Their approach, first developed for a general 1-D linear reaction-diffusion-advection PDE, is based on a constructive strategy with a design in the continuous space domain followed by discretization for implementation and simulation. The backstepping method has three main stages:

1. the selection of a target system which verifies the desired properties (typically stability, proven with a Lyapunov function), but still closely resembles the original system;
2. the use of an invertible integral transformation (the backstepping transformation), that maps the original plant into the target system in the appropriate functional spaces;
3. and the kernel equations, which are determined from the original and target systems and the transformation, and whose solution determine the kernel of the integral transformation. These equations can be usually proven solvable by transforming them to integral equations and then using the method of successive approximations or numerical methods.

These stages are closely connected. A suitable choice of the target system will result in solvable kernel equations and an invertible transformation. The observer gains is then determined so that the error system is converted into the selected exponentially stable target system by the backstepping transformation. The resulting observer gains stabilize the error system exponentially with a given decay rate. The backstepping observer has been extended to systems described by other types of PDEs (Krstic et al., 2008; Vazquez and Krstic, 2010; Krstic et al., 2011). More recently, systems of coupled PDEs were considered in the backstepping-based boundary control and observer design settings. The most intensive efforts of the current literature seem however to be oriented towards coupled hyperbolic processes of the transport type PDEs (Vazquez et al., 2011; Moura et al., 2013).

The motivation behind the present study is the observer design for the tubular catalytic cracking reactor process (Mohammadi et al., 2012) described by the following coupled mass-balance parabolic PDEs:

$$\frac{\partial x_A}{\partial t}(z,t) = \gamma \frac{\partial^2 x_A}{\partial z^2}(z,t) - \upsilon \frac{\partial x_A}{\partial z}(z,t) - k_0 {x_A}^2,$$
$$\frac{\partial x_B}{\partial t}(z,t) = \gamma \frac{\partial^2 x_B}{\partial z^2}(z,t) - \upsilon \frac{\partial x_B}{\partial z}(z,t) + k_1 {x_A}^2 - k_2 x_B. \quad (1)$$

This application requires an extension of the method which is far from being trivial because the underlying backstepping treatment gives rise to complex developments to find an explicit form of the observer gains using matrix Bessel series. Figure 1 shows the evolution of the actual states (red lines) and the estimated states (blue lines) related to the proposed observer.

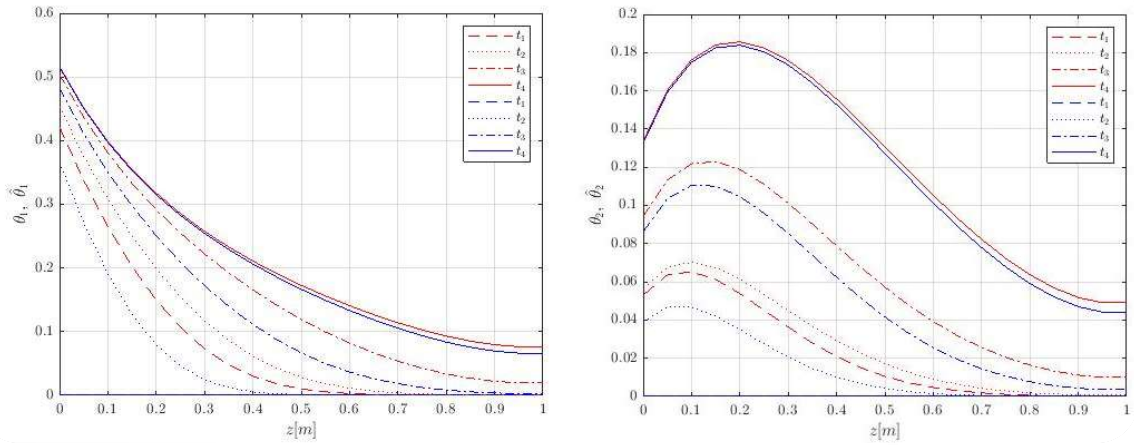

Figure 1: Actual and estimated states for the tubular catalytic cracking reactor.

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