A Robust Quality Measure for Quality-Guided 2D Phase Unwrapping Algorithms Based on Histogram Processing

Ambroise Moreau, Matei Mancas, Thierry Dutoit

Numediart Institute, University of Mons, Belgium

Abstract

Among two-dimensional phase unwrapping algorithms, the quality-guided strategy offers one of the best trade-off between speed and accuracy. it assigns a quality value, also called reliability, to each pixel and removes the wraps on a path that goes from the highest to the lowest quality region. By doing so, challenging regions are unwrapped last, avoiding error propagation on the entire phase map. Strictly sorting the data by quality value is time-consuming but the process can be accelerated if they are roughly sorted in histogram bins that span the quality range instead. Nevertheless, using this histogram sorting scheme on existing quality measures can lead to erroneous results in some cases such as the presence of physical discontinuities in the true unwrapped phase map. In this paper, we propose a quality criterion that meets the requirements of histogram processing. Our method has been tested on challenging synthetic phase map with physical discontinuities and in the presence of noise. It shows encouraging results for data provided by shape measurement methods like phase-shifting profilometry.

Keywords: 2D Phase Unwrapping, Quality-Guided Phase Unwrapping, Image Processing, Shape Measurement, Phase-Shifting Profilometry.

1 Introduction

Phase unwrapping is an inherent problem in many three-dimensional shape measurement methods such as phase-shifting profilometry [Cong et al., 2015], magnetic resonance imaging [Maier et al., 2015], shearography [Van Brug, 1998] or interferometric synthetic aperture radar [Danudirdjo and Hirose, 2015]. These methods recover the phase of a two-dimensional modulated signal to measure physical quantities like ground elevation or surface profile. The retrieved phase map is usually constrained to its principal interval $(-\pi, \pi]$ and shows discontinuities that are not present in the original data. These discontinuities, or phase wraps, have to be removed through phase unwrapping.

For simple phase maps, phase unwrapping is an easy integration problem: the true phase value of each pixel is found by adding multiples of 2π based on the unwrapped neighbours [da Silva Maciel and Albertazzi, 2014]. For real phase maps, the process may face complications because of true discontinuities in the underlying physical quantity, undersampling in local areas or high local variations of signal-to-noise ratio [Herráez et al., 2002]. In these conditions, phase unwrapping becomes path-dependent. To overcome these difficulties, various methods that can be classified as temporal phase unwrapping or spatial phase unwrapping have been proposed in the past decades. As stated by Zhang et al., temporal algorithms are effective and robust but require several wrapped phase maps along the time dimension while spatial approaches works with a single phase map but do not deal well with disjoint regions and true phase discontinuities [Zhang et al., 2014].

Spatial methods can be further divided into global error minimization algorithms, quality-guided phase unwrapping, branch-cut approaches and region-growing methods [Herráez et al., 2002]. Algorithms falling in the second category are known to offer the best trade-off between speed and accuracy. They rely on a quality map to measure the reliability of each pixel and unwrap the phase on a path that goes from the highest to the lowest quality region, avoiding error propagation on the entire phase map. Sorting pixels according to their quality value is a time-consuming task. To speed up the process, Lei et al. proposed a novel method based on histogram processing, but depending on the chosen quality criterion, the final result can be erroneous [Lei et al., 2015].

In this paper, we propose a new quality criterion, designed to be unaffected by the histogram sorting process. The remainder of the paper is organized as follows. Section 2 reviews the related work and gives an overview of existing quality criteria. Section 3 presents the new measure of quality. In Section 4, our algorithm is compared with two other quality criteria: the second differences reliability (SDR) [Herráez et al., 2002] and the random tilt reliability (RTR) [Arevalillo-Herráez et al., 2016]. Finally, Section 5 concludes on our work.

2 State of the Art

Many phase-measuring techniques use the phase of a modulated signal to measure a physical quantity such as the shape of an object. For instance, in phase-shifting profilometry, sinusoidal patterns projected on a scene, and deformed by its height, are observed with a camera which enables to recover the wrapped phase map by using the arctangent function on a combination of the captured images [Cong et al., 2015].

Mathematically, the spatial phase unwrapping of a wrapped pixel a, neighbour of an unwrapped pixel b, can be written as:

$$\Phi_a = \phi_a + 2\pi \cdot r\left(\frac{\Phi_b - \phi_a}{2\pi}\right) \tag{1}$$

where ϕ stands for the wrapped phase map, Φ stands for the unwrapped phase and $r(.)$ is a function that rounds its input to the closest integer. As explained by Itoh, the only requirement of Equation (1) is that the difference of the unwrapped phase between neighbouring pixels should fall within the principal interval $(-\pi, \pi]$ [Itoh, 1982]. The simplest unwrapping algorithm would go through the entire phase map, row after row, and remove the wraps using Equation (1). However, in practical cases, the final result could be wrong for four reasons: (i) sharp differences exceeding the principal interval caused by undersampling; (ii) noise leading to wrong unwrapped values; (iii) invalid area like shadows in FPP leading to error propagation; (iv) true discontinuities in the physical quantity mistaken for continuities [Zhang et al., 2014, Zhao et al., 2011]. Moreover, if one pixel is wrongly unwrapped, the subsequent ones will be erroneous too. Phase unwrapping becomes path-dependent. Quality-guided phase unwrapping solves these problems by assigning a quality value to each pixel from the wrapped phase map and following a path that unwraps problematic pixels in the end.

Depending on the application, the quality parameter can be computed from the raw measurement data or from the wrapped phase map itself [Zhao et al., 2011]. The correlation coefficient map in inSAR, the modulation map and the reliability map from least square fitting in phase-shifting profilometry belong to the first category whereas quality measures like the phase derivative variance map and the first and second phase differences maps belong to the second category. In this paper, we present a new quality criterion and compare it with two existing measures: the second differences reliability (SDR) [Herráez et al., 2002] and the random tilt reliability (RTR) [Arevalillo-Herráez et al., 2016].

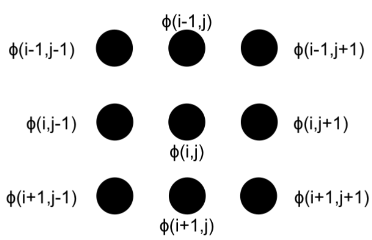


Figure 1: 3x3 window centered on the pixel of interest

SDR is computed on a 3 x 3 window centered on the pixel of interest as illustrated in Figure 1. Its quality, or reliability, is equal to the sum of the squared second differences in the four possible directions. In this way, if the window contains at least one noisy pixel or goes through a physical discontinuity, the SDR is high and accounts for an unreliable region that should be unwrapped last. Note that the logic is inverted as pixels with high SDR are less reliable. Since SDR is computed on the wrapped phase map, the measure can only take value in the interval $[0, 32\pi^2]$. Equations (2)-(5) give the second differences in the four directions and Equation (6) is the quality formula. $W(.)$ is an operator that removes 2π discontinuity between adjacent pixels.

$$H = W(\phi(i, j-1) - \phi(i, j)) - W(\phi(i, j) - \phi(i, j+1)) \tag{2}$$

$$V = W(\phi(i+1, j) - \phi(i, j)) - W(\phi(i, j) - \phi(i+1, j)) \tag{3}$$

$$D_1 = W(\phi(i-1, j-1) - \phi(i, j)) - W(\phi(i, j) - \phi(i+1, j+1)) \tag{4}$$

$$D_2 = W(\phi(i-1, j+1) - \phi(i, j)) - W(\phi(i, j) - \phi(i+1, j-1)) \tag{5}$$

$$SDR = H^2 + V^2 + D_1^2 + D_2^2 \tag{6}$$

The random tilt criterion relies on the notion of residues, also known as poles. They are associated with local inconsistencies and have been used in quality-guided phase unwrapping and branch-cut methods as well. For two-dimensional signals, residues are defined as regions of 2 x 2 pixels with a non-zero sum of unwrapped values around any closed path. To measure the quality of a pixel, RTR computes the probability that a residue appears when a random tilt is added to the wrapped phase. Again, the logic is inverted and pixels with high RTR are less reliable. The measure can only take value in the interval $[0, 4\pi]$.

Although more than one measure of quality exist, the way the unwrapping path is built does not vary as much and can be summed up in five steps:

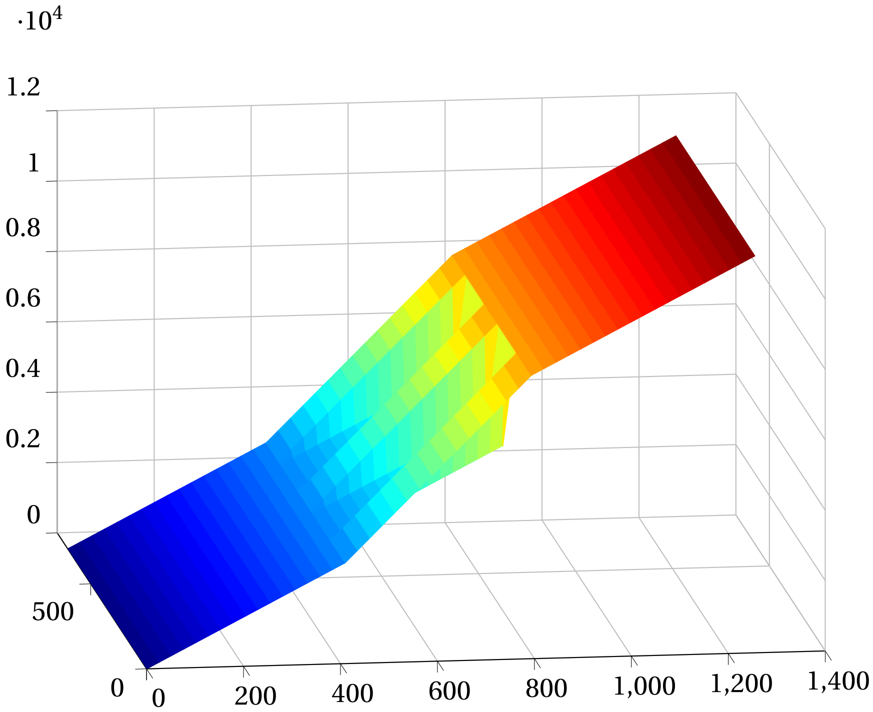
1. Compute the quality map.
2. Assign a quality value to each vertical and horizontal edge i.e the separation between two adjacent pixels. Its quality value is equal to the sum of those of the pixels it is made of.
3. Sort all of the edges according to their quality.
4. Assign each pixel to a different group. The initial number of groups is the same as the number of pixels in the phase map.
5. Go through the sorted edges and unwrap the pixels with respect to the two following rules:
   (a) If the pixels forming the current edge belong to two different groups, unwrap the smallest one and merge them into one.
   (b) If the pixels forming the current edge are already in the same group, do nothing.

The third step of this process is time-consuming if the edges are strictly sorted but it can be boosted by roughly grouping them in histogram bins, as done in the work of Lei et al [Lei et al., 2015]. They applied this sorting scheme to SDR. Due to the general quality distribution, the sorting histogram bins are not of equal size; the ones smaller than a threshold are narrower to enable a finer sorting. Indeed, it is reasonable to expect the majority of pixels to be reliable. Experiments lead to an empirical value of $3\pi^2$ for the threshold and a number of 100 bins.

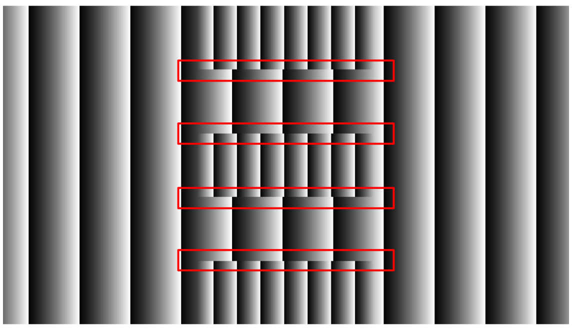
In section 3, we show how the histogram sorting can affect the result of phase unwrapping in some cases and propose a criterion designed to fit this fast sorting procedure.

3 Proposed quality measure

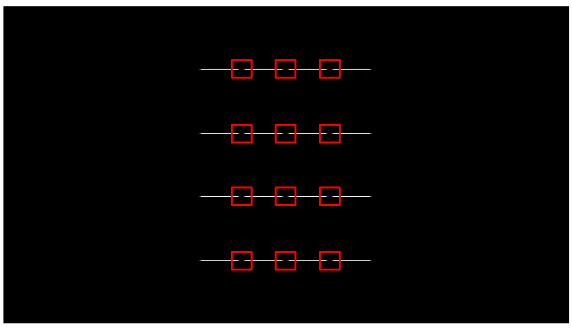
Because of the periodic nature of the wrapped phase map, SDR can lead to variable reliability values along physical discontinuities. Some pixels are wrongly labeled as reliable and modify the unwrapping path if edges are not strictly sorted. To illustrate this behaviour, we applied the two sorting procedures on the synthetic surface shown in Figure 2(a). It contains four lines of discontinuities that should not be crossed by the unwrapping



(a) Synthetic surface



(b) wrapped phase map



(c) SDR along the highlighted discontinuity line

(d) Unwrapped phase map with histogram sorting

(e) Unwrapped phase map with strict sorting

Figure 2: Phase unwrapping of a synthetic phase map

path, but Figure 2(d) shows that it is not the case when edges are sorted in the histogram bins. The twelve falsely reliable spots highlighted in Figure 2(c) are characterized by a SDR value of 0.011 which means they are sorted in the same bin as truly reliable edges. As a result, they are unwrapped too early. Crossing is avoided when edges are strictly sorted, as shown in Figure 2(e), since the falsely reliable regions are unwrapped after all the truly reliable pixels have been visited by the unwrapping path. An efficient way of solving this problem is to use a quality criterion that reduces the variability along physical discontinuities. When taking a closer look at H, V, D_1 and D_2 on one of the discontinuity lines shown in Figure 3, it appears they are all piecewise linear. Therefore, their first derivatives are constant and meet the requirement of low variability along discontinuities when using histogram sorting.

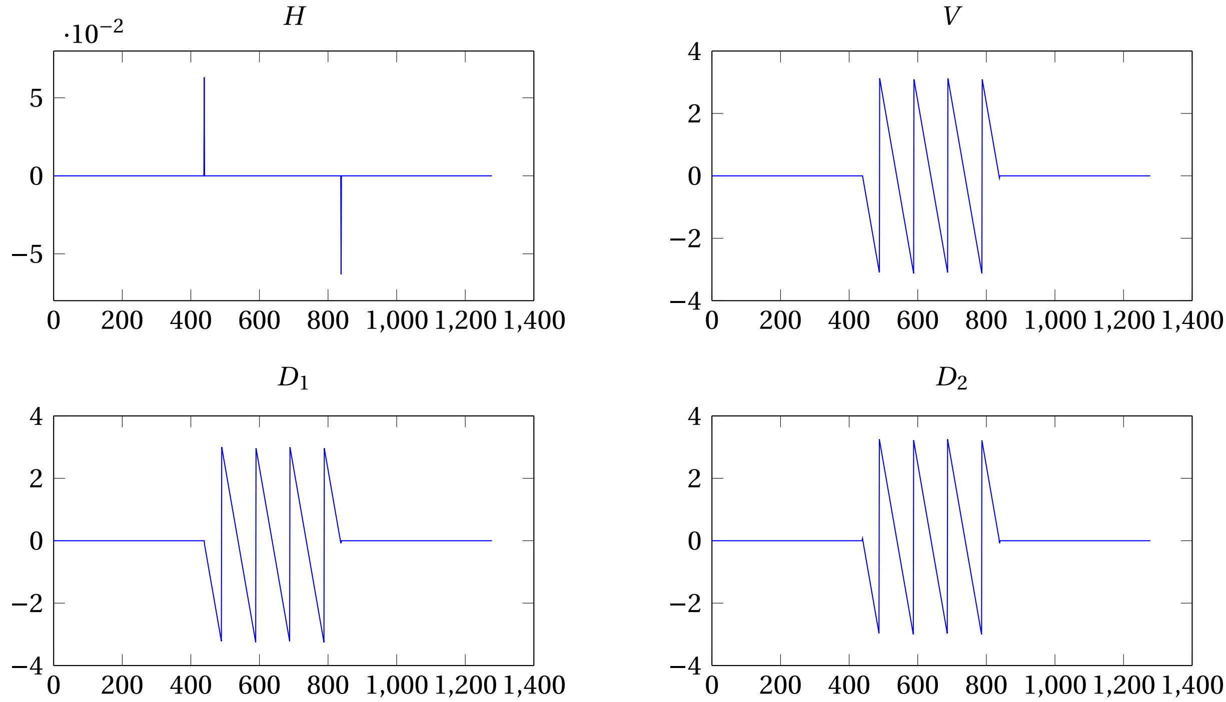


Figure 3: Second differences in the four directions along a discontinuity line

This observation leads to the definition of the *first derivative of second differences reliability* ($FDSDR$) quality criterion given by Equation (7).

$$FDSDR(i,j) = \left|W\left(D_1(i,j+1) - D_1(i,j-1)\right)\right| + \left|W\left(D_2(i,j+1) - D_2(i,j-1)\right)\right| \quad (7)$$

As the information carried by H and V is also found in D_1 and D_2, they are omitted from the measure to reduce computation time. Since D_1 and D_2 are wrapped between $-\pi$ and π, $FDSDR$ takes its value in the interval $[0, 8\pi]$.

4 Experimental results

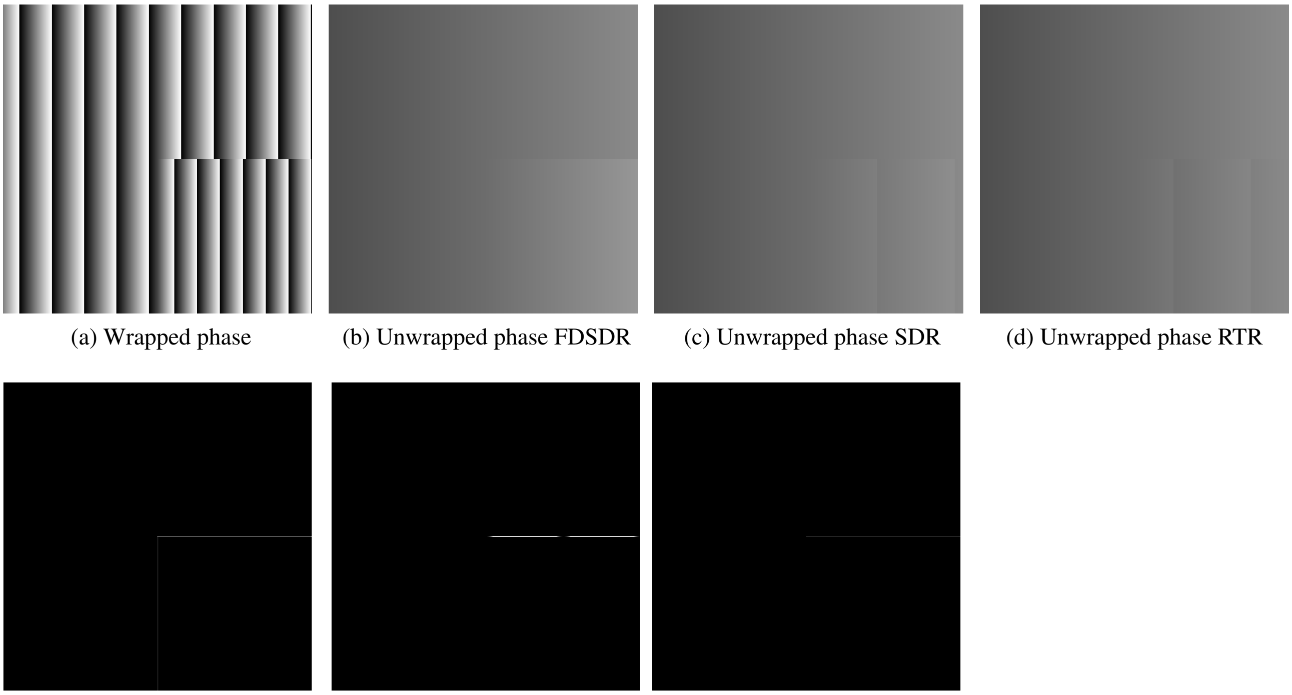
The proposed quality measure has been tested on synthetic data containing discontinuities and noise. SDR, RTR and $FDSDR$ have all been implemented in OpenCV [1]. The hardware used for the tests is a computer with a 2.7 GHz Intel®Core™i5-4210U and 16 Gbytes of RAM.

4.1 Ramps with different slopes and without noise

Our first test was carried out on a phase map of size 720 x 720 divided into two planar regions with different slopes. This profile may seem simple but it is a challenging example that has been used as a reference in some

[1] https://github.com/opencv

previous work [Bioucas-Dias and Valadao, 2007, Ghiglia and Pritt, 1998]. Figure 4 shows the wrapped phase map (fig. 4(a)) , the result of phase unwrapping with histogram sorting applied on $FDSDR$ (Figure 4(b)), SDR (Figure 4(c)) and RTR (Figure 4(d)) and the corresponding quality maps (Figures 4(e), 4(f), 4(g)). The histogram was divided into twelve small bins and a large one. The threshold was set to π for $FDSDR$, $4\pi^2$ for SDR and 0.5π for RTR. In these conditions, unwrapping failed for SDR and RTR, discontinuities are present in the lower part of the phase maps because the unwrapping path crossed the discontinuity line. On average, unwrapping with FDSDR took 0.48 s when edges were sorted in the histogram while it took 0.59 s when they were strictly sorted. The sorting algorithm we used is the one implemented in the standard C++ library. As seen on Figure 4(e) and 4(g), $FDSDR$ and SDR are constant on the discontinuity line but unwrapping still fails when RTR is used as the quality measure. Indeed, the difference between reliable and unreliable pixels for $FDSDR$ is higher than the same difference for RTR which leads to a better sorting.



(a) Wrapped phase (b) Unwrapped phase FDSDR (c) Unwrapped phase SDR (d) Unwrapped phase RTR

(e) FDSDR map (f) SDR map (g) RTR map

Figure 4: Phase unwrapping of a simulated surface with two planar regions

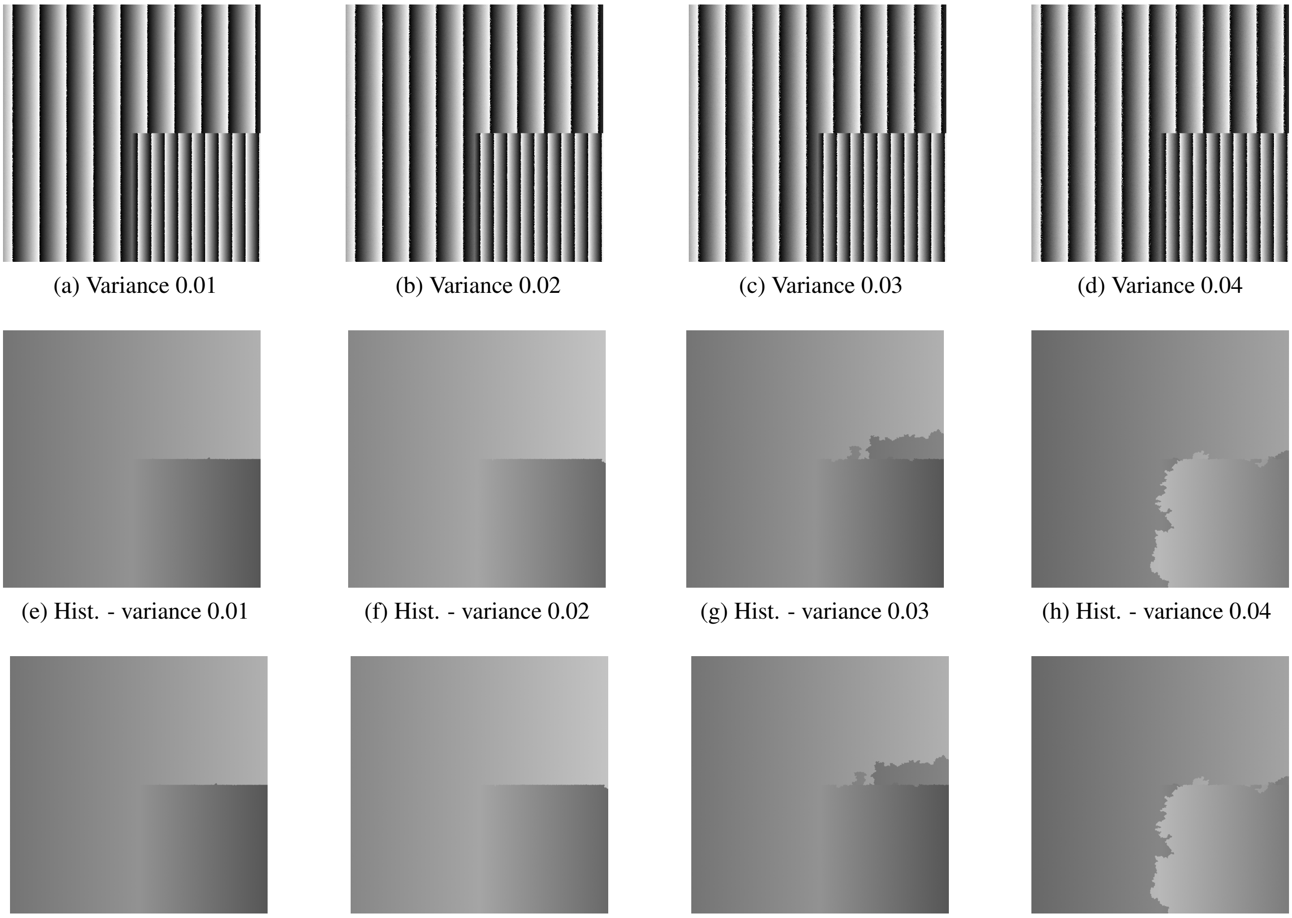
4.2 Ramps with different slopes and with noise

The ability to unwrap a noisy phase map is an important feature for any unwrapping algorithm. The second phase map was also divided into two planar regions with different slopes but this time, Gaussian white noise generated in Matlab was added to the data. Four levels of variance have been tested. Table 1 gives the mean processing time over ten unwrapping of the same phase map along with the minimal number of small bins required to get the best result. Except for the lowest noise level, processing times are equivalent when histogram processing is used. This observation does not hold when edges are strictly sorted. As announced by Lei et al. in their work, the number of small bins required to get the best result increases with the noise level but it does not impact the time performances [Lei et al., 2015]. On the other hand, the number of large bins has no influence on the unwrapped phase map.

Figure 5 shows the result of phase unwrapping for the four levels of noise. As expected, error propagation also increases with the noise level. An interesting observation is that the unwrapped phase map is the same for the two sorting procedures which means histogram sorting improves the processing speed without compromising the final result.

Noise level	0.01	0.02	0.03	0.04
$FDSDR$ sorted (s)	0.69	0.70	0.70	0.70
$FDSDR$ histogram (s)	0.50	0.59	0.59	0.59
# of small bins	21	96	130	150

Table 1: Phase unwrapping for different levels of noise



(a) Variance 0.01 (b) Variance 0.02 (c) Variance 0.03 (d) Variance 0.04

(e) Hist. - variance 0.01 (f) Hist. - variance 0.02 (g) Hist. - variance 0.03 (h) Hist. - variance 0.04

(i) Sorted - variance 0.01 (j) Sorted - variance 0.02 (k) Sorted - variance 0.03 (l) Sorted - variance 0.04

Figure 5: Phase unwrapping of a simulated surface with two planar regions for different levels of noise

5 Conclusion

In this paper, we have presented a new quality measure designed to work with the histogram sorting process proposed by Lei et al [Lei et al., 2015]. Based on the observation that H, V, D_1 and D_2 are piecewise linear, it reduces the quality variability along discontinuities by computing the first derivative of D_1 and D_2. Thus, the unwrapping path is less likely to cross discontinuity lines.

We have successfully tested our method on challenging examples. Histogram processing applied to our quality measure gives as good results as strict sorting while requiring less processing time. Further testing could be made about the histogram distribution since the number of large bins does not seem to impact the outcome of unwrapping. Moreover, the threshold is still empirical.

Finally, histogram processing could be applied to other measures of quality, sharing similar properties with $FDSDR$, meaning a low variability along discontinuity lines and a high difference of quality between reliable and unreliable pixels.

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