Holographic Lorentz and Carroll Frames

Arnaud DELFANTE

Unit of Physics of the Universe, Fields and Gravitation

University of Mons (Belgium)

October 3rd, 2022 – Quantum Gravity Around the Corner

Based on [A. Campoleoni, L. Ciambelli, AD, C. Marteau, P. M. Petropoulos, R. Ruzziconi (2208.07575)]

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Outline

I. **Plan and Motivations**

II. Covariant Bondi gauge in AdS and holographic frames

III. Flat limit and boundary Carroll frames

IV. Summary

I. Plan and Motivations

- Study of the classical phase space of 3D asymptotically AdS gravity:

  Select the allowed metric fluctuations at infinity [Brown-Henneaux '86]

- No requirement to fix any particular gauge but it is often convenient

  For example: Fefferman–Graham, Bondi gauge

- In this talk: covariant Bondi gauge, allow for a smooth flat-space limit

  Originally from fluid/gravity correspondence

  Study holographically Lorentz and Carroll-boost anomalies

Outline

II. Covariant Bondi gauge in AdS

- Key idea: relax the AdS Bondi gauge → dependence on the boundary dyad

$$\mathrm{d}s^2_{\mathrm{AdS}} = \frac{2}{k^2}\,\mathrm{u}\,(\mathrm{d}r + r\,\mathrm{A}) + r^2 g_{\mu\nu}\,\mathrm{d}x^\mu \mathrm{d}x^\nu + \frac{8\pi\mathcal{G}}{k^4}\,\mathrm{u}\,(\varepsilon\,\mathrm{u} + \chi\,{*\mathrm{u}})$$

- Boundary metric and Cartan frame:

$$g_{\mu\nu} = \frac{1}{k^2}\left(-u_\mu u_\nu + {*u_\mu}{*u_\nu}\right)$$

Weyl connection: [Loganayagam '08]

$$\mathrm{A} = \frac{1}{k^2}\left(\Theta^* {*\mathrm{u}} - \Theta\,\mathrm{u}\right), \qquad \Theta = \nabla_\mu u^\mu, \qquad \Theta^* = \nabla_\mu {*u^\mu}$$

- Energy-momentum tensor: [Brown-York '93]

$$\mathrm{T} = \mathrm{T}(\varepsilon, \chi)\ : \qquad \nabla_\mu T^{\mu\nu} = 0\,, \qquad T^\mu{}_\mu = \frac{R}{16\pi\mathcal{G}k}$$

II. Covariant Bondi gauge in AdS: residual symmetries

- Asymptotic Killing vectors: [Ciambelli-Marteau-Petropoulos-Ruzziconi '20]

$$v = \left(\xi^\mu - \frac{1}{k^2 r}\,\eta * u^\mu\right)\partial_\mu + \left(r\,\sigma + \frac{1}{k^2}\left(*u^\nu\,\partial_\nu\eta + \Theta^*\eta\right) + \frac{4\pi\mathcal{G}}{k^2 r}\,\chi\,\eta\right)\partial_r$$

$\hookrightarrow$ bdy diffeomorphisms $\xi^\mu(x)$, Weyl rescalings $\sigma(x)$ and Lorentz boosts $\eta(x)$

$$\delta_{(\xi,\sigma,\eta)}\mathrm{u} = \mathcal{L}_\xi \mathrm{u} + \sigma\,\mathrm{u} + \eta *\mathrm{u}\,, \quad \delta_{(\xi,\sigma,\eta)} *\mathrm{u} = \mathcal{L}_\xi *\mathrm{u} + \sigma *\mathrm{u} + \eta\,\mathrm{u}$$

where

$$\delta_{(\xi,\sigma,\eta)} g_{\mu\nu} = \mathcal{L}_\xi g_{\mu\nu} + 2\,\sigma\,g_{\mu\nu}$$

and

$$\begin{pmatrix}\mathrm{u}'\\ *\mathrm{u}'\end{pmatrix} = \begin{pmatrix}\cosh\eta & \sinh\eta\\ \sinh\eta & \cosh\eta\end{pmatrix}\begin{pmatrix}\mathrm{u}\\ *\mathrm{u}\end{pmatrix}$$

- Question: What are the asymptotic symmetries?

II. Covariant Bondi gauge in AdS: symplectic structure

- Einstein–Hilbert presymplectic potential: [Iyer-Wald '94]

$$\Theta_{\text{EH}}[G;\delta G] = \frac{\sqrt{-G}}{32\pi\mathcal{G}}\left[\nabla^N \delta G_{PN}\, G^{PM} - \nabla^M \delta G_{PN}\, G^{PN}\right] \epsilon_{MQS}\, \mathrm{d}x^Q \wedge \mathrm{d}x^S$$

  Radial divergences: need for renormalization

$$\Theta^{(r)}_{\text{EH}}[G;\delta G] = r^2\,\Theta_{(2)} + r\,\Theta_{(1)} + \Theta_{(0)} + \mathcal{O}\left(r^{-1}\right)$$

  Ambiguous definition:

$$\Theta_{\text{EH}}[G;\delta G] \to \Theta_{\text{EH}}[G;\delta G] + \delta Z[G] - \mathrm{d}\,Y[G;\delta G]$$

- Choices of prescription:
  i. same results as obtained in FFG [de Haro-Solodukhin-Skenderis (2000)]
  ii. presymplectic potential that remains finite in the flat-space limit

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II. Covariant Bondi gauge in AdS: surface charges

- Conformal gauge: conformally flat bdy metric $(x^{\pm} = \phi \pm k\, u)$

$$\mathrm{d}s^2 = \mathrm{e}^{2\varphi}\, \mathrm{d}x^+ \mathrm{d}x^-$$

Parametrization of the Cartan frame: $(\varphi = \varphi(x^+, x^-),\ \zeta = \zeta(x^+, x^-))$

$$\mathrm{u} = -\frac{k}{2}\, \mathrm{e}^{\varphi}\left(\mathrm{e}^{\zeta}\, \mathrm{d}x^+ - \mathrm{e}^{-\zeta}\, \mathrm{d}x^-\right), \qquad \ast\mathrm{u} = \frac{k}{2}\, \mathrm{e}^{\varphi}\left(\mathrm{e}^{\zeta}\, \mathrm{d}x^+ + \mathrm{e}^{-\zeta}\, \mathrm{d}x^-\right)$$

- Charges associated with the Weyl–Lorentz symmetries: $(\delta_v \varphi = \varpi,\ \delta_v \zeta = h)$

$$Q_{(\varpi, h)} = \frac{1}{4\pi \mathcal{G} k} \int_0^{2\pi} \mathrm{d}\phi \left(h\, (\partial_- - \partial_+)\, \zeta\right)$$

$\hookrightarrow$ integrable and non-conserved: Lorentz is anomalous, Weyl is pure gauge

II. Covariant Bondi gauge in AdS: anomalies

- Anomaly in the Lorentz symmetry in the dual theory $(F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu)$

$$\delta_{(\xi,\sigma,\eta)} S_{\mathrm{L}} = \int \left(\eta \, \frac{F}{8\pi\mathcal{G}} \right) \mathsf{Vol}_{\partial\mathcal{M}}$$

$\hookrightarrow$ flat limit: yes

- If we choose the first prescription $\rightarrow$ anomaly in the Weyl symmetry in the dual theory [Alessio-Barnich-Ciambelli-Mao-Ruzziconi '20]

$$\delta_{(\xi,\sigma,\eta)} S_{\mathrm{W}} = \int \left(\sigma \, \frac{R}{8\pi\mathcal{G}} \right) \mathsf{Vol}_{\partial\mathcal{M}}$$

$\hookrightarrow$ flat limit: no

- Displacement of the anomaly: two different representatives in the same cohomology class $\rightarrow$ BRST formulation [Ciambelli-Leigh-Jia (to appear)]

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III. Flat limit: surface charges and anomalies

- Key idea: timelike AdS bdy $\underset{k\to 0}{\longrightarrow}$ null manifold, Carrollian geometry
- Conformal gauge: parametrization of the Carrollian dyad $(\beta = \lim_{k\to 0} \frac{\zeta}{k})$

$$\mu = \lim_{k\to 0} \frac{u}{k^2} = -e^{\varphi}\left(du + \beta\, d\phi\right), \quad \mu^* = \lim_{k\to 0} \frac{*u}{k} = e^{\varphi}\, d\phi$$

Charges associated with the Weyl–boost symmetries: $(\delta_v \varphi = \varpi,\ \delta_v \beta = \tilde{h})$

$$Q_{(\varpi,\tilde{h})} = \frac{1}{4\pi\mathcal{G}} \int_0^{2\pi} d\phi \left(\partial_u \tilde{h}\, \beta\right)$$

↪ integrable, non-conserved: Carroll boost is anomalous, Weyl is pure gauge

- Anomalies: $(\mathcal{A} = \lim_{k\to 0} A,\ \mathcal{F}_{\mu\nu} = \partial_\mu \mathcal{A}_\nu - \partial_\nu \mathcal{A}_\mu,\ \lambda = \lim_{k\to 0} \frac{\eta}{k})$

$$\delta_{(\xi,\sigma,\lambda)} S_{\mathrm{C}} = \int \left(\lambda \frac{\mathcal{F}}{8\pi\mathcal{G}}\right) \mathrm{vol}_{\partial\mathcal{M}}$$

↪ new holographic prediction, calling for further investigation

Outline

IV. Summary

Main goal:

- Explore the charges of 3D (AdS or flat) gravity in covariant Bondi gauge

  $\hookrightarrow$ bdy diffeomorphisms, Weyl rescalings and local frame boosts

Results:

- Divergences in the symplectic structure
- Renormalization via ambiguities
- Surface charges and anomalies
- New holographic Carrollian prediction

Future possibilities:

- Relate to asymptotic corner group [Donnelly-Freidel '16, Freidel-Geiller-Pranzetti '20, Ciambelli-Leigh-Pai '21]
- Connect to the celestial holography proposal [Strominger '17, Pasterski-Pate-Raclariu '21, Donnay-Fiorucci-Herfray-Ruzziconi '22]
- Extension to higher dimensions [Petkou-Petropoulos-Betancour-Siampos '22]

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"I'm Late", Alice in Wonderland, White Rabbit, by Sir John Tenniel

Thank you for listening!

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