Infinite distances in multicritical CFTs and higher-spin holography

Andrea Campoleoni

Physique de l'Univers, Champs et Gravitation

UMONS
Université de Mons

I. Basile, A.C., S. Pekar, E. Skvortsov,
2209.14379

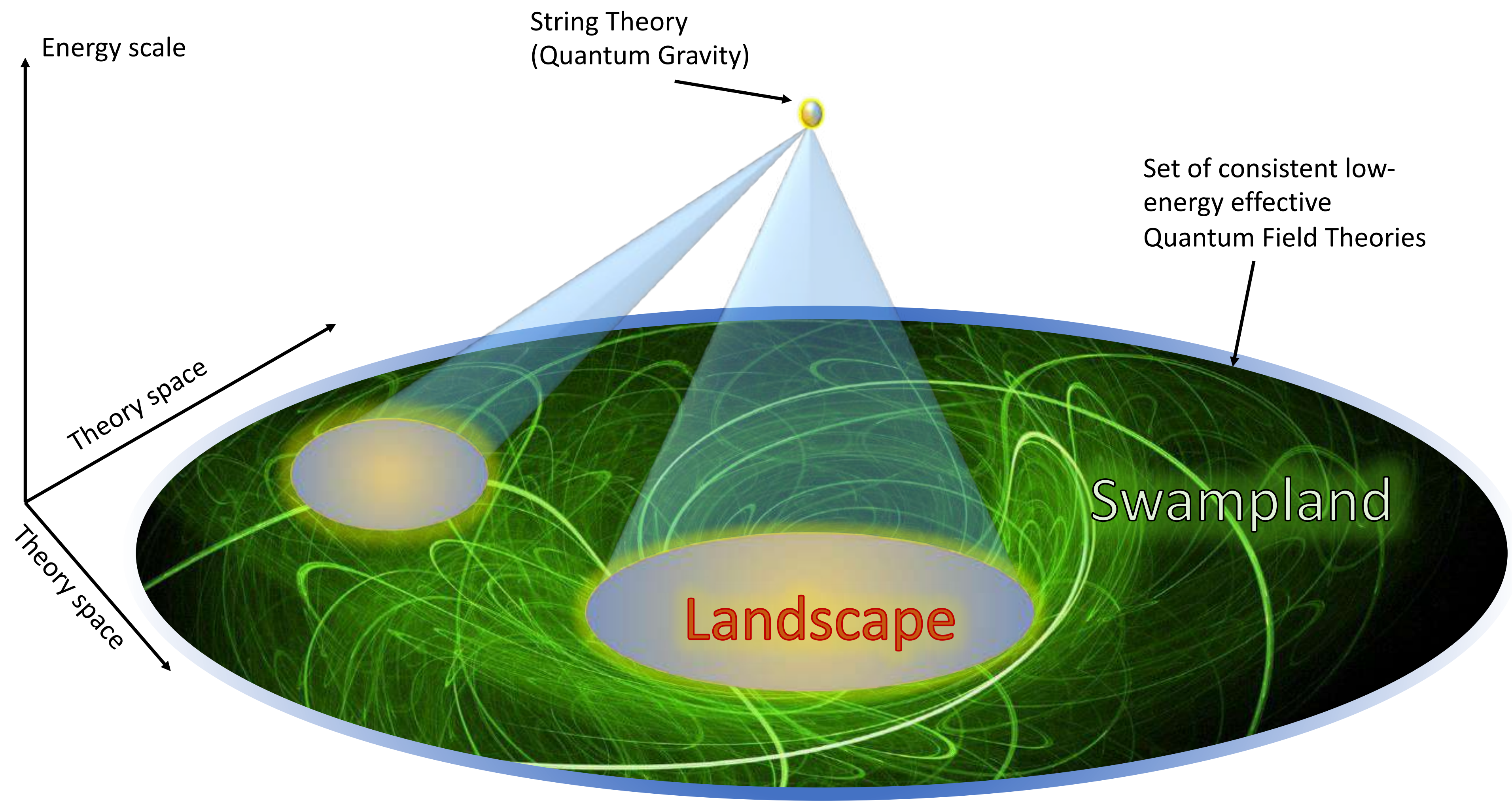
Higher Spin Gravity and its Applications, APCTP Pohang, 16/10/2022

fnrs
LA LIBERTÉ DE CHERCHER

The landscape and the swampland

- How to distinguish effective field theories (EFT) that can be completed into quantum gravity in the UV (***landscape***) from those that don't (***swampland***)?

Vafa, Douglas (2005) and many others afterwards…



[E. Palti, The Swampland: Introduction and Review, 1903.06239]

1. A biased tour into the swampland

The swampland program

- A network of ***conjectures*** about the constraints that EFT living in the landscape must satisfy:
  - Distance (or duality) conjecture — Ooguri, Vafa (2006)
  - Weak gravity conjecture — Arkani-Hamed, Motl, Nicolis, Vafa (2006)
  - No global symmetries conjecture — Banks, Seiberg (2010) [Banks, Dixon (1988)]
  - Completeness conjecture — [Polchinski (2003)]
  - Emergent proposal — Grimm, Palti, Valenzuela (2018)
  - de Sitter conjecture — Obied, Ooguri, Spodyneiko, Vafa (2018)
  - *and counting…*

See, e.g., E. Palti, The Swampland: Introduction and Review, 1903.06239

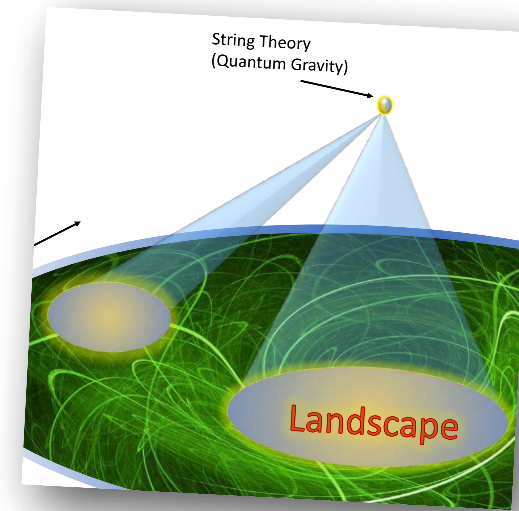
The swampland program

- A network of ***conjectures*** about the constraints that EFT living in the landscape must satisfy:
  - Distance (or duality) conjecture — Ooguri, Vafa (2006)
  - Weak gravity conjecture — Arkani-Hamed, Motl, Nicolis, Vafa (2006)
  - No global symmetries conjecture — Banks, Seiberg (2010) [Banks, Dixon (1988)]
  - Completeness conjecture — [Polchinski (2003)]
  - Emergent proposal — Grimm, Palti, Valenzuela (2018)
  - de Sitter conjecture — Obied, Ooguri, Spodyneiko, Vafa (2018)
  - *and counting…*

Here focus on the distance conjecture

Higher-spin gravity landscape?

- Most of the swampland conjectures have been *inspired by* and *checked in* **String Theory**



- Two versions of the swampland program:
  - Shaping the landscape of EFT resulting from string compactifications
  - Shaping the landscape of all EFT that can be UV completed into quantum gravity (M-theory may not be the only option!)

- A natural non-stringy candidate: **Higher Spin Gravity**

- *What are the swampland conjectures corroborated by Higher Spin Gravity and how to interpret possible mismatches?*

The swampland distance conjecture (SDC)

- Simplest example: *compactifications on a circle*, $X^d \simeq X^d + 1$
  - $ds^2 \equiv G_{MN} dX^M dX^N = e^{2\alpha\phi} g_{\mu\nu} dX^\mu dX^\nu + e^{2\beta\phi} \left(dX^d\right)^2 ,$
  - α and β constants, while ϕ specifies the radius: $2\pi R \equiv \int_0^1 \sqrt{G_{dd}} dX^d = e^{\beta\phi}$
  - ϕ is a dynamical field: $\int d^D X \sqrt{-G} R^D = \int d^d X \sqrt{-g} \left[R^d - \frac{1}{2} (\partial\phi)^2\right]$
- Fields and strings behave differently:
  - $\Psi\left(X^M\right) = \sum_{n=-\infty}^{\infty} \psi_n\left(X^\mu\right) e^{2\pi i n X^d} ,$
  - $X^M_{(s)}(\tau, \sigma) = x^\mu + \alpha' p^M \tau + \frac{\alpha'}{2}\left(p_L^M - p_R^M\right)\sigma + \text{oscillators}$

winding modes

with $X^d_{(s)}(\sigma + 2\pi, \tau) = X^d_{(s)}(\sigma, \tau) + w 2\pi R$

The swampland distance conjecture (SDC)

- Masses of the modes:

Field Theory (Kaluza Klein)	String Theory (KK + winding)
$M_n^2 = \left(\frac{n}{R}\right)^2 \left(\frac{1}{2\pi R}\right)^{\frac{2}{d-2}}$	$(M_{n,w})^2 = \left(\frac{1}{2\pi R}\right)^{\frac{2}{d-2}} \left(\frac{n}{R}\right)^2 + (2\pi R)^{\frac{2}{d-2}} \left(\frac{wR}{\alpha_0'}\right)^2$

- The expectation values of the field ϕ defines the field space

$$\mathcal{M}_\phi \ : \ -\infty < \phi < \infty$$

T-duality: $R \leftrightarrow \frac{\sqrt{\alpha'}}{R}$

- Two mass scales into the game:

$$M_{KK} \sim e^{\alpha\phi} \,, \quad M_w \sim e^{-\alpha\phi} \quad \Rightarrow \ \forall \ \Delta\phi \ \exists \quad M\left(\phi_i + \Delta\phi\right) \sim M\left(\phi_i\right) e^{-\alpha|\Delta\phi|}$$

The swampland distance conjecture (SDC)

- Very "stringy" property, that triggered… Ooguri, Vafa (2006)

Swampland Distance Conjecture [4]

- *Consider a theory, coupled to gravity, with a moduli space $\mathcal{M}$ which is parametrized by the expectation values of some field ϕ^i which have no potential. Starting from any point $P \in \mathcal{M}$ there exists another point $Q \in \mathcal{M}$ such that the geodesic distance between P and Q, denoted $d(P,Q)$, is infinite.*
- *There exists an infinite tower of states, with an associated mass scale M, such that*

$$M(Q) \sim M(P)\, e^{-\alpha d(P,Q)} \,, \qquad (3.79)$$

where α is some positive constant.

- Geodesic distance:

$$S = \int d^d x \sqrt{-g} \left[\frac{R}{2} - g_{ij}\left(\phi^i\right) \partial\phi^i \partial\phi^j + \ldots \right] \quad \Rightarrow \quad d(P,Q) \equiv \int_\gamma \left(g_{ij} \frac{\partial\phi^i}{\partial s} \frac{\partial\phi^j}{\partial s} \right)^{\frac{1}{2}} ds$$

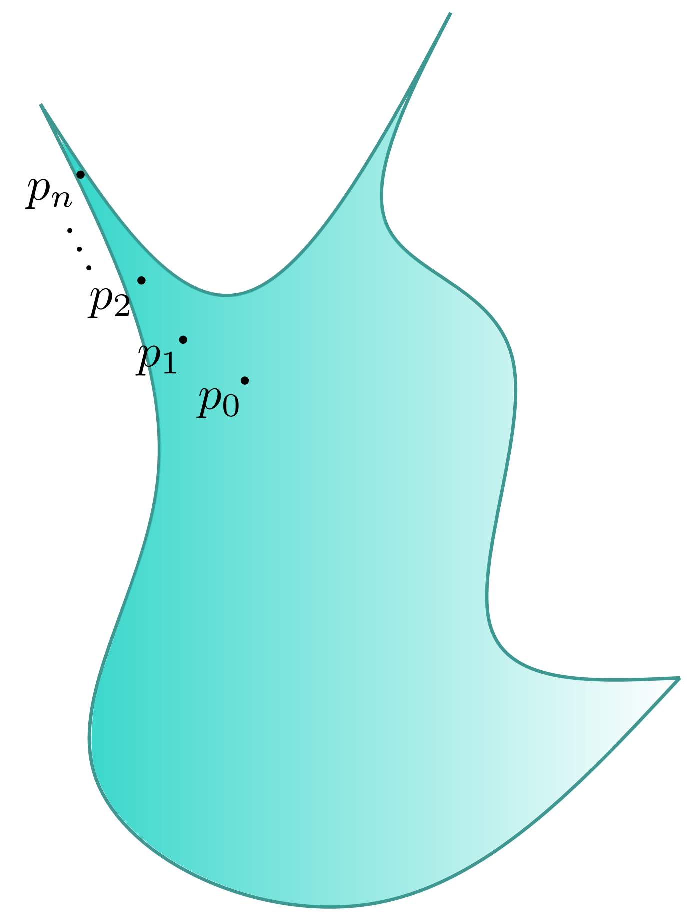
2. AdS swampland & holography

The CFT distance conjecture

- What about AdS?
  - When $LM_{\text{Pl}} \to \infty$ a similar phenomenon is expected, related to "decompactification" (the radii of AdS and of the internal manifold are related) **[not today…]**

    Lüst, Palti, Vafa (2019)
  - One can also *keep* LM_{Pl} *fixed* and move around the moduli space
  - A similar behaviour as in flat space is expected
  - The effective field theory description breaks down at the "corners" of moduli space !
  - One can ***use the dual CFT description*** to explore the moduli space

    Baume, Calderón Infante (2021); Perlmutter, Rastelli, Vafa, Valenzuela (2021)

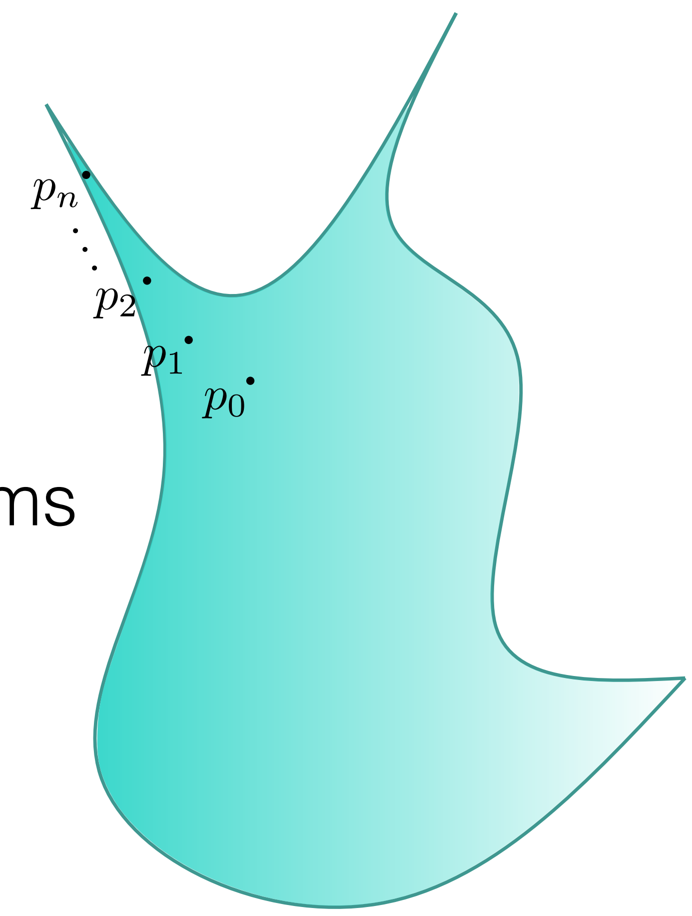


The CFT distance conjecture

Baume, Calderón Infante (2021); Perlmutter, Rastelli, Vafa, Valenzuela (2021)

- What is the counterpart of the geodesic distance in moduli space?
  - Bulk moduli space ⇔ exactly marginal couplings in the CFT
  - Exactly marginal couplings span the conformal manifold: $\delta S = t^i \int d^d x\, \mathcal{O}_i$
  - Zamolodchikov metric on the conformal manifold:

$$|x-y|^{2d}\langle \mathcal{O}_i(x)\mathcal{O}_j(y)\rangle = g_{ij}(t^i)$$



- Distance conjecture(s) can be reformulated in terms of CFT data (*whether a conformal manifold exist!*)
  - masses ⇔ anomalous dimensions of HS operators

Improved CFT distance conjecture(s)?

- The previous setup is very effective to study superconformal field theories and their bulk duals

- *How one can tackle models that do not admit a conformal manifold?* (e.g. our beloved higher spins…)

- IDEA: consider the space of fixed points of the RG flow and compute distances along the RG flow using the ***quantum information metric*** Stout (2021)

- *Within this approach one can also "discrete theory spaces"!*
  - Well adapted to higher-spin holography, where higher-spin symmetry is recovered in the limit of large rank N

Computing distances in theory space

- Theory with *n* couplings g^a corresponding to operators $\hat{\Phi}_a(x)$

O'Connor, Stephens (1993); Dolan (1997)

  - Consider $\tilde{\Phi}_a(x) = \hat{\Phi}_a(x) - \langle \hat{\Phi}_a(x) \rangle$ and define

$$G_{ab} = \int d^D x \langle \tilde{\Phi}_a(x) \tilde{\Phi}_b(0) \rangle$$

  - This object transforms like a *metric* under coordinate transformations in the coupling space!

$$g^a \to g^{a'}(x) \quad \Rightarrow \quad G_{ab} \to G_{a'b'} = \frac{\partial g^c}{\partial g^{a'}} \frac{\partial g^d}{\partial g^{b'}} G_{cd}$$

  - If the $\hat{\Phi}_a(x)$ are exactly marginal deformations of a given CFT then this metric coincides with the Zamolodchikov one

Computing distances in theory space

- The previous metric can be recovered as follows

O'Connor, Stephens (1993); Dolan (1997)

  - Compute the free energy

$$W(g) = -\ln Z(g) \quad \text{where} \quad Z(g) = \int \mathcal{D}\varphi e^{-S[\varphi]}$$

  - Consider $dW = \partial_a W dg^a$ and $dS = \partial_a S dg^a$ and define

$$ds^2 = \langle (dS - dW) \otimes (dS - dW) \rangle$$

- If the action is linear in the couplings then

$$G_{ab} = -\partial_a \partial_b w \qquad \text{with} \quad w = \frac{1}{V} W$$

in practice we'll have to compute the free energy...

Halfway summary

- CFT distance conjectures (in a broad sense):

  1. In any theory space, HS symmetries emerge only at infinite distance
  2. All CFTs at infinite distance display HS symmetry
  3. The anomalous dimensions of the HS currents vanish exponentially fast in the distance

- We found a rather general way to measure distances on theory spaces

- Next goal: test these ideas in HS holography, i.e. for Chern-Simons vector models

  - Challenge: find a way to interpolate between different values of the rank N of the gauge group

3. Higher spin swampland

[*aka "what we actually did ourselves"*]

3.1 Choose the theory space

Multicritical vector models

- We propose to consider ***multicritical vector models***

Calabrese, Pelissetto, Vicari (2022)

  - Field content: $\phi_1\,,\,\dots\,,\,\phi_k$ with ϕ_a in the vector repr. of $O(N_a)$
  - Action: $$S = \int d^d y \left(\frac{1}{2} (\partial \phi_a)^2 + \frac{1}{2} r_a \, \phi_a^2 + \frac{\lambda_{ab}}{N} \phi_a^2 \, \phi_b^2 \right)$$
  - A bit of notation… $N = \sum_a N_a\,,\ x_a = \frac{N_a}{N}\,,\ r_a = \mu^2 g_a\,,\ \lambda_{ab} = \mu^{4-d} g_{ab}$

- Why?
  - There are flows in which factors “fuse”: $O(N_a) \times O(N_b) \longrightarrow O(N_a + N_b)$
  - Possibility to modify *N* following the RG flow! Possibility to measure distances between different values of *N*

Identifying the “trajectories”: beta functions

- We work with N ≫ 1 and d = 4 − ϵ (or d = 2 + ϵ for fermions)
- Beta functions for the *bosonic* models:

$$B \equiv \frac{2\Omega_{d-1}}{(2\pi)^d} = \frac{1}{4\pi^2} + \mathcal{O}(\epsilon)$$

$$\mu \frac{dg_a}{d\mu} = -2g_a - 2B\left(x_b - \frac{4B}{N}\delta_{ab}\right)\frac{g_{ab}}{1+g_a}\,,$$

$$\mu \frac{dg_{ab}}{d\mu} = -\epsilon\, g_{ab} + 2B\left(x_c + \frac{2}{N}(\delta_{ac}+\delta_{bc})\right)\frac{g_{ac}\,g_{bc}}{(1+g_c)^2} + \frac{8B}{N}\frac{g_{ab}^2}{(1+g_a)^2(1+g_b)^2}$$

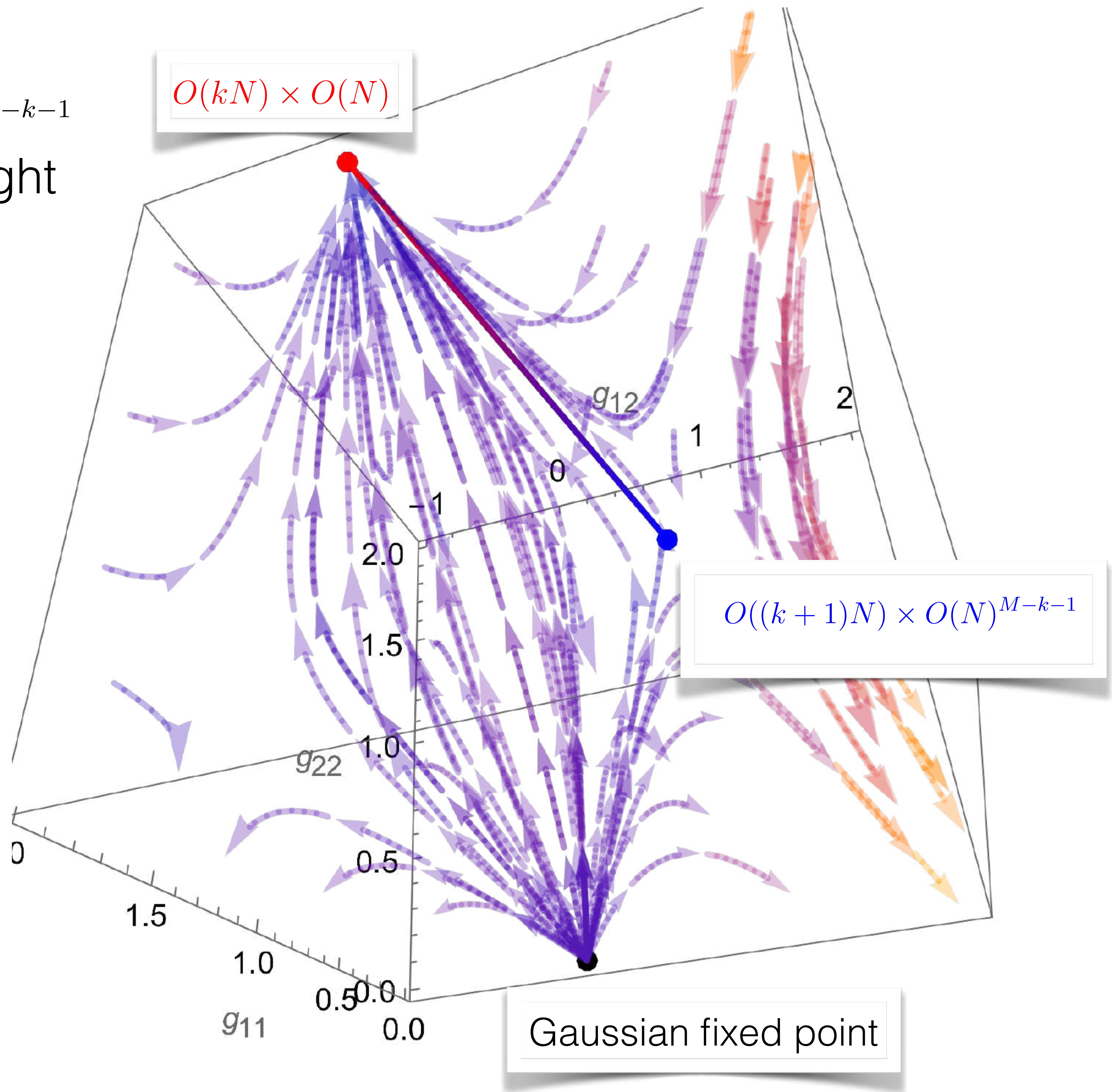
- Focus on $O(N)^M$ models and on flows that follow this pattern

$$O(MN) \longrightarrow O((M-1)N)\times O(N) \longrightarrow O((M-2)N)\times O(N)^2$$
$$\longrightarrow \ldots \longrightarrow O(N)\times O(N)^{M-1} \longrightarrow O(N)^M$$

- At each step the flow is essentially bicritical

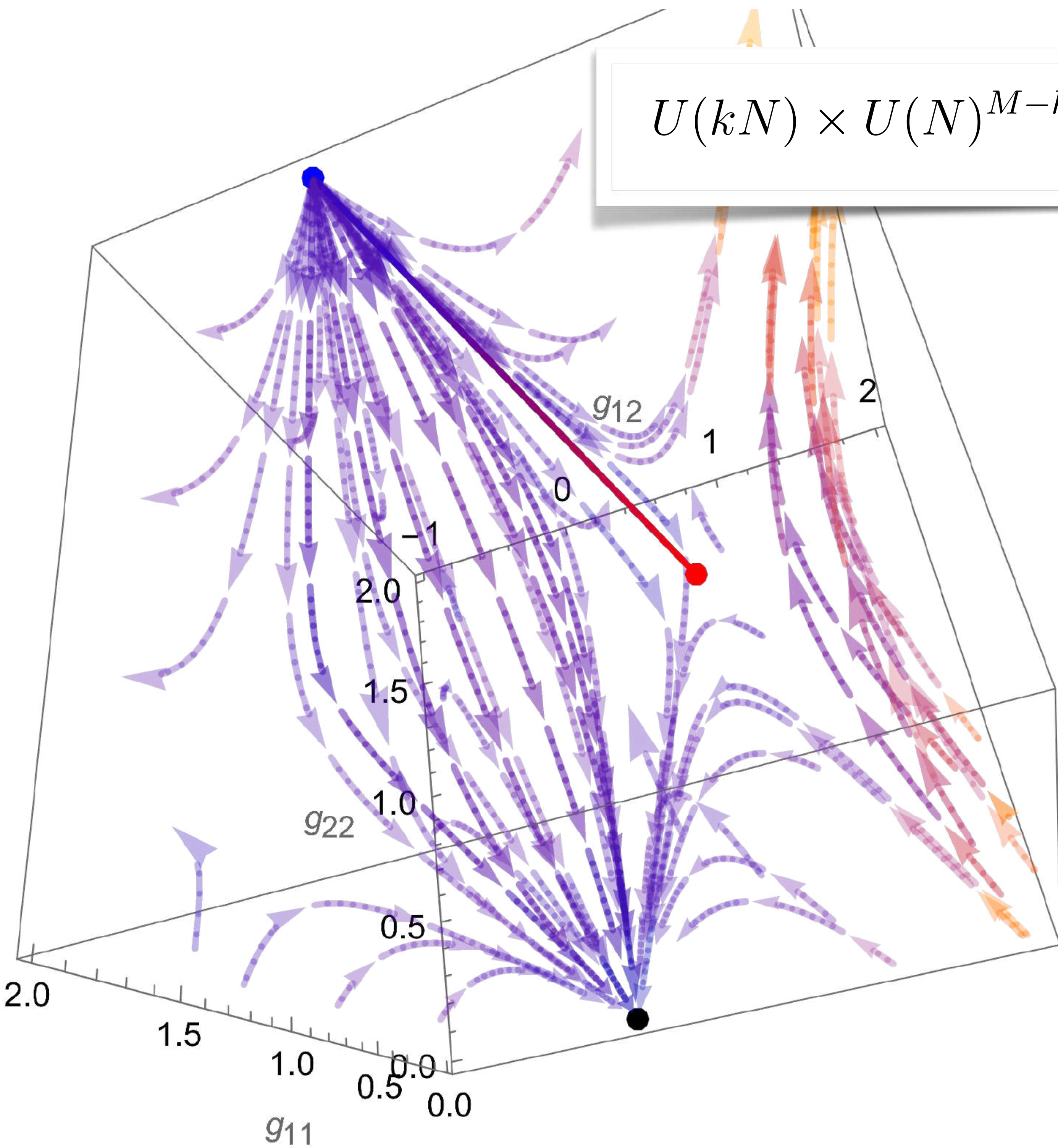
The bicritical RG flow for bosons

At large *N*, the trajectory connecting $O((k+1)N) \times O(N)^{M-k-1}$ with $O(kN) \times O(N)^{M-k}$ is a straight line



The bicritical RG flow for fermions

$$U(kN) \times U(N)^{M-k} \longrightarrow U((k+1)N) \times U(N)^{M-k-1}$$



Similar story…

3.2 Compute distances

along the RG flow

Computing the information metric

- We resort to the usual Hubbard-Stratonovich trick
  - We rewrite the quartic interaction as $\sigma_a \phi_a^2$
  - Integrating out the ϕ_a we get

$$S_{\rm eff} = -\frac{N}{4}(g^{-1})^{ab}\int d^d y\, \sigma_a\, \sigma_b + \frac{N}{2}\, x_a\, {\rm Tr}\log\left(-\Box + r_a + 2\,\sigma_a\right)$$

- For σ_a constant, the saddle point condition gives gap equations

$$\sigma_a = 2\, g_{ab}\, x_b\, \dot{L}(r_b + 2\sigma_b)$$

$$L(r) \equiv \int^{\Lambda_{\rm UV}} \frac{d^d p}{(2\pi)^d}\, \log\left(p^2 + r\right)$$

- Denoting as $\overline{\sigma}_a(\mathbf{r}, \mathbf{g})$ the solution:

$$G_{(ab)(cd)} = -\frac{\partial}{\partial g_{ab}}\frac{\partial}{\partial g_{cd}} V_{\rm eff}(\overline{\sigma}(\mathbf{0}, \mathbf{g}))$$

Computing distances along the flow

- For bosons, we computed the full metric in d = 3 numerically in the large-N expansion and analytically in $d = 4 - \epsilon$ for $\epsilon \ll 1$

- The length of the RG trajectory is

$$\Delta_{\text{ss}}^{\text{bdry}} = \int_0^1 \sqrt{G_{vv}(u)}\, du$$

[v is the direction of the line connecting the two fixed points of the bicritical RG flow]

- In analogy with the analysis of conformal manifolds we propose to measure distances as

$$\Delta_{\text{ss}}^{\text{bulk}} \sim \frac{1}{\sqrt{C_T}} \int_0^1 \sqrt{G_{vv}(u)}\, du$$

$$\langle T_{ab}(x) T_{cd}(0) \rangle = \frac{C_T}{x^{2d}}\, \mathcal{I}_{ab,cd}(x)$$

  - in the large-N limit, C_T doesn't vary at leading order in a single step, but it varies at the subleading one

Computing distances along the flow

- The trajectories we have chosen are those along which C_T *varies the least* (strictly speaking the previous expression makes sense only if C_T is constant)

- This provides a criterion to interpret our trajectories as "geodesics" in theory space

- To probe large distances in theory space we eventually have to consider the full flow $O(MN) \to \cdots \to O(N)^M$ with $M \gg N \gg 1$

$$\Delta^{\text{bulk}} \sim \sum_{k=1}^{M-1} \Delta_k^{\text{bulk}} \sim \frac{\epsilon}{N} \sqrt{\log \frac{1}{\epsilon}} \sqrt{M}$$

unusual dependence on the square root of the parameter in theory space

Contrasting with anomalous dimensions

- How to give an holographic interpretation to this computation?
  - Let's impose the singlet constraint on a diagonal *O(N)*
- Anomalous dimensions of HS currents
  - At the decoupling point $O(N)^M$ they scale as $\gamma_{\text{HS}} \sim \frac{1}{N}$
  - They gradually decrease up to $\gamma_{\text{HS}} \sim \frac{1}{MN}$

$$\frac{\gamma_{\text{HS}}^{O(NM)}}{\gamma_{\text{HS}}^{O(N)^M}} \,, \; \frac{\gamma_{\text{HS}}^{U(NM)}}{\gamma_{\text{HS}}^{U(N)^M}} \sim \frac{\mathcal{O}(1)}{M} \qquad\qquad \Delta \sim \sqrt{M}$$

- **The distance conjecture is violated!**

$$\frac{\gamma_{\text{HS}}^{O(NM)}}{\gamma_{\text{HS}}^{O(N)^M}} \,, \; \frac{\gamma_{\text{HS}}^{U(NM)}}{\gamma_{\text{HS}}^{U(N)^M}} \sim \frac{\mathcal{O}(1)}{\Delta^2}$$

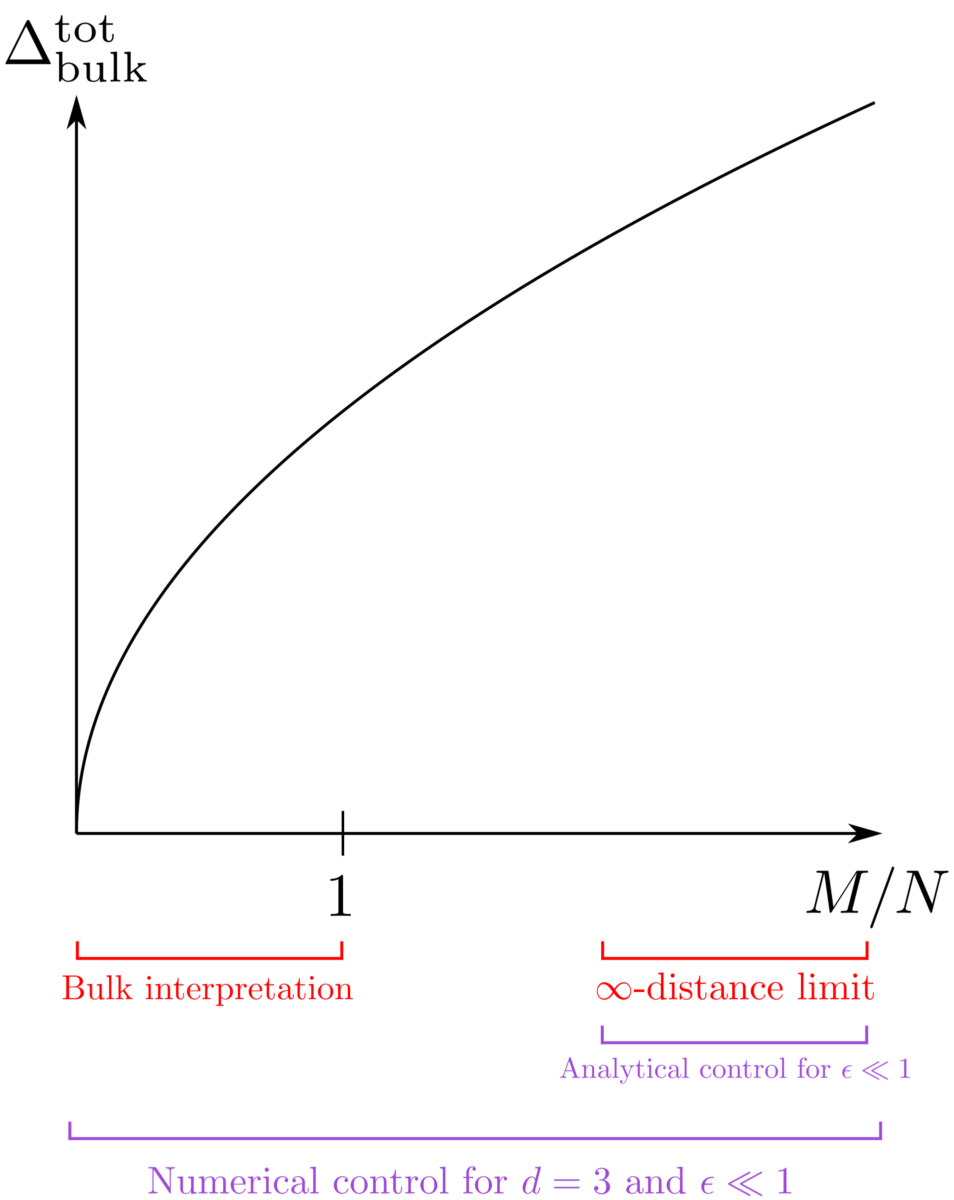
Validity of the holographic interpretation

- Since we impose the singlet constraint only on the diagonal *O(N)*, our HS currents display Chan-Paton factors:

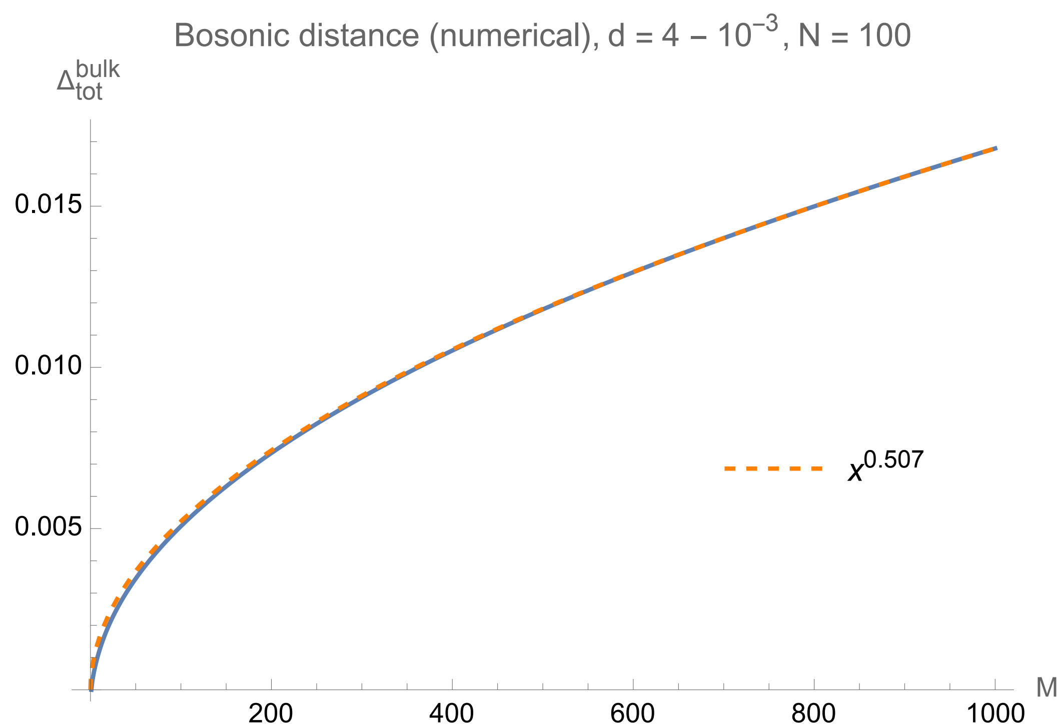
$$J_s{}^{ab} = \phi_i^a \partial^s \phi_j^b \delta^{ij} + \dots$$

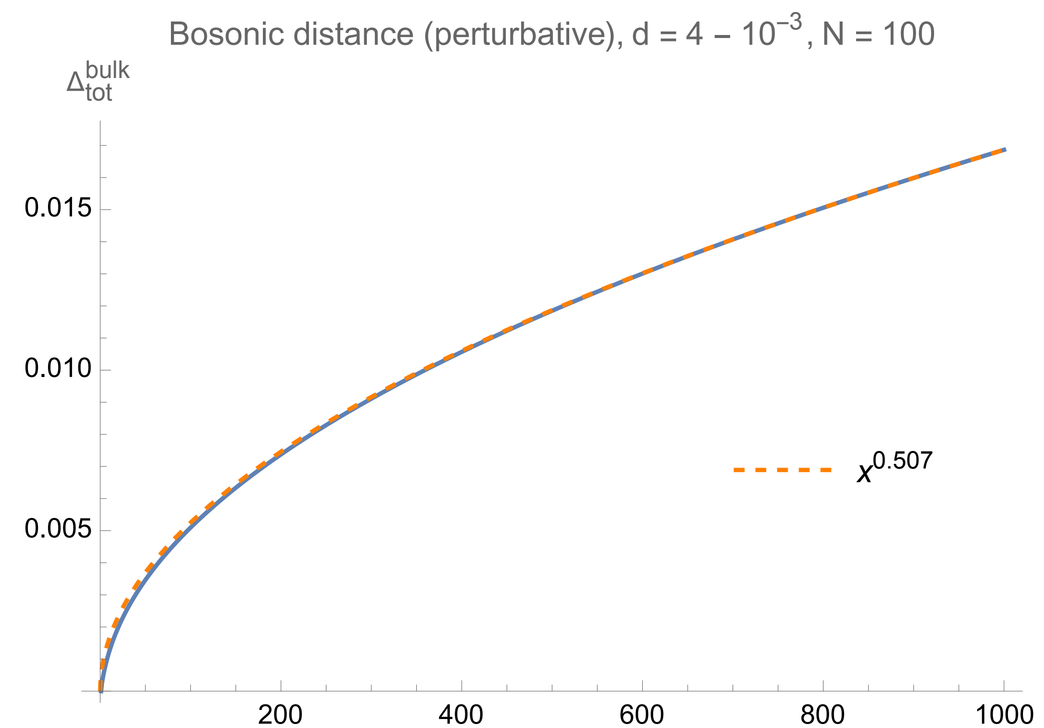
- When M >> N some of the bulk field seem not to be independent anymore

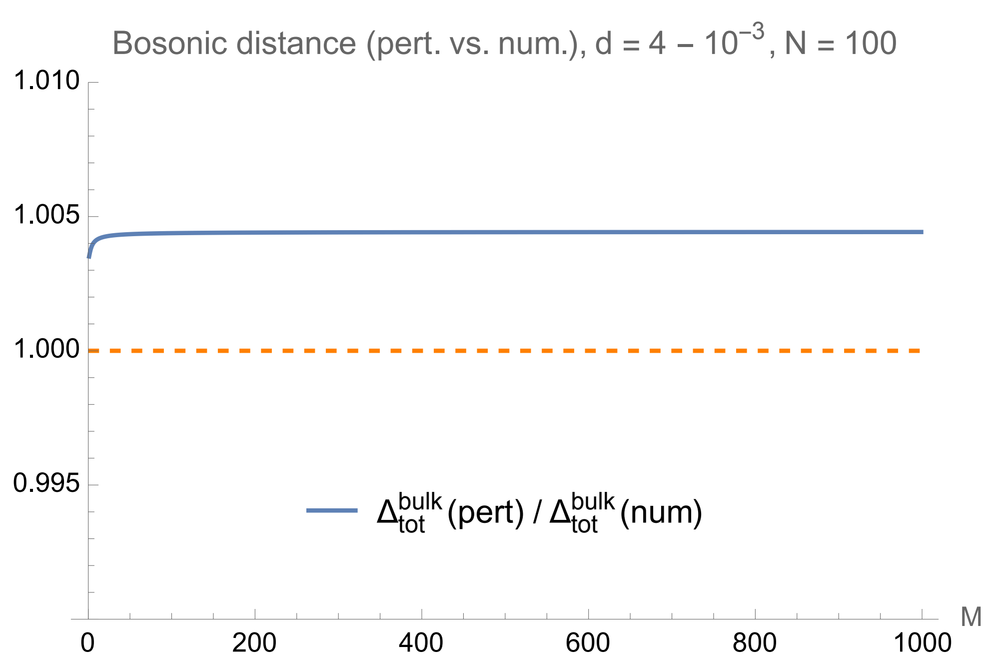
- Bulk interpretation for M >> N?



Validity of the numerical approximations







where we can compare the numerical approach with the epsilon expansion we get an error of less than 5×10^{-3}

in the fermionic case the numerical approach is much reliable, but we obtain qualitatively similar results

Exploring other directions in the theory space

- Standard way to impose the singlet constraint in d=3: coupling to a Chern-Simons action
- In the large-N limit with $\lambda \equiv \frac{N}{k} \equiv \frac{N}{k_{\mathrm{CS}} + N} \in (0,1)$ fixed one has a line of fixed points
- Higher-spin symmetry is present at the edges of the interval (free/critical bosons or fermions)
- How information distance behave in the λ space?

$$ds^2 \overset{\frac{1}{\lambda}, N \gg 1}{\sim} \lambda^2 \, d\left(\frac{1}{\lambda}\right) \otimes d\left(\frac{1}{\lambda}\right) = \frac{d\lambda \otimes d\lambda}{\lambda^2}$$

the distance at $\lambda = 0$ is <u>infinite</u> and one can argue that the same is true at $\lambda = 1$ by a duality argument

Chern-Simons vector models

- This time the distance diverges logarithmically in the parameter!

$$\Delta(\lambda, \lambda_0) \sim c \log \frac{\lambda_0}{\lambda}$$

- For $\lambda \ll 1$ the anomalous dimensions scale as

Giombi, Gurucharan, Kirilin, Prakash, Skvortsov (2016)

$$\gamma_s \overset{\frac{1}{\lambda}, N \gg 1}{\sim} \frac{(a_s + b_s)\pi^2}{8N} \lambda^2$$

- The exponential falloff in the distance is recovered!

$$\frac{\gamma_s(\lambda)}{\gamma_s(\lambda_0)} \sim e^{-\frac{2}{c}\Delta(\lambda, \lambda_0)}$$

Interpretation: the matrix-like nature of the degrees of freedom in Chern-Simons may be responsible for the stringy decay

Summary

- Using the quantum information metric one can holographically explore regions of the (CFT) theory space in which the effective field theory description may break down

- Using multicritical vector models we identified a theory space including the usual vector models enjoying higher spin symmetry

- *The higher-spin points lies at infinite information distance from any generic point in the theory space* ***OK!***

- The anomalous dimensions of higher-spin currents <u>don't</u> fall off exponentially in the distance ***[violation of a part of the SDC]***

- *Reaching the higher-spin points by varying the parameter λ in Chern-Simons vector models give an esponential fall off* ***OK!***

Conclusion

- The emergence of higher-spin symmetries at infinite distance in moduli space / conformal manifold / theory space seems a very robust feature
- *The exponential fall off seems instead a characteristic of string models or, holographically, of matrix-like degrees of freedom*

Outlook

- Holographic (dual?) description of multicritical vector models?
- Testing other swampland conjectures in higher spin gravity?