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Pricing Electricity in Residential Communities Using Game-Theoretical Billings

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***Abstract*—By sharing common assets such as the power grid, prosumers are closely interrelated by their actions and interests. Game theory provides powerful tools for increased coordination among the prosumers to optimize the energy resources. However, depending on the prosumer profiles and the market rules, the individual bills may notably differ and prove to be unfair. In this work, we analyze the outcomes of three relevant game-theoretical billing methods, which are innovatively transposed to the day-ahead scheduling of energy exchange within a liberalized residential community dominated by distributed energy resources. The first two approaches rely on a (static) daily billing scheme, while the third considers a multi-temporal (continuous) billing. The Nash equilibria are computed using distributed algorithms, hence ensuring individual decision-making and avoiding third-party dependencies. The cost distributions are assessed using both a qualitative and a quantitative comparison based on various prosumer profiles in a modern smart grid. It is shown that, depending on the billing option, either the contribution towards the entity (i.e., the ability to improve the global solution) or the individual empowerment (i.e., the ability to bargain) can be preferentially incentivized.**

***Index Terms*—Billing, distributed algorithms, energy efficiency, energy communities, fairness, game theory, renewable energy, smart grid, energy storage, Vickey-Clarke-Groves.**

I. Introduction

In recent years, electricity systems have changed significantly. The desired energy transition towards a decarbonized system has led to the emergence of distributed energy resources (DERs), with local generation and new flexibility mechanisms at the end-user level, such as electric vehicles and energy storage [1]. This transition opens the way to decentralized operations with new strategies that proactively manage the demand side and overall improve the system operations by mitigating local grid constraints and empowering the prosumers [2]. Future solutions to enhance coordination within distribution systems may consider a more or less structured organization of the prosumers [3].

In this regard, an increasingly popular solution is to turn to collectives of prosumers. Much of the current literature on this topic focuses on the design of various types of market entities [4] that adopt local implementations of spot markets or peer-to-peer mechanisms [5]–[10]. However, network costs are poorly accounted for, as they usually arise as an output of the allocation mechanism to meet capacity constraints and pose a major fairness problem [11], e.g., distribution locational marginal prices, which discriminate prices based on the location [12]–[14]. There are, however, developing endogenous grid fees [15], [16], which are not directly based on physics. Moreover, inherent market imperfections (such as market power) tend to be exacerbated in such a context with a limited number of participants, which may prevent the maximization of the actual social welfare. Instead, it is possible to opt for a tariff structure that better reflects the actual incurred costs. This takes into account the use of resources, i.e., wear and tear as well as power losses, and thus considers the costs as inputs. Naturally, these costs are non-linear and the aggregate power flow scheme should be considered to implement accurate billing.

Building on demand-side management mechanisms, implementing billings that reflect the incurred costs arise as a particularly appropriate denominator in local energy collectives to engage individuals while taking advantage of local and physical realities. In such an interdependent environment, the cost allocation is challenging because it is subject to strategy. Game theory is a convenient framework to enable fair and efficient resource allocation under a controlled common objective [17]. A number of contributions consider aggregative cost functions for the operation of a bounded network. For example, [18]–[24] formulate day-ahead energy consumption scheduling games where each end-user optimizes its own costs. An alternative approach is to enforce a predefined consensus on the billing key through cooperative game-theoretical methods [25]–[27]. Most of the literature on demand-side management does not consider the current context of liberalized electricity markets. In [28], an adaptation of the energy consumption scheduling game to liberalized electricity markets is proposed. Here, the prosumers retain the freedom to choose their retail supplier and the costs of the common low-voltage network are taken into account through a billing scheme that reflects the incurred costs. However, [28] is limited to one type of cost allocation and does not account for energy storage.

In addition to these methodological considerations, it is essential to qualify and quantify the cost allocations with respect to the prosumer profiles. Indeed, technological advances have profoundly changed the nature of the energy load and have transformed the end-users into full-fledged actors. Their associated flexibility and the constraints they face have a major impact on the outcomes of demand-side management programs. Currently, there is a flagrant lack of perspective on the application of one or another billing method in terms of

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cost allocation and load profiles. In this regard, most works are limited to highlighting global performance and assessing fairness in a single dimension. For example, [24] examines daily and hourly billing using an elementary framework in which only flexible equipment can be mobilized (ignoring individual energy generation and storage resources) and in which a single utility can be contracted.

In this paper, we study the day-ahead scheduling of energy exchange in a residential community. In particular, we transpose three billing schemes to the liberalized European context, taking into account individual energy generation and storage. Particular attention is paid to the faithful reproduction of the incurred costs, i.e., commodity and grid costs. The main novelty lies in combining these methods and their evaluation with a comparison of cost allocation among the prosumers. Furthermore, we identify different incentives that target different objectives and thus address fairness in different ways. Specifically, we provide the following contributions.

- We propose three different billing methods in a liberalized residential community. In such a context, each end-user is free to choose its electricity supplier (contracted for a defined period), while the network costs are charged by the transmission and distribution system operators to the community as a whole. This approach differs from most of the literature, which assumes that a single utility charges a single cost encompassing commodity and grid costs. The first two billing methods use daily proportional billing that considers a single distribution key for the entire time horizon, i.e., one day: one of them uses a distribution key that is directly proportional to the net load, while the other passes on the relative contribution through a Vickrey-Clarke-Groves (VCG) mechanism. In the third billing method, the cost is distributed proportionally to the net load in each time slot of the scheduling horizon. The three billing methods offer different incentives to the prosumers depending on their load profiles.
- We charge prices for each billing that reflect the actual incurred costs. Their inherent non-linearity creates interdependence among the end-users and thus requires coordinated load scheduling schemes. We adopt a game-theoretical approach whereby each prosumer optimizes its day-ahead energy consumption scheduling depending on the applied billing. In this respect, the Nash equilibria are computed using the asynchronous best-response algorithm in presence of the first two billings methods (based on daily proportional billing) and the proximal decomposition algorithm in presence of the third billing method.
- We provide a comprehensive analysis of the above billing methods and evaluate their performance in the realistic context of a modern and liberalized residential community. The prosumer profiles feature potential energy generation and storage as well as electrical devices that have significant scheduling flexibility, such as electric vehicles and heat pumps. A qualitative comparison shows how the prosumers are impacted depending on their load profiles and billing methods. In addition, a benchmark with a representative mix of prosumer profiles based on a real-world database, i.e., the Pecan Street project [29], provides a quantitative assessment.

Our results show that, for the three considered billing methods, the degree of flexibility that characterizes a prosumer has a strong impact on fairness and inefficiency, i.e., the deviation from the social optimum. The billing method affects the way flexibility is evaluated, either by focusing on individual empowerment (continuous proportional billing) or by considering the entire unit under study (VCG mechanism). Moreover, the Nash equilibrium of the continuous proportional billing does not deviate significantly from the social optimum under expected operating conditions.

The remainder of this paper is organized as follows. Section II introduces the prosumer load model, pricing items, and billing schemes. Section III characterizes the underlying energy consumption scheduling games and provides insights into the computation of their Nash equilibria. Section IV analyzes the cost allocation under the different billing schemes for different prosumer profiles and evaluates the fairness and cost effectiveness in a typical test case. Finally, Section V provides the concluding remarks.

II. System Structure and Methodology

This paper considers a modern power grid where each prosumer is connected to a bi-directional communication infrastructure (using, e.g., a smart meter). We assume a liberalized framework, i.e., the power generated by the electricity producers flows through the transmission grid and the distribution network, and is sold by an electricity supplier to the end-users. We focus our attention on the demand side, where the prosumers are connected to the distribution network. More specifically, we consider a set of prosumers sharing the same low-voltage network, i.e., located behind the same low-voltage distribution transformer. These prosumers need to coordinate their energy consumption scheduling with all the other end-users since they share the same network and have interdependent costs (see Section II-D). Together, they thus act as a community.

A. Demand-Side Model

Let $\mathcal{N} = \{1, \ldots, N\}$ denote the set of prosumers of the considered network, i.e., the community, and let $\mathcal{T} = \{1, \ldots, T\}$ be the set of time slots in the time period of analysis, i.e., the next day. Here, each time slot $t \in \mathcal{T}$ has duration $\Delta\tau$, which depends on the time granularity adopted in the analysis. Each prosumer $n \in \mathcal{N}$ is characterized by the net load vector $\mathbf{l}_n = (l_n^1, \ldots, l_n^T)$ that includes various load components, some of which hold scheduling possibilities.

First, we describe the load of critical devices, for which each prosumer n does not allow any time flexibility (e.g., kitchen appliances, lighting, and multimedia equipment) using the *non-flexible load vector* $\mathbf{k}_n = (k_n^1, \ldots, k_n^T)$. Second, each prosumer n may allow a set $\mathcal{A}_n$ of its appliances (e.g., electric vehicles and heat pumps) to operate with some time flexibility. In this setting, each flexible appliance $a \in \mathcal{A}_n$ is characterized by the *scheduling vector* $\mathbf{x}_{n,a} = (x_{n,a}^1, \ldots, x_{n,a}^T)$ and the resulting *flexible load vector* is given by $\mathbf{x}_n = \sum_{a \in \mathcal{A}_n} \mathbf{x}_{n,a}$ (see the flexible load model in Section II-B). Third, each

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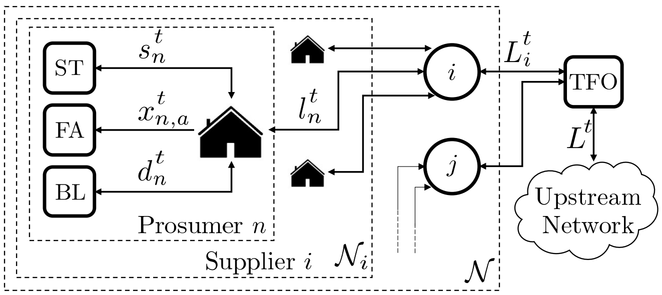

Fig. 1. Demand-side model: prosumer n, who contracted supplier i, owns an energy storage system (ST), flexible appliances (FA), and is characterized by a baseload (BL) grouping non-flexible consumption and PV generation. All the prosumers across all the suppliers behind the same transformer (TFO) form the residential community.

prosumer n may own a photovoltaic (PV) system and the related *generation vector* is denoted by $\mathbf{g}_n = (g_n^1, \ldots, g_n^T)$. Lastly, each prosumer n may have an energy storage system whose charging and discharging schedule is described by the *storage vector* $\mathbf{s}_n = (s_n^1, \ldots, s_n^T)$ (see the storage load model in Section II-C). The non-flexible load and the PV generation cannot be controlled and, in the context of a deterministic approach, they can thus be treated as state variables for the day-ahead procedure, as further explained in Section III. Hence, we group the state variables in the *baseload vector* $\mathbf{d}_n = \mathbf{k}_n - \mathbf{g}_n$ and express the net load vector as

$$\mathbf{l}_n = \mathbf{d}_n + \mathbf{x}_n + \mathbf{s}_n, \tag{1}$$

where $\mathbf{x}_n$ and $\mathbf{s}_n$ represent the decision variables. Furthermore, we use Λ_n to denote the set of decision variables of prosumer n, including $\bigcup_{a\in\mathcal{A}_n} \mathbf{x}_{n,a}$ and $\mathbf{s}_n$, and Λ^t to denote the set of decision variables at time slot t. Finally, the set of all the decision variables across all the prosumers and time slots is defined as $\Lambda = \{\Lambda_n\}_{n\in\mathcal{N}} = \{\Lambda^t\}_{t\in\mathcal{T}}$.

B. Flexible Load Model

The flexible devices can be controlled and scheduled at the most opportune time within the time constraints dictated by the individual requirements. In this regard, the prosumers can specify one or more time ranges during which they allow a possible operation. For each time slot t, we thus define the binary parameter $\delta_{n,a}^t \in \{0,1\}$ indicating this possibility, with $\boldsymbol{\delta}_{n,a} = (\delta_{n,a}^1, \ldots, \delta_{n,a}^T)$. Without loss of generality, we consider only appliances that are flexible in both time and power consumption level to avoid the introduction of binary decision variables required by equipment with a fixed consumption cycle. Hence, for a flexible appliance $a \in \mathcal{A}_n$, the predetermined energy $E_{n,a}$ must be delivered within the permissible time ranges, i.e.,

$$\boldsymbol{\delta}_{n,a}^{\mathrm{T}} \mathbf{x}_{n,a} \Delta\tau = E_{n,a}, \tag{2}$$

and is subject to a maximum power level $M_{n,a}$, i.e.,

$$\mathbf{0} \leq \mathbf{x}_{n,a} \leq M_{n,a} \boldsymbol{\delta}_{n,a}. \tag{3}$$

Note that the constraint in (3) restricts the energy consumption to permissible time slots.

C. Storage Load Model

We adopt a storage model that encompasses charging and discharging efficiencies, leakage rate, capacity, and maximum charging and discharging levels. Considering prosumer n, let us express the charging and discharging efficiencies through the vector $\boldsymbol{\beta}_n = (\beta_n^{(+)}, \beta_n^{(-)})^{\mathrm{T}}$ and write the charging and discharging schedule at time slot t using the vector $\boldsymbol{\sigma}_n^t \triangleq \big(\max(s_n^t, 0), \min(s_n^t, 0)\big)$.[1] Hence, the state of charge of the battery is constrained as

$$0 \leq \sum_{t=1}^{\bar{t}} \alpha_n^{\bar{t}-t} \boldsymbol{\beta}_n^{\mathrm{T}} \boldsymbol{\sigma}_n^t \Delta\tau + \alpha_n^{\bar{t}} E_{\mathrm{st}}^0 \leq E_{\mathrm{st}}^{\max}, \quad \forall \bar{t} \in \mathcal{T}, \tag{4}$$

where α_n is the leakage rate, E_{st}^0 is the initial state of charge, and $E_{\mathrm{st}}^{\max}$ is the storage capacity. In addition, the state of charge of the battery is subject to maximum charging and discharging levels, respectively denoted by $M_n^{(+)}$ and $M_n^{(-)}$. By considering the charging and discharging efficiencies, we have

$$-M_n^{(-)}\mathbf{1} \leq \beta_n^{(-)} \mathbf{s}_n, \tag{5}$$

$$\beta_n^{(+)} \mathbf{s}_n \leq M_n^{(+)}\mathbf{1}. \tag{6}$$

D. Cost Structure and Billing Models

In the context of liberalized electricity markets, the cost of electricity supply originates from the following two components.

- **Commodity costs.** They are the costs billed by the electricity supplier. To better reflect the incurred costs deriving from the possible generation capacity and the market position of the supplier for the next day, we adopt a real-time pricing scheme [30]. In this context, the suppliers can tailor their price each day and for each time slot freely, although they have to guarantee some limits (such as maximum price variations and preferred time slots) to maintain legibility in their billing plan offer. Hence, each prosumer can choose a supplier from the set $\mathcal{S} = \{1, \ldots, S\}$ that best corresponds to its consumption habits. Accordingly, each supplier $i \in \mathcal{S}$ is associated with a subset of customers $\mathcal{N}_i \subset \mathcal{N}$, with $\bigcup_i \mathcal{N}_i = \mathcal{N}$. However, it is assumed that only the positive net load $l_n^{t+} = \max(l_n^t, 0)$ is accounted for billing, i.e., the supplier does not buy energy back from the prosumers. The reasoning is that the prosumers usually own generation systems that have close to zero marginal generation costs. Therefore, the cost that supplier i applies to its customer $n \in \mathcal{N}_i$ at time slot t is given by

$$C_{\mathrm{supp},i}^t(l_n^t) = \gamma_{\mathrm{com},i}^t l_n^{t+} \Delta\tau, \tag{7}$$

where $\gamma_{\mathrm{com},i}^t$ is the corresponding price in €/kWh. It should be noted that the choice of a suitable supplier for each prosumer can benefit either the whole community or a single individual depending on the billing method, as discussed in Section IV. Alternatively, the commodity costs could be easily substituted by the day-ahead spot

[1] Note that the charging and discharging operations are mutually exclusive.

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prices if the prosumers were connected to the wholesale market, either through the supplier or directly without any intermediary.

- **Grid costs.** They are the costs covering the use of the transmission and distribution grids. It is essential to pass on the incurred costs directly to the end-users because system operators have very little control over the power flows. We recall that the costs are accounted for based on the use of physical resources, i.e., wear and tear as well as power losses. Due to the inherent non-linear nature of these costs, they can be determined only based on the aggregate load of all the prosumers, denoted by $L^t = \sum_{n\in\mathcal{N}} l_n^t$. Without loss of generality, we apply the quadratic cost function as in [18]–[21], i.e.,

$$C_{\text{grid}}^t = \gamma_{\text{grid}}(L^t\Delta\tau)^2, \tag{8}$$

where γ_{grid} represents the grid cost coefficient in €/kWh^2. It allows us to express the non-linearity and the underlying interdependence across the prosumers.

From the cost functions in (7) and (8), it is possible to establish billing models that help to achieve specific performance objectives, where efficiency and fairness occur to be among the most commonly sought ones. In this paper, we use and compare three different billing methods (see Section IV), each holding advantages and drawbacks regarding these objectives. They originate from the following two billing classes.

- **Daily proportional billing.** It consists in issuing bills proportionally to the costs aggregated across both prosumers and time (on a daily basis). The cumulative cost function is denoted by

$$f(\Lambda) = \sum_{t\in\mathcal{T}} \left(\sum_{i\in S} \gamma_{\text{com},i}^t L_i^{t+}\Delta\tau + \gamma_{\text{grid}}(L^t\Delta\tau)^2 \right), \tag{9}$$

where $L_i^{t+} = \sum_{n\in\mathcal{N}_i} l_n^{t+}$. This type of billing is optimal from the system's perspective in the sense that the prosumers are incentivized to minimize the cumulative cost $f(\Lambda)$, as discussed in Section III. Under a daily proportional distribution, the billing function is given by

$$b_n(\Lambda) = \frac{w_n}{\sum_{m\in\mathcal{N}} w_m} f(\Lambda), \tag{10}$$

where w_n represents the weight of prosumer n in the bill. However, it is a challenging task to redistribute these costs among the prosumers by implementing a fair set of weights. Indeed, the aggregative nature of the grid costs makes it difficult to fairly identify the individual contributions. A straightforward choice of the weights is obtained by considering the cumulative net positive load of each prosumer n, i.e., $\sum_{t\in\mathcal{T}} l_n^{t+}$. Its minimum possible value, denoted by $\mathrm{L}_n^\star$, is considered so as to keep the weight independent from the actual solution set Λ. Such a weight choice ensures that there is no possible strategy on the distribution key. The main flaw of this cost distribution is that it does not link one's bill directly to its choice of supplier (i.e., the prosumers who negotiate good contracts with their supplier are not incentivized) and to its flexibility level. Another choice of the weights is obtained by considering the relative contribution of each prosumer using the normalized VCG mechanism [23] or the Shapley value [31]. It is the first option that is developed further in this paper because of its reduced computation needs (see Section IV-A).

- **Continuous proportional billing.** It consists in issuing bills proportionally to the costs aggregated across prosumers for each time slot t. The cost function is denoted by

$$f^t(\Lambda^t) = \sum_{i\in S} \gamma_{\text{com},i}^t L_i^{t+}\Delta\tau + \gamma_{\text{grid}}(L^t\Delta\tau)^2. \tag{11}$$

Under a continuous proportional distribution, the billing function is given by

$$b_n(\Lambda) = \sum_{t\in\mathcal{T}} \frac{l_n^{t+}}{L^{t+}} f^t(\Lambda^t), \tag{12}$$

which accounts for the amount of energy exchanged by an individual and directly charges the underlying cost [19]–[24]. Although it does not yield the social (global) optimum, the inefficiency is expected to be more than compensated by the higher overall flexibility, leading to reduced costs. Indeed, the continuous proportional billing tends to be more rewarding than the daily proportional billing, since planning power exchanges at the preferential times of the dynamic pricing is better valued in the individual bills [32]. Incentivization is thus more effective in this context.

III. Energy Exchange Scheduling

Due to the competition arising from the shared use of the network, it is appropriate to adopt a game-theoretical formulation to characterize potential strategic behaviors. This section introduces the game formulation and shows how the underlying Nash equilibria are computed.

A. Game Formulation

Non-cooperative game theory is a powerful mathematical framework for modeling the interactions between selfish individuals competing for a common resource [33], [34]. It is thus meaningful to formulate the energy exchange scheduling problem under the conditions presented in Section II as a non-cooperative game, independently of the chosen billing method. In this context, each prosumer $n \in \mathcal{N}$ is a player who competes against the others by choosing its strategy profile Λ_n to minimize its objective function $b_n(\Lambda)$, which is defined either as in (10) or as in (12).

Let $\Lambda_{-n} = \{\Lambda_m\}_{m\in\mathcal{N}\setminus\{n\}}$ denote the set of all the strategy profiles except those of player n. We formally define the game by the tuple $\mathcal{G} = \langle\Omega, \mathbf{b}\rangle$, where $\Omega = \prod_{n\in\mathcal{N}} \Omega_n$ is the joint strategy set, with Ω_n being the individual strategy set of prosumer n, and $\mathbf{b} = \big(b_1(\Lambda_1, \Lambda_{-1}), \ldots, b_N(\Lambda_N, \Lambda_{-N})\big)$ is the vector of all the objective functions. Hence, each player n aims at solving the following optimization problem, given Λ_{-n}:

$$\begin{aligned} \underset{\Lambda_n}{\text{minimize}} \quad & b_n(\Lambda_n, \Lambda_{-n}) \\ \text{s.t.} \quad & \Lambda_n \in \Omega_n \end{aligned} \qquad \forall n \in \mathcal{N}. \tag{13}$$

The solution of $\mathcal{G}$ is given by the well-known concept of Nash equilibrium, which is a feasible strategy profile $\Lambda^\star = \{\Lambda_n^\star\}_{n\in\mathcal{N}}$ with the property that no single player n can benefit by unilaterally deviating from $\Lambda^\star$ if all the other players act according to $\Lambda^\star_{-n} = \{\Lambda_m^\star\}_{m\in\mathcal{N}\setminus\{n\}}$.

Proposition 1. *The game $\mathcal{G}$ has a non-empty and compact set of Nash equilibria.*

Proof: Building on [33], [34], the above is guaranteed when the following conditions hold for each player n: *i)* , the individual strategy set Ω_n is compact and convex; and *ii)* the objective function $b_n(\Lambda_n, \Lambda_{-n})$ is convex for any feasible Λ_{-n}. The first condition is easily verified since Ω_n consists of the linear equalities and inequalities (1)–(6), whereas the second condition can be proved by showing that the Hessian matrix of $b_n(\Lambda)$, defined either as in (10) or as in (12), is positive semidefinite. ■

Proposition 2. *All the Nash equilibria of $\mathcal{G}$ yield the same values of the objective functions.*

Proof: Building on [19], [20], it is easy to show that there exists an infinity of strategy profiles producing the same net load vectors $\mathbf{l}_n$. ■

B. Nash Equilibrium Computation

In the case of a daily proportional billing, if the strategy of each prosumer n is computed by minimizing the billing function (10) via the (asynchronous) best-response algorithm (see, e.g., [18]), the cumulative cost (9) can either decrease or remain constant. In this setting, the Nash equilibrium is reached when no player can decrease its bill, i.e., when the cumulative cost is minimized. Hence, in this specific case, it is possible to consider the Nash equilibrium as the solution of the system optimization problem

$$\begin{aligned} \underset{\Lambda}{\text{minimize}} \quad & f(\Lambda) \text{ as in (9)} \\ \text{s.t.} \quad & \text{(1)–(5)}, \end{aligned} \tag{14}$$

which can be solved either via a centralized algorithm or by using a distributed implementation based, for instance, on the alternative direction method of multipliers (ADMM) [35].

On the other hand, pure best-response algorithms cannot be used in the case of continuous proportional billing because minimizing the billing function in (12) for any prosumer n cannot guarantee that the cumulative cost in (11) is not increased. However, one can use more sophisticated distributed schemes such as the proximal decomposition algorithm or the proximal-point method [36], which are guaranteed to converge under some technical conditions (the latter additionally requires the strict monotonicity of the cost function). For instance, the convergence conditions for the proximal decomposition algorithm can be conveniently derived by resorting to variational inequality theory, as done in [19]–[21] (see also [33], [34]).

In the case of daily proportional billing, we adopt the asynchronous best-response algorithm described in Algorithm 1. At each iteration, player n either computes $\Lambda_n^\star$ according to (13) or keeps the previous solution $\Lambda_n^{(q)}$. The iterations at which player n computes a new solution are specified by the set $\mathcal{Q}_n$.

Algorithm 1 Asynchronous Best-Response Algorithm

Data: Choose a feasible starting point Λ^0. Set $q = 0$.

while a suitable termination criterion is not satisfied **do**
  for $n \in \mathcal{N}$ **do**

$$\Lambda_n^{(q+1)} = \begin{cases} \Lambda_n^\star \in \underset{\Lambda_n\in\Omega_n}{\operatorname{argmin}}\, b_n(\Lambda_n, \Lambda_{-n}^{(q)}), & \text{if } q \in \mathcal{Q}_n \\ \Lambda_n^{(q)}, & \text{otherwise} \end{cases} \tag{16}$$

  end for
  $q \leftarrow q + 1$.
end while

Algorithm 2 Proximal Decomposition Algorithm

Data: Choose a feasible starting point $\Lambda^{(0)}$. Set $q = 0$, $\kappa > 0$, and the initial centroid $\bar{\Lambda} = \{\bar{\Lambda}_n\}_{n\in\mathcal{N}} = \mathbf{0}$.

while a suitable termination criterion is not satisfied **do**
  for $n \in \mathcal{N}$ **do**

$$\Lambda_n^{(q+1)} \in \underset{\Lambda_n\in\Omega_n}{\operatorname{argmin}} \{b_n(\Lambda_n, \Lambda_{-n}^{(q)}) + \frac{\kappa}{2}\|\Lambda_n - \bar{\Lambda}_n\|^2\} \tag{17}$$

  end for
  if the Nash equilibrium has been reached **then**
    each player $n \in \mathcal{N}$ updates its centroid: $\bar{\Lambda}_n = \Lambda_n^{(q+1)}$.
  end if
  $q \leftarrow q + 1$.
end while

In the case of continuous proportional billing, we adopt the proximal decomposition algorithm described in Algorithm 2, which is based on the regularized game

$$\begin{aligned} \underset{\Lambda_n}{\text{minimize}} \quad & b_n(\Lambda_n, \Lambda_{-n}) + \frac{\kappa}{2}\|\Lambda_n - \Lambda_n^{(q)}\|^2 \\ \text{s.t.} \quad & \Lambda_n \in \Omega_n \end{aligned} \qquad \forall n \in \mathcal{N}. \tag{15}$$

For a sufficiently large regularization parameter $\kappa > 0$, (15) has a unique solution that can be computed in the same way as in the best-response algorithm. Both Algorithm 1 and Algorithm 2 have desirable privacy-preserving properties, since only the aggregate load is necessary for each prosumer to compute its solution at each iteration. Furthermore, the distributed nature of the algorithms does not require the intervention of a third party. The analysis of the convergence of the proposed algorithms is beyond the scope of this paper; we refer to [18]–[21] for more details.

IV. Comparative Study

As a basis for our different test cases, we use residential consumption data from the Pecan Street project [29]. The dataset covers 6 months of electricity consumption and PV generation of 25 homes in the state of New York, USA. For illustrative purposes, the test cases consider two energy suppliers offering opposite daily price profiles. Depending on the prosumer characteristics, one may opt for overall cheaper night or day rates. In practice, however, there may be more possible suppliers meeting more specific energy consumption and generation profiles. In this respect, guided selection may be considered to further optimize the bill, e.g., based on the consumption history through a recommendation algorithm.

A. Billing and Prosumer Loads

In our comparative study, we use three different cost distribution schemes, each holding different interesting properties. The first two are variant of the daily proportional billing, for which the asynchronous best-response algorithm described in Algorithm 1 is adopted. The third is a variant of the continuous proportional billing, for which the proximal decomposition algorithm described in Algorithm 2 is adopted.

1) **Net load proportional billing [Net].** The first variant of the daily proportional billing distributes the bill proportionally to the minimum possible cumulative positive net load $\mathrm{L}_n^\star$. In fact, using this quantity instead of the actual cumulative positive net load ensures that an individual cannot be penalized for improving the social cost. For example, a prosumer may be asked to inject its PV generation into the network instead of storing the energy for future personal use in order to prevent load peaks in the grid and decrease the overall bill. Hence, we choose each weight such that $w_n = \mathrm{L}_n^\star,\ \forall n \in \mathcal{N}$, which yields (cf. (10))
$$b_n^{\mathrm{Net}} = \frac{\mathrm{L}_n^\star}{\sum_{m=1}^N \mathrm{L}_m^\star} f(\Lambda). \quad (18)$$
Note that accounting for the positive net load encourages the self-consumption of locally produced electricity. This is in line with the costs structure that assumes no purchasing price for the injection.

2) **Marginal cost billing [VCG].** The second variant of the daily proportional billing is shown to be the only VCG mechanism ensuring a dominant truthful strategy [37], i.e., that is independent of the other participants' strategies. Like all VCG mechanisms, it achieves a socially optimal solution. Here, we choose each weight such that $w_n = \mathrm{C}_\mathcal{N}^\star - \mathrm{C}_{\mathcal{N}\setminus\{n\}}^\star,\ \forall n \in \mathcal{N}$, where $\mathrm{C}_\mathcal{N}^\star$ is the optimal cost achieved by the whole set of prosumers by solving (14), which yields (cf. (10))
$$b_n^{\mathrm{VCG}} = \frac{\mathrm{C}_\mathcal{N}^\star - \mathrm{C}_{\mathcal{N}\setminus\{n\}}^\star}{\sum_{m\in\mathcal{N}} \mathrm{C}_\mathcal{N}^\star - \mathrm{C}_{\mathcal{N}\setminus\{m\}}^\star} f(\Lambda). \quad (19)$$
This scheme requires the additional solution of N optimization problems to obtain each $\mathrm{C}_{\mathcal{N}\setminus\{n\}}^\star$, i.e., the optimal cost achieved by all the prosumers except n. This can be conveniently done in a distributed fashion via ADMM, as in the sharing problem presented in [35]. Note that the marginal cost pricing can also be formalized as a cooperative game. Indeed, in such a VCG mechanism, a predefined consensus can be reached and the maximum value (i.e., the savings) is achieved when the grand coalition (i.e., the whole community) cooperates.

3) **Continuous proportional billing [CP].** The variant of the continuous proportional billing integrates the cost considerations inherent to a liberalized context, which yields (cf. (12))
$$b_n^{\mathrm{CP}} = \sum_{t\in\mathcal{T}} \left(\gamma_{\mathrm{com},i}^t l_n^{t+} \Delta\tau + \gamma_{\mathrm{grid}} l_n^t L^t (\Delta\tau)^2\right). \quad (20)$$
Unlike the first two methods, this billing directly accounts for the contracted supplier through $\gamma_{\mathrm{com},i}^t$ (see (7)).

In addition, for each of the scenarios described above, the prosumers can own one or more of the following devices.

- *Photovoltaic facility (PV)*: it is considered as a negative non-flexible load.
- *Energy storage system (ST)*: it can store possible individual surplus PV generation or store energy during cheaper hours.
- *Electric vehicle (EV)*: two time windows (morning and evening) are made available for charging, each with its energy constraint; thus, it is considered as a partially flexible load.
- *Heat pump (HP)*: for simplicity, and without loss of generality, it is considered as a fully flexible load.

The parameters considered for each of the above flexible devices are based on modern specifications and can be found in [38].

B. Profiles and Cost Allocation

In this section, we aim at characterizing how the nature of the load impacts the allocation of the electricity bill. Based on the flexible devices detailed in the previous section, i.e., PV, ST, EV, and HP, we define four different qualitative scenarios and compare their billing outcomes for each billing scheme. Each scenario considers a reference consumer, whose baseload corresponds to the average load over the dataset, and a prosumer with the same baseload who additionally owns one or more flexible devices. Both are assumed to have contracted a supplier that offers a cheaper night rate for the commodity. Note that this setting aims at providing a generic and simple comparison, whereas a more representative and detailed benchmark is presented in Section IV-C.

Fig. 2 considers the scenarios where the prosumer is equipped with: (a) PV, (b) EV+HP+ST, (c) PV+EV+HP+ST, and (d) PV+EV+HP+ST. Each subplot shows the different billing schemes in the x-axis and the prices or costs in the y-axis obtained by the consumer (left bar) and the prosumer (right bar). Here, the prices are obtained as the division of the cost by the respective minimum positive net load $\mathrm{L}_n^\star$, which is detailed for each scenario in Table. IV-B. Furthermore, we explicitly show the prices and costs associated with the grid components (dark color) and the commodity components (light color).

Fig. 2(a) depicts the outcome of a scenario with no flexible loads and for which the prosumer is distinguished only by the presence of a PV system generating the average production obtained on the dataset. As expected, the net load proportional billing b_n^{Net} in (18) features the same price for both the consumer and the prosumer, which in turn leads to a significant cost reduction for the PV holder because of the reduced

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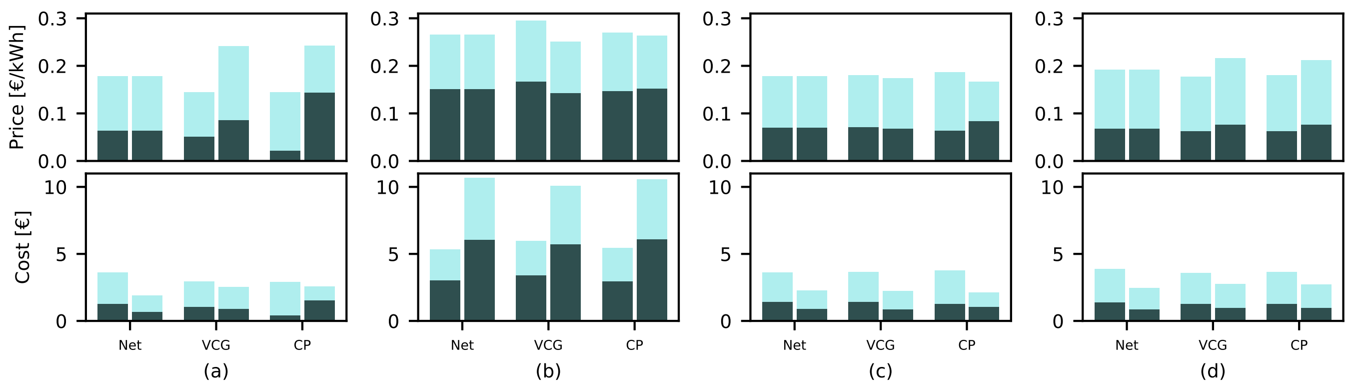

Fig. 2. Comparison of the prices (top figures) and costs (bottom figures) for the three considered billing methods, i.e., b_n^{Net} in (18), b_n^{VCG} in (19), and b_n^{CP} in (20). Each subplot considers a reference consumer (left bar) and a prosumer (right bar) equipped with: (a) PV, (b) EV+HP+ST, (c) PV+EV+HP+ST, and (d) PV+EV+HP+ST (with different supplier). The prices and costs are divided into grid components (dark color) and commodity components (light color).

[kWh]	(a)	(b)	(c)	(d)
Consumer (left bars)	20.185			
Prosumer (right bars)	10.545	40.185	12.792	12.792

TABLE I
Minimum positive net load $\mathrm{L}_n^\star$ (imports) of the consumer and the prosumer for each scenario illustrated in Fig. 2.

[%]	PV	HP	ST	All	Day rate
Net	High PV (15.0 c€/kWh)				
VCG	0.2	-7.0	-11.3	-8.5	-2.6
CP	-4.6	-6.1	-6.7	-3.9	0.7
Net	Low PV (22.4 c€/kWh)				
VCG	1.6	-0.5	0.8	0.6	-3.1
CP	0.6	-1.5	-0.2	-0.4	-2.1

TABLE II
Variation of the mean energy price for the three considered billing methods in function of the load characteristics and suppliers for days with high and low PV generation.

expected net load. This cost reduction is less pronounced for the other two discriminative cost distributions and their incurred price is therefore higher. Note that such a higher price originates from an increase in both commodity and grid prices for the marginal cost billing b_n^{VCG} in (19), whereas it is due only to an increased grid price for the continuous proportional billing b_n^{CP} in (20). The latter billing scheme represents the only cost distribution where the commodity and grid costs are individually computed. By consuming little energy from imports during the day and benefiting from cheaper night rates, the prosumer can directly assert a claim on the reduction of the commodity costs. However, it is penalized in terms of grid costs by the overall higher impact on the network. In general, the absence of flexibility negatively impacts the outcomes of PV holders under b_n^{VCG} and b_n^{CP}.

In Fig. 2(b), the prosumer holds flexible devices, namely an EV, a HP, and an ST (but no PV), and the price obtained with the discriminative billings is lower than the one applied to the reference consumer. This is expected because the flexibility of these DERs, which help to avoid consumption peaks, is valued through a lower price. Since the cost function is increasing and convex, the prices and the costs are all higher than in the previous scenario because of the increased load.

In Fig. 2(c), PV and flexible loads are combined and the resulting total cost is notably reduced. This is even more evident for the prosumer as its energy needs are partially covered by the PV. Indeed, the discriminative billings reward the efforts of the prosumer by producing lower prices for its remaining imports. As previously discussed, the continuous proportional billing directly reflects the higher impact of the prosumer on the reduction of the commodity costs and the increase of the grid costs.

Lastly, Fig. 2(d) illustrates the major role played by the choice of the supplier. Here, we assume the same setting as in Fig. 2(c) except that a supplier offering cheaper day rates is chosen instead of one offering cheaper night rates. Indeed, while the choice of Fig. 2(c) leads to cheaper prices for the prosumer, the choice of Fig. 2(d) results in increased prices. Here, we assume 0.08 €/kWh and 0.16 €/kWh for the cheap and normal commodity prices, respectively.

C. Case Study

For a more quantitative and representative assessment of how the considered billing schemes can impact a modern smart grid, we present the following case study considering a combination of profiles. These are based on various projections of the expected load characteristics of a modern residential community within a decade. We consider 50 prosumers corresponding to residential homes whose loads are sampled from the dataset [38]. Here, each prosumer can be equipped with PV, ST, EV, and HP. Besides, they are assigned to a supplier offering cheaper night or day rates. The 20 days with highest and lowest PV generation are considered to resemble sunny and cloudy days, respectively. The mean prices obtained through the global optimization, i.e., $\overline{\gamma} = \mathrm{C}_{\mathcal{N}}^\star / \sum_{t\in\mathcal{T}} L^t$, are 15.0 c€/kWh and 22.4 c€/kWh, respectively.

For the days with high and low PV generation, Table II summarizes the variation of the mean price obtained depending on their profile. For instance, the first column compares the

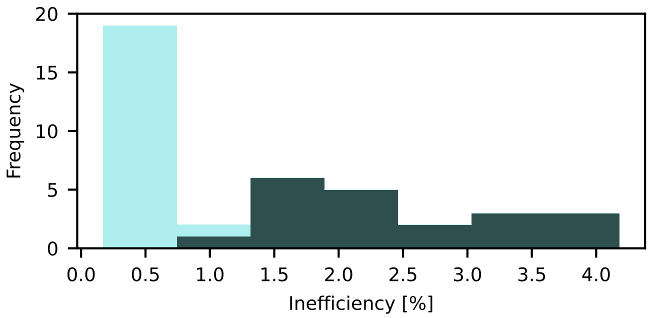

Fig. 3. Histogram of the inefficiency for the 20 days with highest PV generation (dark color) and lowest PV generation (light color).

difference between the mean prices obtained by the prosumers with a PV ($\overline{\gamma}_{\rm PV}$) and without a PV ($\overline{\gamma}_{\rm noPV}$), i.e., $(\overline{\gamma}_{\rm PV} - \overline{\gamma}_{\rm noPV})/\overline{\gamma}_{\rm noPV}$. The remaining columns show the variation depending on whether the prosumers do or do not own an HP, an ST, or all these devices. The difference between the prosumers who have contracted a supplier offering cheaper night or day rates is also shown. We recall that the prices obtained with a net load proportional billing remain equal regardless of the conditions.

The most striking feature is the considerable impact of the PV generation. For the days with low PV generation, the billing schemes are very weakly discriminating. This is expected as the potential extra load tends to increase the grid costs because of the higher imports, even though it can be scheduled at the most opportune time, i.e., during the hours with cheaper commodity costs. It is only by contracting the supplier with cheaper day rates that the prosumers can obtain better mean prices. They are indeed less numerous (see [38]) and tend to cancel out the negative impact on the grid costs caused by the preferred scheduling of the flexible appliances during night hours.

The discriminative billing schemes exhibit their characteristics mostly in the days with high PV generation. The marginal cost billing (VCG mechanism) rewards the flexibility mobilized by the prosumers as a whole. Indeed, the flexible loads and energy storage allow to shave off the backflows that would be generated by surplus PV generation, hence resulting in significant grid costs savings. Interestingly, the prosumers equipped with PV benefit from lower mean prices with respect to their counterparts only if continuous proportional billing is considered. In fact, through a VCG mechanism, the prosumers equipped with PV cannot claim any contribution in the decrease of the total bill as PV on its own is not a source of flexibility. On the other hand, the continuous proportional billing gives more space for strategy and leads to some inefficiency.

As the continuous proportional billing yields a suboptimal solution, it is interesting to quantify the inefficiency expressed as $(\sum_{n\in\mathcal{N}} b_n^{\rm CP} - {\rm C}^\star_{\mathcal{N}})/{\rm C}^\star_{\mathcal{N}}$. Fig. 3 depicts the histogram of this metric for the same 20 days with highest and lowest PV generation considered above. The right tail of the histogram results from the days with high PV generation (dark color) whereas the days with low PV generation lead to very little inefficiency (light color). This is further verified by considering no PV generation at all under the same conditions: in this case, the inefficiency is smaller than 0.01%. The reason for the decreased efficiency during the days with high PV generation is that the grid component of (20) is subject to strategy. For instance, the PV owners, in comparison with a socially optimal billing, tend to increase their self-consumption by just the right amount in order not to cancel the aggregate net load and remain positive. By doing so, they benefit from a payment, i.e., negative cost, because of their individual negative net load. The days with high PV generation thus provide more bargaining power to the PV owners. Note that the mean daily inefficiency for the entire dataset is 1.97%.

[c€/kWh]	High PV	Low PV
As in Section IV-C	15.0	22.4
Reduced price gap	15.7	21.6
Flat rate	16.8	21.4

TABLE III
Mean energy price for different supplier rates.

D. Impact of Supplier Rate Schemes

In the previous case study, we considered two simple rate schemes for the commodity, i.e., cheaper night or day rates, to emphasize the role of PV generation. Indeed, this represents a major non-flexible power source and the associated power flows have a notable impact on the network (and thus on the costs). As shown in the previous section, cheaper night rates lead to lower overall energy prices because the energy is cheaper when the PV panels cannot produce.

Here, we reproduce the case study of Section IV-C with two different tariff schemes. The first considers a lower night-day gap with 0.10 €/kWh and 0.14 €/kWh for the cheap and normal commodity prices, respectively, whereas the second consists in a flat rate with a price of 0.12 €/kWh for the whole day. Table III shows that both the alternative tariff schemes lead to an overall increase in the mean energy price. Indeed, the days with high PV generation are affected by the more expensive night rates and the days with low PV generation tend to cancel out such an increase because of the reduced day prices. Besides, it can be observed that the bargaining power of the PV owners is reduced since their relative weight in the bill is higher. All the other observations remain similar.

E. Discussion

VCG mechanisms, when used for distributing costs, are often presented as intrinsically fair [24], [39]. In reality, this claim is highly dependent on the context and the adopted framework, e.g., possible cooperative mechanisms. The marginal cost billing, although ensuring a truthful dominant strategy, does not give much decision-making power. Indeed, the social optimum is imposed and mobilizes more or less the resources of each individual without any further consideration. This may be little stimulating for the prosumers to increase their maximal flexibility. On the other hand, the continuous proportional billing gives direct negotiation power to the prosumers, which leads to a deviation from the social optimum. However, it

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	Efficiency	Empowerment	Tractability
Net	+	Neutral	++
VCG	+	+	Neutral
CP	Neutral	++	+

TABLE IV
Summary of the advantages (+) and strong advantages (++) of the three considered billing methods.

is reasonable to assume that such a billing would generate higher flexibility levels, which more than compensates for the inherent inefficiency. Another issue is that, although the true cost philosophy is preserved at the aggregated level, it is not the case at the individual level where some prosumers can benefit from payments while their marginal cost is zero. An interesting framework adaptation, in line with our goal of optimizing the use of physical resources (see Section II), consists in considering the surplus PV generation as free energy made available locally to a defined community [31]. In this context, the PV owners are regarded as positive contributors, whereas the recipients are regarded as having a neutral impact even though they benefit from reducing their overall demand. However, the game-theoretical framework required for such a decentralized approach is more demanding and beyond the scope of this paper. On another note, the net load proportional billing does not discriminate against individuals at all, which might lead to little incentivization of flexibility.

Table IV summarizes the important considerations highlighted throughout this paper for the three considered billing methods. Here, efficiency refers to the definition provided in Section IV-C; in this context, a billing is fully efficient if the social optimum is reached. Empowerment indicates if the billing tends to reward proactive behaviors by the prosumers allowing more flexibility. Lastly, tractability evaluates qualitatively the computation burden and the mathematical complexity underlying each billing method. As shown in Section IV-C, the continuous proportional billing leads to very little inefficiency compared to the social optimum, which is featured by the two other methods. The marginal cost billing appears to be the one with the best potential for mobilizing individual efforts, which justifies the strong advantages in terms of empowerment. On the other hand, the flexibility is poorly incentivized under a net proportional billing, which is, however, the least demanding solution in terms of computational complexity. The marginal cost billing requires a significantly higher computational burden because it needs to solve $N+1$ optimization problems, while the distributed algorithms of the continuous proportional billing need more attention for their convergence. Recall that all the distributed implementations have the advantage to enforce privacy by requiring to share only the aggregate load.

V. Conclusion

Acknowledging the modern context of liberalized electricity networks and increased penetration of DERs, this paper proposes and analyzes three different game-theoretical billing methods for the day-ahead scheduling of flexible appliances in a residential community. The applied cost structure that is used for billing the prosumers intends to reflect an accurate image of the mobilized energy resources. We present the game formulation for each billing method, namely, the net proportional billing, the marginal cost billing, and the continuous proportional billings, which can be solved using efficient distributed algorithms. Then, the results obtained from solving the games allow to derive qualitative and quantitative considerations highlighting various efficiency and fairness features.

The net load proportional and marginal cost billings are both socially efficient as they distribute the optimal aggregate cost of the whole low-voltage entity. The first does not discriminate the prosumers according to their load profile, which may be perceived as little stimulating for consenting to a significant level of maximal flexibility, whereas the second is sensitive to the cost structure and the characteristics of the individual load. The continuous proportional billing, although not socially optimal, shows very little inefficiency. Besides, it accounts for their waiver of empowerment at the benefit of the global solution.

The net load proportional billing is the most straightforward cost distribution. Although it has very limited incentive to consent flexibility, it has the advantage to be the most egalitarian. The marginal cost billing is highly relevant when the effective contribution towards the entity is promoted. Under such a scheme, the interaction framework would be more relevant if cooperation is taken a step further by considering community resources (e.g., mutualization of excess resources) so as to have more representative distributions. On the other hand, the continuous proportional billing represents a good trade-off between higher coordination needs and self-determination.

An interesting prospect for future work is to address the constraints of partially or entirely meshed networks. Evaluating power flows in such a context becomes essential because the congestions may affect one or several branches. Note that the underlying shared constraints introduced by the power flow equations would lead to a generalized Nash equilibrium problem [9], [21].

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