# Tailoring of electric dipoles for highly directional excitation in parity-time symmetric waveguides

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The flexibility of photonic structures offers an ideal platform for exploring parity-time (PT) symmetry phenomena. In these platforms, electric dipoles are often used as accurate models for electromagnetic sources, and elliptical dipoles were shown to provide for directional mode excitation. Here, we tailor the polarization of an electric dipole to cancel one of the modes of two coupled PT-symmetric waveguides. This creates a contrast between wave propagation on both sides of the dipole, which manifests differently depending on the unique features of the modes in the various PT regimes.

## Context

Electric dipole sources have been used for several years in integrated photonics as compact electromagnetic sources, due to their efficient coupling to photonic guided modes [1,2]. The near-field directionality of circularly polarized electric dipoles has recently been demonstrated, by taking advantage of constructive or destructive interference of different evanescent waves. [3,4] Coupling waveguides to these dipoles can lead to directional excitation of the waveguide modes. However, the directionality is lost if the dipole is at the center of an inversion-symmetric photonic structure like coupled waveguides. In order to restore the contrasting properties between two sides, we exploit the unique characteristics of parity-time-symmetric coupled waveguides.

Parity-time (PT) symmetry can be realized in coupled waveguides by using a balanced profile of their imaginary refractive index, with one waveguide made of a gain material and the other with an equal amount of loss. [5] The uniqueness of these structures stems from the two regimes in which they can operate depending on the value of the gain-loss parameter  $\gamma$ , that defines the absolute imaginary part of the refractive index in the waveguides. The transition between these two regimes occurs at the exceptional point (EP), which is located at a certain value of  $\gamma$  dependent on the structure geometry. In the PT-symmetric regime ( $\gamma < \gamma_{EP}$ ), both modes of the structure propagate without any gain or loss, whereas in the PT-broken regime ( $\gamma > \gamma_{EP}$ ) one mode benefits from the gain and is amplified while the other experiences losses and decays.

In this work, we take advantage of the characteristics of such a PT-symmetric structure with tailored dipoles to create various types of left-right contrasted wave propagation.

#### Structure



Figure 1 - Schema of the photonic structure used in the simulations. The dielectric gain and loss materials are represented in orange and green respectively and the air in white. The location of the dipole is marked by a red dot.

In our structure, the electric dipole is placed in the center of the air layer separating two PT-symmetric slab waveguides. The waveguide made of the gain material with refractive

index  $n = 2 + i\gamma$  is at the top, and the lossy guide with index  $n = 2 - i\gamma$  is at the bottom (fig. 1). The value of the gain-loss parameter  $\gamma$  at the EP for our structure is = 0.1231. We use the CAMFR (CAvity Modeling Framework, [6]) eigenmode expansion Maxwell equations' solver to numerically simulate our setup for values of  $\gamma$  ranging from 0 to 0.2.

## **Dipole-mode coupling theory**

To create the desired left-right contrasted propagation, we need to directionally excite a mode of the structure using the dipole source. Therefore, we first need to understand how a dipole couples to a photonic mode.

The excitation amplitude  $(A_m)$  of a mode *m* by a source of dipole moment  $\vec{p}$  is given by

$$A_m \propto \vec{p}. \vec{E_m}(\vec{r_0}) \tag{1}$$

where  $\overrightarrow{r_0}$  is the dipole position and  $\overrightarrow{E_m}$  is the electric field associated to mode m. [6] According to the Maxwell equations, the longitudinal electric field component  $(E_z)$  of two identical modes propagating in opposite directions need to have opposite signs. We define  $E_{mx}$  and  $E_{mz}$  as the field components of the right-side mode (z > 0) at the dipole position  $\overrightarrow{r_0}$  and develop the scalar products of equations (1):

$$A_m \propto p_x E_{mx} \pm p_z E_{mz} \tag{2}$$

with the + sign corresponding to a mode propagating on the left (z < 0) and the - sign to a mode on the right of the dipole (z > 0).

By setting  $A_m$  to 0, equation 2 shows that the mode can be excited on one side while being canceled on the other side if

$$p_x E_{mx} = \mp p_z E_{mz}.\tag{3}$$

The mode will be canceled on the left side in the – case and on the right side in the + case. Since in our structure the modes electric field and the dipole moment can be complex, the condition with their polar expressions then becomes  $|p_x|e^{i\phi_{px}}|E_x|e^{i\phi_{Ex}} = \mp |p_z|e^{i\phi_{pz}}|E_z|e^{i\phi_{Ez}}$  which is fulfilled if

 $|p_x||E_x| = |p_z||E_z| \text{ and } \phi_{px} - \phi_{pz} = \phi_{Ez} - \phi_{Ex}(+\pi)$ (4) where  $\pi$  needs to be added if one cancels the mode on the left side.

### **Mode analysis**

As shown by the theory, the field of a mode at the dipole position is a crucial variable to directionally excite this mode. Therefore, we next study the evolution of the modal field in (0,0) as a function of the gain-loss parameter  $\gamma$ .

Depending on the value of  $\gamma$ , the electric field profile of the modes along x exhibits different symmetries: symmetric or antisymmetric in the PT-symmetric regime and asymmetric in the PT-broken regime. These result in different phases and moduli of the electric field of the modes at the dipole position, since an antisymmetric component will not contribute to the field at the dipole position (x = 0). For both modes, in figure 2, we represent the quantities that intervene in the cancellation condition (eq. 4): the moduli of  $E_x$  and  $E_z$  as well as the phase difference between  $E_z$  and  $E_x$  as a function of  $\gamma$ .

In the PT-symmetric regime ( $\gamma < 0.1231$ ), figure 2(a) shows that the x and z field components have an equal complex phase, as  $\phi_{Ez} - \phi_{Ex} = 0$ . This is due to the peculiar symmetries of the modes in this regime: the symmetric parts of their x and z field components are both real for mode 1 and imaginary for mode 2. Figure 2(b) also shows that the relative intensities of  $E_x$  and  $E_z$  vary with  $\gamma$ , differently for modes 1 and 2. At the EP ( $\gamma = 0.1231$ ), the modes are defective, meaning that in addition to their eigenvalues



- propagation constants - being equal, their profiles are identical. Their fields are thus equal at the dipole position.

Figure 2 – (a) Phase difference and (b) moduli of the x and z electric field components of modes 1 (blue) and 2 (orange) at the dipole location for  $\gamma = 0$  to 0.20. The exceptional point is marked by a grey dashed line.

In the PT-broken regime ( $\gamma > 0.1231$ ), it can be seen on figure 2(b) that the field moduli of modes 1 and 2 are equal component by component (identical blue and orange results). This is once again explained by the symmetries of the modes. In this regime, the electric fields of the two modes are asymmetric, one mode being stronger in the gain guide while the other is more in the loss guide, but their profiles are mirrored with respect to x = 0. Modes 1 and 2 thus have equal values of |E| at the source. Their profiles are also complex conjugates in this regime, which explains the opposite phase differences in figure 2(a).

## Dipole tailoring and resulting contrasting behaviors

Considering the evolution of the phase and modulus of the modal field with  $\gamma$ , we can now calculate the dipole polarization required to cancel the desired mode on one side using equation (4). We choose to cancel mode 2 on the left side.



Figure 3 – (a) Characteristics of the dipole tailored for the desired contrast at each  $\gamma$ . (b-e) Magnetic field absolute value |H| in the structure. The dipole, tailored for each  $\gamma$ , is in (0,0). The insets represent (d) |H| or (e) log |H| at x=0. (f) The dipole tailored for  $\gamma = 0.15$  is used, with an added phase of  $\pi$  compared to image (e).

In the PT-symmetric regime, since it has been seen in Fig. 2(a) that  $\phi_{Ez} - \phi_{Ex} = 0$  for both modes, the phase difference of the adequate dipole (purple in fig. 3(a)) is  $\phi_{px} - \phi_{pz} = \pi$  for any value of  $\gamma$  below 0.1231. However, the ratio between  $|p_x|$  and  $|p_z|$  (green in fig. 3(a)) needs to be adjusted to account for the variation in  $|E_x|$  and  $|E_z|$ . In the PTbroken regime, the dipole phase difference needs to be adjusted for each  $\gamma$  as the mode phase difference varies, but the x-to-z component ratio of the dipole moment remains almost constant as  $|E_x|$  and  $|E_z|$  evolve with similar tendencies as a function of  $\gamma$ .

Using the dipole tailored for each  $\gamma$  to excite the modes of the structure, we observe and compare the wave propagation on the left and right sides.

In the PT-symmetric regime (fig. 3(b,c)), mode 2 is one of the two propagating modes of the structure. Removing this mode on the left produces a uniform field profile, while exciting both modes on the right causes a beating, thus creating a contrast in the wave propagation between the two sides of the dipole. The beating pattern observed on the right also changes as y varies since the propagation constants – the eigenvalues – vary with the PT conditions. Close to the EP, the modes become defective. As seen in figure 3(d), both modes are then canceled on the left, while the step of the beating pattern becomes almost infinite as the propagation constants are nearly equal. The system behaves as a single waveguide, which results in a near-complete directionality. In the PT-broken regime (fig. 3(e)), mode 2 is the gain mode. Removing it on the left makes the field considerably smaller than on the right side, as the gain mode remains on the right making the field explode. It is also important to note that for any  $\gamma$  the contrast can be reversed between the two sides by adding a phase of  $\pi$  to the phase difference between the components of the adequate dipole (fig. 3(f)). This feature could be used with the beating contrast in a directional coupler arrangement, as it would enable the selective excitation of any of the four ports. It could also lead to directional lasing if used with the directional amplification.

## Conclusion

By tailoring the polarization of a dipole to the properties of the modes of PT-symmetric coupled waveguides, a directional mode excitation is achieved in both PT regimes. This creates a left-right contrast in the wave propagation in the structure, with the presence of either beating in the PT-symmetric regime or field amplification on a chosen side of the dipole. Our results also highlight a feature of the EP: the defectiveness of the modes in this unique configuration enables a near-complete directional excitation.

In the end, these various types of contrasting phenomena may offer new possibilities for integrated photonics applications, routing setups, and lasing behavior.

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## References

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