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TEACHING PROPORTIONALITY: TEACHERS' CONCEPTIONS AND REPORTED PRACTICES*

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Abstract

Proportionality in mathematics occupies an important place not only in teaching but also in everyday life. Roblin (2015) tells us that it is not an easy thing to teach. This article aims to understand teachers' teaching methods in proportionality.

To achieve this, a questionnaire distributed to primary and secondary school teachers will allow us to analyze their conceptions and their declared practices when teaching proportionality and this, according to their level of teaching: primary or secondary.

Our results show that the level of difficulty of teaching proportionality perceived by the teachers is high. Our analyses will be followed by a discussion and recommendations for the training and teaching of proportionality.

Key words: *Design; Reported practices; Proportionality; Proportional reasoning; Teaching.*

1. Introduction

Proportionality is one of the fundamental mathematical concepts in Belgian compulsory education. Its learning starts in elementary school and continues in the first level of secondary education. Proportionality is used to solve various problems from everyday life, from various fields such as physics or economics (University of Ontario, 2016). Indeed, proportional reasoning is used in particular, to calculate our shopping, convert foreign currencies, follow recipes and adapt them to our needs. For this reason, proportionality occupies an important place in mathematics learning (Sokona, 1989). From then on, mastering proportional reasoning is an important part

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of understanding and applying mathematics (Ministry of Education, 2012). Lamon (2005) estimates that over "90% of high school students lack the reasoning skills to fully understand mathematics" (p. 10). Although this notion is central, students have many difficulties with this reasoning (Bertheleu *et al.*, 1997; Comin, 2002; Dupuis & Pluvinage, 1981).

In light of these findings and the ubiquity of proportionality in everyday life, this leads us to question the teaching methods used when learning proportionality. Consequently, we are going to investigate the teachers' conceptions and their declared practices when teaching proportionality within the framework of this survey. For this purpose, we have developed a questionnaire based on the pedagogical literature to investigate whether "teachers' conceptions of teaching proportionality differ according to level" and whether "teachers' reported practices in learning proportionality differ according to level".

In order to verify these two assertions, a theoretical review of the literature allowing to define proportionality and to study its different aspects at the learning level is first presented. Then, the methodological framework of our research is explained. Finally, a statistical analysis of the results, a discussion and recommendations are presented.

2. Review of the literature

The purpose of this first section is to present the theoretical framework that delimits our research. First, the notions of proportionality and proportional reasoning will be explained through the pedagogical literature. We will then present the main difficulties encountered by the students. Finally, we will also develop a recommendation made by some authors to bring non-proportional situations as early as possible and to contrast them with proportional situations.

2.1. Definition

Proportionality is defined as "a particular relationship between two quantities (or rather their measurements) or between two sequences of numbers. These two sequences of numbers (whether or not associated with quantities) must be multiples of each other" and quantities are described as "...a characteristic of an object that allows it to be compared to others" (Daro, Geron & Stegen, 2007, p. 20). Proportional reasoning is the ability to think about and compare multiplicative relationships, symbolically represented as ratios between quantities (Van de Walle & Lovin, 2008). According to Deblois (2011), a statement in which a proportionality situation is emphasized is a proportion problem.

2.2. Representations of data in a proportional situation

When solving a proportionality problem situation, the choice of procedure can be influenced by the four techniques for representing a directly proportional situation (Daro *et al.*, 2007):

- Arrow patterns called "sagittal graphs".

Example: 4 kilograms of apples → €4.80

- Sentences.

Example: 4 kilograms of apples cost 4.80€.

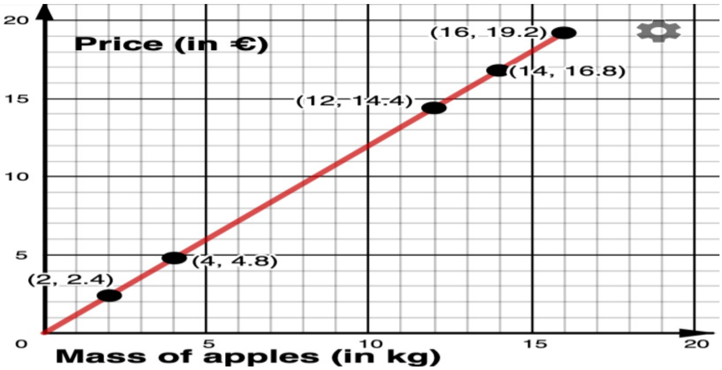
- Tables in rows or columns.

Example:

Mass of apples (in kg)	4	2	12	16	14
Price (in €)	4,80	2,40	14,40	19,20	16,80

- Graphics.

Example:



The most commonly used forms of representation are the number table and the sagittal graph. The graph is the most complex form because it requires a range of skills. Its use is often limited to reading and interpreting the data it represents (Colomb *et al.*, 2001). Furthermore, Duval (1993) emphasizes the importance of using different registers of semiotic representations in the same procedure.

2.3. Proportional reasoning

Teaching proportionality is not easy because this procedure seems, on the one hand, to be perceived as difficult for students (Boisnard *et al.*, 1994) and, on the other hand, there are several methods for solving proportionality problems (Roblin, 2015; Simard, 2012b). The use of these solution methods depends on the problem itself. Indeed, the choice of the solving procedure comes from the student's interpretation of the problem.

Consequently, learning proportionality is based on situations that allow for unconscious proportional reasoning (Colomb *et al.*, 2001; Daro *et al.*, 2007). In solving these situations, the teacher can support the students' task by encouraging them to verbalize their approach. In this sense, proportionality is not a subject in its own right, but a tool for solving everyday situations (Colomb *et al.*, 2001). Proportional reasoning can be defined as "...multiplicative reasoning used routinely in everyday life" (Oliveira, 2008, p. 9). According to Oliveira (2008), proportional reasoning is not limited to the method used to solve the problem. We agree with Post, Behr, and Lesh (1988) that thinking is crucial when dealing with this concept. Thus, a problem involving direct proportionality (e.g., the mass of apples purchased and the price to be paid) can be approached by qualitative reasoning of this type: "If I buy more apples, will I pay more or less? By approaching the problem from this angle, the student, taking into account the relationship between the quantities, is no longer, on the one hand, in a simple treatment of sets of data and, on the other hand, envisages a relevant procedure allowing to treat it. Once the answer has been found, its plausibility must then be judged by the student. Gnass (2000) specifies that qualitative reasoning makes the understanding of the problem better, which will consequently lead the student to identify a wider range of strategies enabling him to solve the proposed problem.

2.4. Students' difficulties with proportionality

In 1981, Dupuis and Pluvinage presented this concept as the sequence of the 4 arithmetic operations. Indeed, proportionality is very important, on the one hand, because of its essential place in mathematics, and on the other hand, several fields use this concept. However, students' mastery of proportionality is not combined with a J-curve as defined by the authors. Thus, it is not mastered by all students after a certain period of time (Lambrecht, 2016).

According to Bergeaut, Billy, Cailhol *et al.* (2013), the main difficulties of students in proportionality situations are: recognizing a proportionality situation (due to the implicit nature of the statements); identifying the quantities to be related and sorting the data associated with a quantity; identifying the linearity relations between data of the same quantities; cognitive overload (intrinsic load) when the statement has several proportionality relations; choosing the appropriate procedure to arrive at the solution of the problem posed.

Another difficulty often encountered by students is the incorrect use of an additive procedure, as in the case of the Brousseau puzzle where students must construct an enlargement of the puzzle. The instruction given is that a segment whose length measurement is 4 units becomes 7 units in the enlarged puzzle. Some students will add 3 units to each segment. When students realize that the puzzle is no longer a square and that the pieces no longer fit together, they question the procedure used (Oliveira, 2008). According to Hersant (2001), the geometric framework is not to be neglected. Indeed, Brousseau (1996) notes that this framework has the advantage of quickly determining the validity of the construction procedures used thanks to the properties of similarities.

However, Comin (1992) draws our attention to the fact that knowing the numerical values of quantities does not guarantee that the student will be able to carry out the expected construction. According to Comin, this would explain a lower success rate compared to other proportionality problems. In his thesis, Adjiage (1999) notes that students also use an additive procedure in this type of exercise: "In the original drawing, the mast measures 4 cm and the bridge measures 9 cm. On the enlargement, the mast is 7 cm. How big is the bridge? Some people realize that there is a 3 cm difference between the original mast and the enlarged mast. They add 3 cm to the deck and find 12 cm for the enlarged deck. When the quantities are of the same nature and are measured in the same unit, this can lead to errors such as those highlighted above (Hersant, 2001).

2.5. Confronting non-proportionality

An overuse of strategies to solve proportionality problems has been demonstrated in non-proportionality situations, notably with students from the second year of primary to the second year of secondary school by De Bock, Van Dooren, Janssens & Verschaffel (2007). The authors attribute this behavior to a phenomenon they call "linearity illusion" and this systematic recourse is caused by a low cognitive investment of the students in problem solving.

According to Gille (2008), the inconsistent use of procedures intended for proportionality in non-proportional contexts is rooted in a meager presentation of non-proportional situations to students. In an article explaining the mathematical foundations of proportionality, Simard (2012a) explains that it is necessary for a student to be able to recognize proportionality problems. A student may be able to apply strategies to solve proportionality problems, but unable to determine when to use them as they may very well use these strategies incorrectly. Thus, the teacher must not only introduce students to the strategies, but also allow them to develop the ability to identify proportionality. This means that they should not be presented with proportionality situations exclusively, but also with non-proportionality situations. This is also recommended by Daro *et al.* (2007), who even recommend confronting non-proportionality situations as early as possible in order to get them to analyze the statement and avoid the abusive use of procedures inappropriate for non-proportionality situations.

3. Background of the research

The purpose of our survey is to describe the conceptions and reported practices of teaching proportionality in primary and secondary schools in French-speaking Belgium. The importance of proportionality in mathematics raises several questions about teachers' conceptions of this mathematical object and their reported practices. Indeed, the difficulties encountered by students in this domain and the results of the national certification tests at age 14 lead us to question the teachers' knowledge and teaching methods in proportionality. Based on our theoretical review, we hypothesize that teachers implement practices based on their conceptions of proportionality and their level of instruction. We attempted to answer the following two research questions:

- *What are teachers' conceptions of teaching and learning proportionality based on their grade level?*
- *What are teachers' reported practices for teaching and learning proportionality by grade level?*

The declared teaching practices are specified by the teacher himself or herself, who makes the information available during an interview or by filling out a questionnaire (Marcel, Olry, Rothier-Bautzer and Sonntag, 2002). These are therefore different from the practices observed in a real context when the teacher is in action in the classroom. In order to access these conceptions and reported practices for teaching proportionality, we took into account several dimensions identified in the literature that we feel are relevant.

4. Methodology

In order to collect and analyze teachers' reported conceptions and practices on proportionality teaching, we developed a questionnaire on Google Forms. This questionnaire was distributed to primary and secondary schools in French-speaking Belgium from February 22 to March 25, 2021. The questionnaire was sent out by contacting the principals by email and sharing it on different groups on digital social networks dedicated to teachers. At the end of this distribution, 179 responses were recorded. This is a casual sample, made up of teachers who volunteered and were willing to help us with our research.

The questionnaire sent to the schools is composed of a first section, comprising 6 items, which concern the respondents' identifying information. The second section consists of 17 open and closed questions, dealing with several themes. These themes study the usefulness of proportionality for the citizen, the level of difficulty of this notion, the forms of data representation used in the teaching of proportionality. The teachers' point of view is also asked about the type of reasoning used, the level of mastery and the type of errors made by the students. Some questions are accompanied by a link to an illustration, explanation, video or additional information to help teachers better understand the concepts or tools referred to in the questions.

The online survey alternates between continuum, multiple choice, and open-ended questions to capture teachers' perceptions. The continuums explore the level of confidence ("very unconfident" to "very confident"), usefulness ("very unhelpful" to "very helpful"), degree of difficulty ("very easy" to "very difficult") or level of agreement ("strongly disagree" to "strongly agree"). The 5-level continuums focus instead on frequency ("never" to "very often").

5. Results

This section is devoted to the presentation and analysis of the results of our survey. First, we conducted a descriptive analysis of our sample. Next, we conducted a descriptive and an inferential analysis for each of our research questions. To perform our analyses, we used Excel and Jasp software. Since we are comparing two levels of education and our data are not normally distributed, we chose a non-parametric Mann-Whitney U procedure.

5.1. Descriptive analysis of our sample

Of the 179 respondents, the sample consisted of 43 males, 134 females, and 2 subjects who did not specify their gender. With regard to seniority, Figure 1 shows that almost half of our respondents have significant teaching experience. As for the level of teaching, 102 respondents teach at the elementary level while 77 subjects are secondary school teachers.

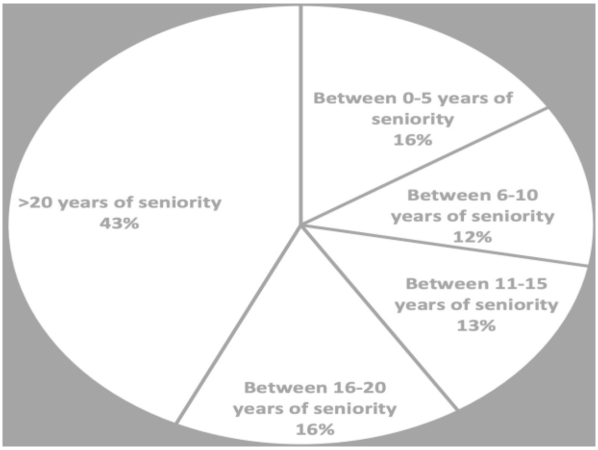


Figure 1. Distribution of teachers by years of service

5.2. Teachers' conceptions of teaching proportionality

The purpose of this subsection is to answer the research question, "*What are teachers' conceptions of teaching proportionality as a function of their level of teaching?*" To do so, we rely on 4 items investigating the level of didactic confidence, the level of usefulness, the level of learning difficulty, and students' misuse of proportionality solving strategies in non-proportionality situations. Table 1 presents, by grade level, the results for the different items considered.

Table 1. Teacher conceptions by grade level

	Primary level	Secondary level	U value of the Mann-Whitney test
Level of confidence in teaching	$\underline{x}$ = 1.922 CV = 32%.	$\underline{x}$ = 2.156 CV = 28%.	U = 3257 $p = 0.018$
Level of social utility of proportionality for the citizen	$\underline{x}$ = 2.186 CV = 29%.	$\underline{x}$ = 2.377 CV = 26%.	U = 3299 $p = 0.037$
Level of difficulty of students learning proportionality	$\underline{x}$ = 1.506 CV = 54%.	$\underline{x}$ = 2.392 CV = 40%.	U = 5213 $p < 0.001$
Misuse of strategies to solve proportionality problems in non-proportionality situations	$\underline{x}$ = 1.176 CV = 45%.	$\underline{x}$ = 1.481 CV = 42%.	U = 2849 $p < 0.001$

For the item devoted to the level of confidence at the didactic level, we observe a higher average for secondary school teachers ($\underline{x}$ = 2.156), meaning that these teachers are more confident about proportionality at the didactic level than primary school teachers ($\underline{x}$ = 1.922). A reading of the coefficients of variation (CV) indicates that the degree of agreement is also more homogeneous for secondary school teachers.

From an inferential point of view, we observe that there is a significant difference for the level of confidence on the didactic level according to the level of teaching (U = 3257, p = 0.018).

Concerning the level of usefulness of proportionality on the social level, secondary school teachers agree more that proportionality is useful for the citizen ($\underline{x}$ = 2.377). Furthermore, the coefficient of variation shows a greater homogeneity of response (CV = 26%). Inferentially, this difference in views between teachers of different grade levels is statistically confirmed (U = 3299, p = 0.037).

In terms of the level of difficulty in learning proportionality, the level of difficulty expressed by high school teachers was higher ($\underline{x}$ = 2.392) than that identified by elementary school teachers ($\underline{x}$ = 1.506). Inferential analysis shows that there is a statistically significant difference for the level of difficulty in learning proportionality as a function of teaching level (U = 5213, p < 0.001). In terms of dispersion, the coefficients of variation indicate less variability for high school teachers.

With respect to the last variable, secondary school teachers agreed more with this item ($\underline{x}$ = 1.481) than primary school teachers ($\underline{x}$ = 1.176). The nonparametric Mann-Whitney U procedure tells us that there is a statistically significant difference between teachers' conceptions according to their level of teaching regarding students' misuse of proportionality strategies in non-proportional situations (U = 2849, p < 0.001).

5.3. Teachers' reported practices in teaching proportionality

The purpose of this subsection is to answer the research question, "What are teachers' reported practices when teaching proportionality based on their grade level?". First, we asked teachers about the types of data organization presented in class.

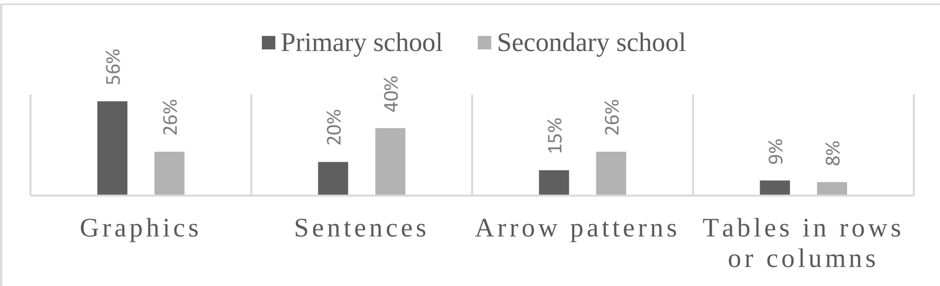


Figure 2. Most frequently used form of data organization

Regarding the form of data organization used most frequently, Figure 2 shows that the majority of teachers select tables in rows or columns. This choice is justified by the answers given to the open-ended question, which highlights the simplicity, ease, clarity, and presence in the course syllabus and tests of this form of data organization. They also specify that this form is the one usually used. It should also be noted that

41% of primary school teachers chose the arrow diagram as the second most used form. This form is said to be more visual and easier to use for elementary students.

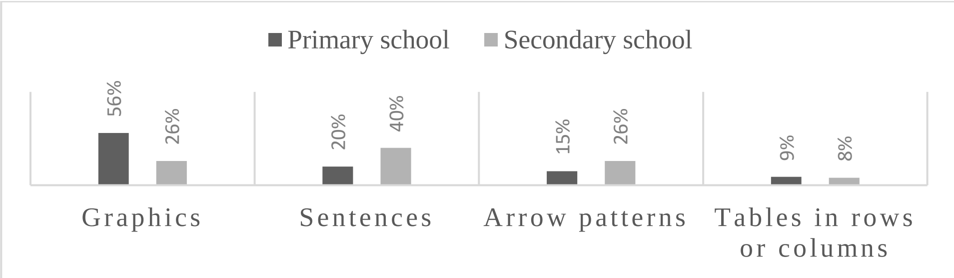


Figure 3. Form of data organization used least often

For the least frequently used form of data organization, we observe discrepancies between elementary and secondary teachers. These discrepancies are objectified by an open-ended question. For primary school teachers, graphs are generally not used very often because the students are too young to draw them and the interpretations are not obvious (comprehension, reading and abstraction). As for secondary school teachers, sentences are, in majority, less frequently used due to a lack of representativeness but also because they are complex to understand and are rather implemented in problems as a conclusion.

Secondly, we questioned teachers on different practices highlighted in the literature. Concerning the frequency of use of an online exercise platform to teach proportionality, it is higher among secondary school teachers ($\underline{x}$ = 0.610 vs. $\underline{x}$ = 0.265). This observed difference is statistically significant by teaching level (U = 2944.5, $p < 0.001$).

This difference can be partially explained by the analysis of the item below, constructed as a MCQ with several possible answers, which characterizes in which cases online exercise platforms are used.

We note some discrepancies in the use of online exercise platforms in primary and secondary education. Indeed, this type of tool is mainly used to choose proportionality exercises to be done in class in basic education whereas in secondary education, these software are generally used to give proportionality exercises to be done at home. The frequency of using video problems to teach proportionality was also higher among secondary school teachers ($\underline{x}$ = 0.403 vs. $\underline{x}$ = 0.186).

At the inferential level, there is a statistically significant difference between the frequencies of use of video problems according to teaching level (U = 3358, $p = 0.015$).

Table 2. Rationale for using exercise platforms

	Primary education	Secondary education
To prepare my work on proportionality in class (presentation, supports, videos, explanation)	28	23
To prepare only the part of the course on proportionality in the classroom (student support)	27	14
To choose proportionality exercises to do in class	42	24
To give proportionality exercises to do at home	25	40
To introduce proportionality	17	11

This finding can be justified by the item below, presented in the form of a multiple-choice test, which analyzes when video problems are used. Table 3 shows that the majority of basic school teachers use video problems to prepare work and choose exercises. On the other hand, secondary school teachers mainly use video problems to introduce proportionality. These differences in use may, in part, explain the differences in the frequency of use of video problems according to the level of instruction.

Table 3. Rationale for using video problems

	Primary education	Secondary education
To prepare my work on proportionality in class (presentation, supports, videos, explanation)	21	14
To prepare only the part of the course on proportionality in the classroom (student support)	11	12
To choose proportionality exercises to do in class	22	16
To give proportionality exercises to do at home	8	12
To introduce proportionality	17	19

With regard to the use of qualitative reasoning, the descriptive analysis shows that primary school teachers use it more often ($\underline{x}$ = 2,735) than secondary school teachers ($\underline{x}$ = 2.675). However, this difference in usage frequencies is not statically significant inferentially as a function of teaching level (U = 4042, p = 0.724). In terms of the frequency of proposing plane similarities in proportionality, high school teachers ($\underline{x}$ = 1.636) propose this type of exercise significantly (U = 3166, p = 0.022) more often than primary school teachers ($\underline{x}$ = 1.275). Moreover, the analysis of the item that studies the level of difficulty of this type of task for the students corroborates this result. Indeed, we note that the primary school teachers ($\underline{x}$ = 2.176) consider this type of task more difficult than secondary school teachers ($\underline{x}$ = 1.883). This divergence of opinion between teachers, according to their level of teaching, concerning the level of difficulty of enlargements or reductions of plane figures in proportionality is statistically significant (U = 4783.5, p = 0.007). A Spearman correlation between these two items testifies that the more difficult teachers perceive this task to be, the less they offer it to their students, regardless of their teaching level

(Rho = -0.518, p < 0.001). With respect to the presentation of non-proportionality situations, secondary school teachers ($\underline{x}$ = 2.494) more regularly presented non-proportionality situations to their students than primary school teachers ($\underline{x}$ = 1.902). As for the inferential analysis, it indicates that there is a statistically significant difference between the frequency of use of non-proportionality situations according to the teaching level (U = 2692.5, p < 0.001).

This conclusion can be justified with the analysis of the next item, on the reasons for the choice of response. The answers given to this open-ended question indicate that, at the level of primary school teachers, the use of non-proportionality situations gives meaning, improves understanding, and allows for variety in the exercises. However, it is difficult to grasp with younger children and is often overlooked due to lack of time. On the other hand, this type of situation is part of the curriculum for secondary education and develops critical thinking while improving understanding. It is also mentioned as a complement to proportionality.

6. Discussion

Given the importance of proportionality in education, it is an indispensable mathematical concept and a good understanding of it is necessary (Oliveira, 2008). However, it is clear that learners have difficulties with this concept (Baldy *et al.*, 2007; Bertheleu *et al.*, 1997; Comin, 2002; Daro *et al.*, 2007; Gille, 2008; Hersant, 2001; Oliveira, 2008). This is evidenced by the unsatisfactory success rates for this concept on the external certification tests at age 14, despite gradual and continuous learning. In our research, we investigated teachers' reported conceptions and practices based on their level of teaching (primary or secondary). Statistical analyses allowed us to answer our two research questions.

First, the conceptions of secondary school teachers differ from those of elementary school teachers in significant ways. It would seem that the level of usefulness of proportionality for the citizen is recognized but in a moderate way by secondary school teachers. Given the importance of the proportional model in many areas but also in everyday life (Colomb *et al.*, 2001), this finding is of concern. Not only is proportionality an essential object in the learning of mathematics (Sokona, 1989), but it is also an indispensable notion in other contexts such as percentages, linear functions, plane similarities, etc. Secondary school teachers also express more confidence from a didactic point of view. The difficulties students have with this concept, as mentioned by various authors (Bertheleu *et al.*, 1997; Comin, 2002; Dupuis and Pluvinage, 1981), are also a reality for the secondary school teachers who participated in our survey. Note that the level of complexity can also be explained by the plurality of different resolution strategies that can be implemented when faced with a proportionality task (Roblin, 2015; Simard, 2012b). In contrast to De Bock and colleagues (2007), only secondary school teachers moderately agreed on the misuse of resolution strategies for proportionality situations in non-proportionality tasks. In terms of teachers' reported practices, the most popular method of representing data is the proportional table. At the primary level, arrow diagrams are also widely used in class. The graph is less presented to primary school students, whereas sentences are used in secondary school. Our results are in line with those of Colomb and his colleagues (2001). The reductive nature of the graph in direct proportionality is also highlighted in the study by Dragone, Temperman and De Lièvre (2020). Secondary school teachers use online platforms significantly more often to provide homework exercises and video problems to introduce proportionality than their elementary school counterparts. The analysis of reported practices reveals, moreover, that teachers often use qualitative reasoning. Qualitative reasoning offers the opportunity to consider the relationship between quantities and avoid simple processing of data sets. This process of reflection (Post, Behr and Lesh, 1988) allows for a better understanding of the problem by the student and the identification of several resolution strategies to deal with it (Gnass, 2000).

We hypothesize that teachers rarely propose "plane similarities" type tasks, given the level of difficulty of these tasks noted by the teachers and, undoubtedly, also due to a lack of didactic avenues. We believe that it is essential to provide teachers with didactic tools for learning proportionality. Since quantities are of the same nature and their measurements are in the same unit (Hersant, 2001), a mistake often made by students is to resort to an additive procedure. Although knowing the numerical values does not guarantee that the student will be able to carry out the geometric construction (Comin, 1992), this framework has the advantage of providing immediate feedback on the construction carried out (Brousseau, 1996) and of leading the student to question the resolution procedure used (Oliveira, 2008). Secondary school teachers make more use of non-proportional situations than primary school teachers. In addition to a low cognitive investment, the illusion of linearity (De Bock, Van Dooren, Janssens & Verschaffel, 2007) could be explained by the fact that few non-proportionality situations are presented to students (Gille, 2008) and handled in a guided manner with them. In terms of developing conditional knowledge, it seems essential that a learner be able to recognize when a situation falls under proportionality or not (Simard, 2012a). For this reason, non-proportional situations should be offered to students in order to allow them to differentiate them from proportionality situations but also to avoid a systematic use of inappropriate solving strategies (Daro *et al.*, 2007).

7. Recommendations for training and teaching proportionality

Following the analysis of the results of this survey, we now attempt to formulate recommendations for training, both initial and continuing, but also for the teaching of proportionality. First of all, we think that it is necessary to highlight the importance of the proportional model in many fields but also in everyday life.

We also recommend proposing non-proportional situations in order to offer students the possibility of distinguishing them from proportional situations and thus avoid resorting to inappropriate strategies. Qualitative reasoning also allows students to reflect on the task by allowing them to better understand it. Furthermore, it seems appropriate to submit tasks related to plane similarities. Despite the difficulties associated with geometric constructions, this framework allows students to verify the accuracy of their construction and, if necessary, to better question the strategy used.

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