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Using temperature as the noise source for stochastic resonance and frequency conversion in non-linear photonic cavities

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Abstract – **Driven and modulated photonic cavities are extensively studied because they exhibit many interesting effects, such as frequency conversion and reciprocity breaking. In particular, in bistable systems, adding noise can lead to stochastic resonance (SR), which corresponds to periodic transitions between stable states, and can be used, for example, to enhance energy harvesting. In this work, we demonstrate that the system temperature can operate as a noise source for stochastic resonance in modulated Kerr cavities. With SR, the outgoing power at the signal frequency is significantly enhanced, up to 40% for certain regimes. This paves the way for new frequency conversion devices using thermal fluctuations as an effective sideband generator.**

I. Introduction

Temporal modulation of photonic structures is widely studied. It presents many exotic effects and various applications like selective frequency conversion [1] or active cooling mechanism [2]. Considering modulated bistable system, stochastic resonance (SR) corresponds to noise-induced transitions between stable states, synchronized with the modulation [3]. As in this work, most SR related studies assume Markov approximation. However, it has been recently employed in optical systems with memory effects [4]. In what follows, we study a non-linear cavity coupled to an external channel. We exploit the noise from thermal fluctuations due to a non-zero temperature of the structure. A periodic modulation is applied to the system in its bistable regime. First, we show that temperature can induce SR. Then, we demonstrate that it can be used as novel frequency-conversion devices. Finally, we provide insight into maximising the conversion of a pump wave into a particular sideband frequency.

II. Numerical model

A non-linear photonic cavity with Kerr effect is coupled to an external port. This system could be implemented with the geometry shown in Fig. 1(a). In such a general situation, the cavity mode amplitude evolution is efficiently described using coupled-mode theory and the Langevin framework [5]. As the structure is driven by a monochromatic pump, using a frame rotating at the pump frequency, the system is described by

$$\begin{cases} \dfrac{\mathrm{d}u}{\mathrm{d}\tau} = \left[j\left(1-|u|^2\right)-\eta\right]u+\sqrt{2\eta^3}s_p+\sqrt{2n_d}\xi_d+\sqrt{2n_e}\xi_e & \text{(1a)} \\ s_+ = s_p+\sqrt{\dfrac{n_e}{\eta^3}}\xi_e & \text{(1b)} \\ s_- = -s_+ + \dfrac{\gamma_e}{\gamma}\sqrt{\dfrac{2}{\eta}}u & \text{(1c)} \\ s_p(\tau) = \sqrt{\zeta_0}\left(1+\delta e^{j\Omega\tau}\right). & \text{(1d)} \end{cases}$$

In Eq.(1a), $|u|^2$ is dimensionless and related to the cavity mode energy. There is dissipation in the system coming either from the coupling to the external port (γ_e) or internal dissipation (γ_d) arising, for example, from coupling to phonons. The total dissipation rate $\gamma = \gamma_e + \gamma_d$ is related to $\eta = \gamma/\delta\omega$ where $\delta\omega = \omega_0 - \omega_p$ is the detuning between the cavity resonance and the pump frequency. Incoming/outgoing dimensionless powers from/into the external channel are given by $|s_+|^2$ and $|s_-|^2$, respectively. In Eq.(1b), the incoming wave s_+ consists of both a pump (with frequency ω_p and amplitude s_p) and thermal radiation ξ_e, arising from the external temperature T_e. The pump amplitude, proportional to ζ_0, is also modulated according to Eq.(1d) with the addition of a small signal at frequency $\omega_p + \Omega$ and amplitude δ. Temperature-induced fluctuations are modelled as additive external and internal noise sources, ξ_e and ξ_d respectively, satisfying

$$\begin{cases} \langle \xi_i^*(\tau)\xi_i(\tau')\rangle = \delta(\tau-\tau') & \text{(2a)} \\ \langle \xi_i(\tau)\xi_i(\tau')\rangle = \langle \xi_i^*(\tau)\xi_i^*(\tau')\rangle = 0 & \text{(2b)} \end{cases}$$

where $i \in \{e,d\}$ and $\langle . \rangle$ means taking the "thermodynamic ensemble average". In Eq.(1a), $n_i = \frac{\gamma_i \alpha k_B T_i}{\delta\omega^2}$ ($i \in \{e,d\}$) where α is the non-linear coefficient, controls the noise intensity which is proportional to the temperatures.

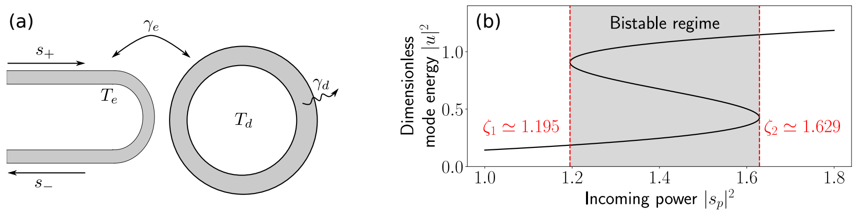

Fig. 1: (a) System geometry. (b) Hysteresis of mode energy according to input power for $\eta = 0.4$.

One can show that the system is bistable for a range of input power, $\zeta_1 < |s_p|^2 < \zeta_2$ and $\eta < \sqrt{3}/3$. The evolution of the mode energy $|u|^2$ with input power $|s_p|^2$ in Fig. 1(b) shows the region of incoming power for which the system is bistable with $\eta = 0.4$. The incoming power $|s_p|^2$, centered between ζ_1 and ζ_2, is modulated according to Eq.(1d), so that the system remains in the bistable regime (delimited by red dashed line in Fig. 1(b)).

III. Results

The parameters are set so that $\eta = 0.4$, the signal detuning $\Delta_s = \Omega/\gamma = 0.01$, the cavity life-time $Q = \omega_0/\gamma = 10^4$, the pump and signal amplitudes, $\zeta_0 = 1.40$ and $\delta = 0.077$ and coupling ratio $\gamma_e/\gamma_d = 1$ (rate matching condition). We consider thermodynamic equilibrium, then temperatures are equal $T_e = T_d = T$.

In Fig. 2(a)-(d), we examine the evolution of mode energy with time for various temperatures. The red noisy line represents the mode energy while the dashed black lines are the system potential energy extrema. With the upper and lower branches being the minima corresponding to stable states and the middle line the local maximum. For small temperature (Fig. 2(a), $T = 1$ K), the noise intensity is very low. The system is stuck in one of the modulated stable states. By increasing the temperature, (Fig. 2(b), $T = 100$ K), the transition probability to the other minimum increases. The system undergoes jumps between its stable states. For even higher temperature (Fig. 2(c), $T = 400$ K), these transitions are periodic with a period equal to the modulation one. This situation corresponds to stochastic resonance. For even higher temperatures (Fig. 2(c), $T = 5000$ K), the system exits the SR regime because transition probability is very high. Jumps between states are no longer periodic.

In Fig. 2(e)-(h), we present the outgoing power spectral density (PSD), normalised by input power at pump frequency. These spectra correspond to situations of Fig. 2(a)-(d). The PSD displays a central peak at pump frequency ω_p, as well as sidebands spaced by one modulation frequency Ω. It shows that at the SR regime (Fig. 2(g)), the outgoing power is maximum with a sidepeak at signal frequency equal to 10% of the incoming power at pump frequency. There is therefore a strong conversion coming from the periodic transitions arising in the cavity.

To understand how the frequency conversion can be enhanced, we examine two figures of merit. The transmission, $|\langle s_-\rangle|^2/|\langle s_+\rangle|^2$, ratio between outgoing and incoming power. And the conversion efficiency defined as $|s_-(\omega_p+\Omega)|^2/|s_+(\omega_p)|^2$, outgoing power at the signal frequency compared to the incoming power at the pump frequency. The temperature is set to $T = 400$ K, corresponding to SR, and we vary the ratio of dissipation rates γ_e/γ_d, while keeping the total dissipation $\gamma = \gamma_e + \gamma_d$ constant. We explore the role of this ratio on the transmission in Fig. 3(a) and the conversion efficiency in Fig.3 (b). When the ratio is small, the transmission is almost unity because the coupling with the cavity is small. Therefore, the frequency conversion efficiency is close to zero. At critical coupling, when both rates are equal, the transmission is minimal. Most of the incoming power is absorbed

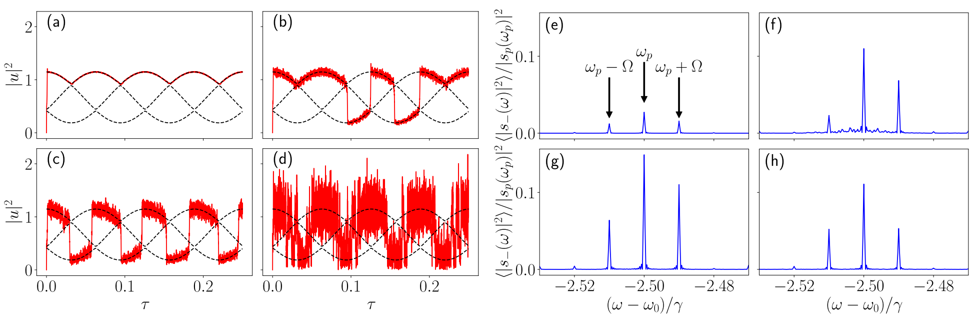

Fig. 2: (a)-(d) Evolution of cavity mode energy with time for various temperatures. (e)-(h) Spectral density of outgoing power corresponding to situation (a)-(d) normalised by incoming power at pump frequency.

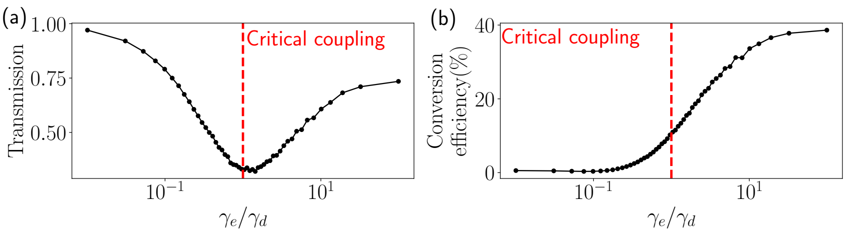

Fig. 3: (a) Transmission versus ratio of dissipation rates. (b) Conversion efficiency versus dissipation rates ratio.

by the cavity. For small internal dissipation or high coupling, the transmission rises again because less energy is lost in the cavity internal dissipation. Then, the conversion efficiency is even higher thanks to the high coupling with the cavity (where SR occurs), reaching almost 40%.

IV. Conclusion

We explore a resonant non-linear, driven cavity coupled to a waveguide. In the bistable regime, by adding a small signal at a frequency close to the pump, the system undergoes dynamic modulation via the Kerr non-linearity. The system temperature can be controlled to induce jumps between states. We demonstrate that for a given range of temperature, these thermally induced transitions become periodic. In such a situation, when the system is at stochastic resonance, the outgoing power and the conversion efficiency are maximised. That conversion can be optimised by increasing the coupling between the cavity and the external port. With an external coupling much larger than the internal dissipation, we observe an efficiency close to 40%. Our results pave the way for enhanced, integrated frequency conversion devices, stimulated via thermal processes.

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