

Chiral approach to massive higher spins

Alexander Ochirov^{1,*} and Evgeny Skvortsov^{2,3,†}

¹*Mathematical Institute, University of Oxford, Woodstock Road, Oxford OX2 6GG, U.K.*

²*Service de Physique de l'Univers, Champs et Gravitation,
Université de Mons, 20 place du Parc, 7000 Mons, Belgium*

³*Lebedev Institute of Physics, Leninsky ave. 53, 119991 Moscow, Russia*

We propose a new, chiral description for massive higher-spin particles in four spacetime dimensions, which facilitates the introduction of consistent interactions. As proof of concept, we formulate three theories, in which higher-spin matter is coupled to electrodynamics, non-Abelian gauge theory or gravity. The theories are chiral and have simple Lagrangians, resulting in Feynman rules analogous to those of massive scalars. Starting from these Feynman rules, we derive tree-level scattering amplitudes with two higher-spin matter particles and any number of positive-helicity photons, gluons or gravitons. The amplitudes reproduce the arbitrary-multiplicity results that were obtained via on-shell recursion in a parity-conserving setting, and which chiral and non-chiral theories thus have in common. The presented theories are currently the only examples of consistent interacting field theories with massive higher-spin fields.

I. INTRODUCTION

The study of higher-spin fields is a formidable subject. In the massless case, such fields possess rich gauge symmetry, starting from the familiar case of electromagnetism, see e.g. [1, 2] for recent reviews. The standard approach is to introduce fields of spin s as symmetric traceless tensors $\Phi^{\mu_1 \dots \mu_s}$ or spinors $\Psi^{\mu_1 \dots \mu_{s-1/2}}$. Massive higher-spin fields [3–6] are more subtle and require a host of auxiliary fields that prevent propagation of unphysical degrees of freedom in interacting theories [7, 8].

Massive higher-spin particles do exist in nature as composite states of elementary particles (see e.g. [9]). One should therefore be able to describe their physics by suitable effective field theories involving an infinite hierarchy of higher-dimensional operators. Although only a finite subset of such operators contribute at any given order in the energy-scale cutoff, it is a laborious task to even list them in a manner consistent with no ghost propagation.

For these reasons, the space of massive higher-spin theories seems vast and riddled with obstacles. It seems plausible, however, that there are hidden gems among such theories, as indicated by recent on-shell studies. For instance, at the level of 3-point scattering amplitudes, various possible on-shell spinor structures for an electromagnetically interacting higher-spin particle of mass m have been classified by Arkani-Hamed, Huang and Huang (AHH), and one out of them was singled out [10] due to its relatively tame behavior in the massless limit, namely

$$\mathcal{A}(1_{\{a\}}, 2^{\{b\}}, 3^+) = \frac{\langle 1_{a_1} 2^{b_1} \rangle \dots \langle 1_{a_{2s}} 2^{b_{2s}} \rangle}{m^{2s}} \mathcal{A}(1, 2, 3^+). \quad (1)$$

Here $\mathcal{A}(1, 2, 3^+)$ is the positive-helicity photon emission amplitude in scalar quantum electrodynamics (QED),

whereas all higher-spin information is encoded into two sets of $SU(2)$ little-group indices, $\{a_1, \dots, a_{2s}\}$ and $\{b_1, \dots, b_{2s}\}$, via $2s$ copies of the same chiral product $\langle 1_a 2^b \rangle := \epsilon_{\alpha\beta} \langle 1_a |^\alpha \langle 2^b |^\beta$ of massive-momentum spinors.

Interestingly, the AHH amplitude may be extended [11, 12] to include $(n-2)$ positive-helicity photons, gluons or gravitons instead of one, while still having the same massive-spin structure as Eq. (1), see e.g. Eq. (32) below for gauge theory. These amplitudes can be derived from mere knowledge of their factorization limits by naïve use of on-shell (BCFW) recursion [13, 14] — in exactly the same way that is known to work for their spin-1/2 counterparts in quantum chromodynamics (QCD) [15]. Unlike mixed-helicity configurations, where the same approach produces answers afflicted by unphysical poles [10, 16], the like-helicity amplitudes involving a pair of higher-spin particles seem absolutely healthy. Another argument in favor of the like-helicity results is that they can be derived [12] using two distinct BCFW constructions: either complex-shifting two massless momenta [14] or one massless and one massive [17, 18].

It is well-known that on-shell recursion fails when the desired amplitudes have bad boundary behavior, i.e. they do not vanish as the complex BCFW-shift parameter z is taken to infinity. This is something that should absolutely be expected from generic effective theories, where the action is built out of higher-dimensional vertex operators, typically with a growing number of derivatives. However, the existence of healthy n -point amplitudes that are constructible from their factorization limits (albeit in a restricted helicity sector) suggests that they should belong to a well-defined massive higher-spin theory — one that is intimately related to the minimally coupled scalar theory and thereby satisfies the boundary-behavior condition allowing for on-shell recursion.

This is precisely the kind of theories that we present in this paper. We start with Sec. II, where we review the common difficulties in higher-spin field theory and show how they are avoided by a new chiral Lagrangian description of a free massive higher-spin field in four dimensions.

* ochirov@maths.ox.ac.uk

† evgeny.skvortsov@umons.ac.be

Minimal gauge interactions are then added to this theory in Sec. III. After that, we present the corresponding chiral theory of gravitationally interacting higher-spin field Sec. IV. Finally, we conclude by discussing some implications of our theories in Sec. V.

II. FREE HIGHER-SPIN THEORY

In this section we discuss the basics of describing a free higher-spin particle. For simplicity, let us temporarily concentrate on integer-spin particles.¹ In the standard approach [4], integer-spin fields constitute symmetric traceless rank- s tensors $\Phi_{\mu_1 \dots \mu_s}$. For instance, in electromagnetism, i.e. the theory of a free massless spin-1 boson, the off-shell field is A_μ , and its connection to the on-shell particle description relies on the notion of polarization vector $\varepsilon_{p,\mu}^\pm$, which converts the Lorentz index into the helicity label. Helicity governs the particle's irreducible representation of the little group $U(1)$ for a given massless momentum p_μ and is additive in the sense that the polarization tensor of e.g. massless spin-2 boson (graviton) is simply $\varepsilon_{p,\mu\nu}^\pm := \varepsilon_{p,\mu}^\pm \varepsilon_{p,\nu}^\pm$.

For $p^2 = m^2 \neq 0$, one can employ symmetric rank- $2s$ tensors as irreducible representations of the massive little group $SU(2)$ with exactly $(2s + 1)$ degrees of freedom. For $s = 1$, we therefore use polarization vector $\varepsilon_{p,\mu}^{a_1 a_2} = \varepsilon_{p,\mu}^{a_2 a_1}$, where a_i are fundamental $SU(2)$ indices, whereas free higher-spin fields are naturally expanded in terms of the polarization tensors

$$\varepsilon_{p,\mu_1 \dots \mu_s}^{a_1 \dots a_{2s}} := \varepsilon_{p,\mu_1}^{(a_1 a_2} \dots \varepsilon_{p,\mu_s}^{a_{2s-1} a_{2s})}. \quad (2)$$

Symmetry in the Lorentz indices is then obvious, and tracelessness $\eta^{\mu_i \mu_j} \varepsilon_{p,\mu_1 \dots \mu_s} = 0$ follows from the spin-1 orthonormality

$$\varepsilon_{p,ab} \cdot \varepsilon_p^{cd} = -\delta_{(a}^{(c} \delta_{b)}^{d)}, \quad \varepsilon_{p,\mu ab} := \varepsilon_{ac} \varepsilon_{bd} \varepsilon_{p,\mu}^{cd} = (\varepsilon_{p,\mu}^{ab})^*, \quad (3)$$

where we have used the fact that $SU(2)$ indices are lowered and raised with the two-dimensional Levi-Civita tensor.

The crucial element of this approach is transversality $p \cdot \varepsilon_p^{\{a\}} = 0$, which is required, roughly speaking, to adjust the number of degrees of freedom on the Lorentz-index side to that on the little-group side (3 in the spin-1 case). In other words, the transversality is required to ensure the irreducibility of the traceless symmetric tensor representation of the Lorentz group under Wigner's little group $\{L \in SO(1, 3) : L^\mu{}_\nu p^\nu = p^\mu\}$.²

¹ The extension of the discussion of the first few paragraphs of Sec. II to half-integer spins is straightforward and can be found e.g. in [16].

² More concretely, for a generic Lorentz transformation $L \in SO(1, 3)$ we have

$$L_\mu{}^\nu \varepsilon_{p,\nu}^{ab} = U^a{}_c(L) U^b{}_d(L) \varepsilon_{(Lp),\mu}^{cd}, \quad (4)$$

where $U(L) \in SU(2)$ in principle depends on the choice of the spin quantization axis for any given p .

Therefore, the free field equations that one needs to impose on (s, s) -representation tensors are

$$(\partial^2 + m^2)\Phi_{\mu_1 \dots \mu_s} = 0, \quad \partial^\mu \Phi_{\mu \mu_2 \dots \mu_s} = 0. \quad (5)$$

We have referred to the symmetric traceless tensor representation by the numbers of its chiral and antichiral $SL(2, \mathbb{C})$ indices upon the application of the spinor map

$$\Phi_{\alpha_1 \dots \alpha_s \dot{\beta}_1 \dots \dot{\beta}_s} := \Phi_{\mu_1 \dots \mu_s} \sigma_{\alpha_1 \dot{\beta}_1}^{\mu_1} \dots \sigma_{\alpha_s \dot{\beta}_s}^{\mu_s}, \quad (6)$$

where $\sigma^\mu = (1, \sigma^1, \sigma^2, \sigma^3)$ are the Pauli matrices. In fact, it is easy to see in this spinor language that, although (s, s) is irreducible under $SL(2, \mathbb{C}) \cong SO(1, 3)$, it is highly reducible under $SU(2) \subset SL(2, \mathbb{C})$ and decomposes into symmetric $SU(2)$ tensors of rank $0, 2, \dots, 2s$.

The Klein-Fock-Gordon equation in Eq. (5) can be obtained from a simple Lagrangian. However, there is no action principle for $s > 1$ that also generates the second equation in Eq. (5), which is required to ensure irreducibility under Wigner's little group, unless a host of auxiliary fields is introduced [3]. For instance, the Singh-Hagen approach [4, 5] relies on introducing $(s - 1)$ symmetric traceless tensor fields of rank $0, 1, \dots, s - 2$. Alternatively, Zinoviev's approach [6] involves s such fields of rank $0, 1, \dots, s - 1$, that are also subject to the double-trace condition $\Phi^{\lambda\mu}{}_{\lambda\mu\mu_5 \dots \mu_{s-k}} = 0$. All these fields vanish on shell but serve to ensure the free-field expansion of $\Phi_{\mu_1 \dots \mu_s}$ in terms of the physical polarization tensors (2).

Here, our radically simple proposal, as inspired by the higher-spin amplitudes (1) and by chiral higher-spin gravity [19–27], is to *take basic fields* $\Phi_{\alpha_1 \dots \alpha_{2s}}$ *in the chiral* $(2s, 0)$ *representation of the Lorentz group* instead of (s, s) . As we will shortly see, this essentially trivializes the transition between the off-shell symmetry group $SL(2, \mathbb{C})$ and the on-shell little-group $SU(2)$.

To be more specific, we employ the massive spinor-helicity formalism [10, 28, 29], which provides perfect building blocks for this construction. Namely, we use chiral and antichiral on-shell spinors $|p\rangle$ and $[p]$, such that³

$$m = 0 \quad \Rightarrow \quad |p\rangle_\alpha [p]_{\dot{\beta}} := p_\mu \sigma_{\alpha\dot{\beta}}^\mu, \quad (7a)$$

$$m \neq 0 \quad \Rightarrow \quad |p^a\rangle_\alpha [p_a]_{\dot{\beta}} := p_\mu \sigma_{\alpha\dot{\beta}}^\mu. \quad (7b)$$

All external wavefunctions of quantum fields can be built out of these spinors. For instance, the massive polarization vector can be constructed as [30, 31]

$$\varepsilon_{p,\mu}^{ab} = \frac{i \langle p^a | \sigma_\mu | p^b \rangle}{\sqrt{2}m} \quad (8)$$

³ The Weyl spinor indices are raised and lowered with two-dimensional Levi-Civita tensors $\epsilon^{\alpha\beta} = \epsilon^{\dot{\alpha}\dot{\beta}} = \epsilon^{ab} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = -\epsilon_{\alpha\beta} = -\epsilon_{\dot{\alpha}\dot{\beta}} = -\epsilon_{ab}$, just as $SU(2)$ indices. Spinor products like [12] := $\epsilon^{\dot{\alpha}\dot{\beta}} [1]_{\dot{\alpha}} [2]_{\dot{\beta}}$ or $\langle 1^a 2^b \rangle$, which appeared in Eq. (1), are hence antisymmetric. The spinors obey an array of properties [10, 15], e.g. the Weyl/Dirac equation $p_\mu \sigma_{\alpha\dot{\beta}}^\mu |p\rangle^{\dot{\beta}} = m |p\rangle_\alpha$.

and all of the desired properties, such as Eq. (3) and transversality, follow automatically.⁴

For a higher-spin field in representation $(2s, 0)$, the free field equations reduce to the Klein-Fock-Gordon equation

$$(\partial^2 + m^2)\Phi_{\alpha_1 \dots \alpha_{2s}} = 0, \quad (11)$$

required to define the mass shell. Indeed, the number of degrees of freedom no longer needs to be artificially reduced, so the only thing that we need from the corresponding external wavefunctions is converting the off-

shell Weyl-spinor indices into the on-shell little-group indices. This is precisely what the massive spinors (7b) are good for, so the external higher-spin wavefunctions are

$$\textcircled{\leftarrow} p, a_1, \dots, a_{2s} = \frac{1}{m^s} |p^{(a_1)}_{\alpha_1} \dots p^{(a_{2s})}_{\alpha_{2s}}. \quad (12)$$

Here the mass prefactor is needed to absorb the mass dimension, which is $1/2$ for momentum spinors. The free-field expansion is therefore

$$\Phi_{\alpha_1 \dots \alpha_{2s}}(x) = \int \frac{d^3 p}{2p^0} \left[\frac{|p^{(a_1)}_{\alpha_1} \dots p^{(a_{2s})}_{\alpha_{2s}}\rangle_{\alpha_{2s}} a_{a_1 \dots a_{2s}}(\vec{p}) e^{-ip \cdot x} + (-1)^{2s} \frac{|p_{(a_1)}_{\alpha_1} \dots p_{(a_{2s})}_{\alpha_{2s}}\rangle_{\alpha_{2s}} a^{\dagger a_1 \dots a_{2s}}(\vec{p}) e^{ip \cdot x}}{m^s} \right] \Big|_{p^0 = \sqrt{\vec{p}^2 + m^2}}, \quad (13)$$

where $\hat{d}^3 p = (2\pi)^{-3} d^3 p$. We have fixed the signs above so that the outgoing external wavefunctions

$$\textcircled{\rightarrow} p, a_1, \dots, a_{2s} = \frac{(-1)^{2s}}{m^s} |p_{(a_1)}_{\alpha_1} \dots p_{(a_{2s})}_{\alpha_{2s}} \rangle_{\alpha_{2s}} \quad (14)$$

are consistent by crossing with their incoming counterparts (12), given the momentum-reversal convention

$$|-p\rangle = -|p\rangle, \quad |-p] = |p]. \quad (15)$$

In particular, the expansion for the spin-1/2 field $\Phi_\alpha(x)$ coincides with the chiral part of the Majorana field $\Psi_M(x)$ as written e.g. in [16]. The standard properties

$$\langle p^a p^b \rangle = -m\epsilon^{ab}, \quad |p^a\rangle_\alpha \langle p_a|^\beta = -m\delta_\alpha^\beta \quad (16)$$

of the massive spinors are then equivalent to the orthonormality and completeness relations for spin $1/2$.⁵

The Lagrangian implying the free field equation (11) is

$$\mathcal{L}_0 = \frac{1}{2} (\partial_\mu \Phi^{\alpha_1 \dots \alpha_{2s}}) (\partial^\mu \Phi_{\alpha_1 \dots \alpha_{2s}}) - \frac{m^2}{2} \Phi^{\alpha_1 \dots \alpha_{2s}} \Phi_{\alpha_1 \dots \alpha_{2s}}. \quad (18)$$

where we treat Φ as a real/hermitian field. The reality has a literal meaning in Euclidean or split signature and should be understood in the sense of Eq. (13) in Minkowski spacetime. The corresponding propagator is

$$\Phi_{\{\alpha_1 \dots \alpha_{2s}\}} \frac{\text{---}}{p} \Phi^{\{\beta_1 \dots \beta_{2s}\}} = \frac{i\delta_{\alpha_1}^{(\beta_1} \dots \delta_{\alpha_{2s}}^{\beta_{2s})}}{p^2 - m^2}. \quad (19)$$

Note that, when all indices are raised, the propagator is automatically antisymmetric for half-integer spins and symmetric for integer spins. Indeed, the spin-statistics theorem is also automatically implemented at the level of (classical) fields:

$$\begin{aligned} \Phi^{\{\alpha\}}(x) \Phi_{\{\alpha\}}(y) &= (-1)^{2s} \Phi_{\{\alpha\}}(x) \Phi^{\{\alpha\}}(y) \\ &= \Phi^{\{\alpha\}}(y) \Phi_{\{\alpha\}}(x), \end{aligned} \quad (20)$$

so they can be manipulated much like scalars.

III. GAUGE THEORY

In this section we include minimal gauge interactions in the chiral higher-spin theory (18). Since complex conjugation switches between the chiral and antichiral representations of the Lorentz group, we choose $O(N)$ as the generic gauge group, which encompasses other interesting cases, such as $SU(N) \subset O(2N)$.

We take the Lagrangian to be simply

$$\mathcal{L}_g = \frac{1}{2} (D_\mu \Phi^{\{\alpha\}})_i (D^\mu \Phi_{\{\alpha\}})_i - \frac{m^2}{2} \Phi_i^{\{\alpha\}} \Phi_i^{\{\alpha\}}, \quad (21)$$

where the covariant derivative

$$D_\mu := \partial_\mu + gA_\mu, \quad A_\mu = A_\mu^A t^A, \quad [t^A, t^B] = f^{ABC} t^C. \quad (22)$$

⁴ In particular, the relationship (4) between Lorentz and little-group transformations for $\epsilon_{p,\mu}^{ab}$ is now a simple consequence of the analogous relations for spinors:

$$S_\alpha^\beta |p^a\rangle_\beta = U^a_b(S) |(Lp)^b\rangle_\alpha, \quad (9a)$$

$$|p^a]_{\dot{\beta}} (S^\dagger)^{\dot{\beta}}_{\dot{\alpha}} = U^a_b(S) |(Lp)^b]_{\dot{\alpha}}. \quad (9b)$$

Here the little-group transformation U depends on S instead of L , but they are of course subject to the two-to-one correspondence $SL(2, \mathbb{C}) \cong SO(1, 3)$:

$$L^\mu_\nu = \frac{1}{2} \text{tr}(\bar{\sigma}^\mu S \sigma_\nu S^\dagger), \quad (10)$$

where $\bar{\sigma}^\mu = (1, -\sigma^1, -\sigma^2, -\sigma^3)$.

⁵ For spin s , the orthonormality and completeness relations are explicitly

$$\frac{(-1)^{2s}}{m^{2s}} \langle p_{(a_1} p^{(b_1)} \dots p_{(a_{2s})} p^{(b_{2s})} \rangle = \delta_{\alpha_1}^{(b_1} \dots \delta_{\alpha_{2s}}^{b_{2s})}, \quad (17a)$$

$$\begin{aligned} \frac{(-1)^{2s}}{m^{2s}} |p^{(a_1)}_{\alpha_1} \dots p^{(a_{2s})}_{\alpha_{2s}}\rangle_{\alpha_{2s}} \langle p_{(a_1)}^{\beta_1} \dots p_{(a_{2s})}^{\beta_{2s}}| &= \delta_{\alpha_1}^{(\beta_1} \dots \delta_{\alpha_{2s}}^{\beta_{2s})} \\ &= \delta_{\alpha_1}^{(\beta_1} \dots \delta_{\alpha_{2s}}^{\beta_{2s})}. \end{aligned} \quad (17b)$$

Note that the propagator numerator in Eq. (19) is consistent with the completeness relation above.

SDYM coupled to our massive higher-spin matter (21). In this chiral theory, all purely gluonic amplitudes automatically vanish at tree level except for Eq. (31a). Amplitudes with matter do not vanish but are significantly simpler than those in the full theory due to the integrability of the self-dual gauge interaction. The non-trivial overlap of the full and the self-dual theories consists of the amplitudes involving a single pair of massive higher-spin particles, to which we have referred in the introduction. Their color-ordered⁹ versions are explicitly [12]

$$A(1^{\{a\}}, 2^+, 3^+, \dots, (n-1)^+, n^{\{b\}}) = \frac{i(1^a n^b)^{\circ 2s}}{m^{2s-2}} \times \frac{[2] \prod_{j=2}^{n-3} \{P_{12\dots j} \not{p}_{j+1} + (s_{12\dots j} - m^2)\} |n-1|}{\prod_{j=2}^{n-2} \langle j|j+1 \rangle (s_{12\dots j} - m^2)}. \quad (32)$$

Here we have written $P_{12\dots j} = p_1 + \dots + p_j$ for momentum sums and $s_{12\dots j}$ for their Lorentz squares. The factors involving slashed matrices in the numerator are $\{(P_{12\dots j}^\mu \bar{\sigma}_\mu^{\alpha\gamma})(p_{j+1}^\nu \sigma_{\nu,\gamma\beta}) + (s_{12\dots j} - m^2) \delta_\beta^\alpha\}$, and their order of multiplication is such that j increases from left to right. The amplitudes (32) are consistent with those for massive scalars first derived in [42], as well as with those for massive quarks [15, 43] and gauged spin-1 matter obtained from Yang-Mills theory via the Higgs mechanism [44]. We have also performed additional numerical checks through 8 points that the Feynman rules (23a) and (23b), in combination with the standard 3- and 4-gluon vertices, give the same answers as the formula (32).

In the important special case of $\text{SO}(2)$, for which $f^{ABC} = 0$ and $t_{ij}^{A=1} = \epsilon^{ij}$, we combine the fields into

$$\Phi^{\{\alpha\}} := \frac{1}{\sqrt{2}} (\Phi_{j=1}^{\{\alpha\}} + i\Phi_{j=2}^{\{\alpha\}}), \quad \widetilde{\Phi}^{\{\alpha\}} := \frac{1}{\sqrt{2}} (\Phi_{j=1}^{\{\alpha\}} - i\Phi_{j=2}^{\{\alpha\}}), \quad (33)$$

where the charge conjugation avoids the chirality switch. The resulting chiral higher-spin QED Lagrangian is

$$\mathcal{L}_Q = (D_\mu \widetilde{\Phi}^{\{\alpha\}})(D^\mu \Phi_{\{\alpha\}}) - m^2 \widetilde{\Phi}^{\{\alpha\}} \Phi_{\{\alpha\}}, \quad (34)$$

where the covariant derivative $D_\mu := \partial_\mu - iQA_\mu$ involves the coupling constant renamed to Q . A straightforward application of the resulting Feynman rules

$$\begin{array}{c} \begin{array}{c} A_3^\mu \\ \diagup \quad \diagdown \\ \widetilde{\Phi}_{1\{\alpha_1\dots\alpha_{2s}\}} \quad \Phi_2^{\{\beta_1\dots\beta_{2s}\}} \end{array} \\ \qquad \qquad \qquad = iQ \delta_{\alpha_1}^{(\beta_1} \dots \delta_{\alpha_{2s})}^{\beta_{2s})} (p_2 - p_1)^\mu, \end{array} \quad (35)$$

$$\begin{array}{c} \begin{array}{c} A_3^\mu \quad A_4^\nu \\ \diagup \quad \diagdown \\ \widetilde{\Phi}_{1\{\alpha_1\dots\alpha_{2s}\}} \quad \Phi_2^{\{\beta_1\dots\beta_{2s}\}} \end{array} \\ \qquad \qquad \qquad = 2iQ^2 \delta_{\alpha_1}^{(\beta_1} \dots \delta_{\alpha_{2s})}^{\beta_{2s})} \eta^{\mu\nu}, \end{array}$$

⁹ Color ordering used in Eq. (32) assumes the standard convention [41] favoring hermitian generators $T^A := i\sqrt{2}t^A = (T^A)^\dagger$, which additionally absorb factors of $\sqrt{2}$. As usual with color ordering, we have relied on the possibility to put all particles in the adjoint representation of an $\mathfrak{su}(N)$ (sub)algebra.

leads to the explicit amplitudes (see e.g. [45])

$$\begin{aligned} A(1^{\{a\}}, 2^+, 3^+, \dots, (n-1)^+, n^{\{b\}}) &= -i(2Q)^{n-2} \quad (36) \\ &\times \frac{\langle 1^a n^b \rangle^{\circ 2s}}{m^{2s}} \sum_{\sigma \in S_{n-2}(\{2,3,\dots,n-1\})} \frac{\prod_{j=2}^{n-1} (P_{1\sigma(2)\dots\sigma(j)} \cdot \varepsilon_{\sigma(j)}^+)}{\prod_{j=2}^{n-2} (s_{1\sigma(2)\dots\sigma(j)} - m^2)}. \end{aligned}$$

We have checked numerically through 8 points that they are consistent with the color-dressed analogue of Eq. (32) under the usual QCD-to-QED projection

$$\begin{aligned} A(1^{\{a\}}, 2, 3, \dots, n-1, n^{\{b\}}) \\ = (\sqrt{2}Q)^{n-2} \sum_{\sigma \in S_{n-2}(\{2,3,\dots,n-1\})} A(1^{\{a\}}, \sigma, n^{\{b\}}). \end{aligned} \quad (37)$$

Note that all gluonic factorization channels non-trivially cancel here, leaving only the massive ones as in Eq. (36).

IV. GRAVITY

In this section we minimally couple our chiral higher-spin theory to gravity. The Lagrangian is

$$\mathcal{L}_G = \sqrt{-g} \left\{ \frac{1}{2} (\nabla_\mu \Phi^{\{\alpha\}}) (\nabla^\mu \Phi_{\{\alpha\}}) - \frac{m^2}{2} \Phi^{\{\alpha\}} \Phi_{\{\alpha\}} \right\}, \quad (38)$$

where we have included the metric dependency of the integration measure. The covariant derivatives may in general act on the Lorentz indices via the spin connection:

$$\nabla_\mu \Phi_{\alpha_1 \dots \alpha_{2s}} = \partial_\mu \Phi_{\alpha_1 \dots \alpha_{2s}} + 2s \omega_{\mu,(\alpha_1}{}^\beta \Phi_{\alpha_2 \dots \alpha_{2s})\beta}. \quad (39)$$

This prevents the vertices from factorizing onto those of the massive-scalar theory. So unlike in the gauge-theoretic case, we cannot make helicity-independent statements of the type (24).

However, much like in gauge theory, there is a clear connection between positive-helicity gravitons and the self-duality condition of the Riemann tensor. Moreover, it is equally true for the spin connection: the chiral part of the spin connection is known to vanish in self-dual gravity (SDGR) [37, 46–48]:

$$\omega_{\mu,\alpha}{}^\beta := \frac{1}{2} \omega_\mu{}^{\hat{\nu}\hat{\rho}} \hat{\sigma}_{\hat{\nu},\alpha\hat{\gamma}} \hat{\sigma}_{\hat{\rho}}^{\hat{\gamma}\beta} \stackrel{\text{SDGR}}{\Rightarrow} 0, \quad (40)$$

where the frame indices are displayed with hats. So restricting to the self-dual gravitational sector, we can easily write the 3-point interaction vertex

$$\begin{array}{c} \begin{array}{c} \mathfrak{h}_{3+}^{\mu\nu} \\ \diagup \quad \diagdown \\ \Phi_{1\{\alpha_1\dots\alpha_{2s}\}} \quad \Phi_2^{\{\beta_1\dots\beta_{2s}\}} \end{array} \\ \qquad \qquad \qquad = i\kappa \delta_{\alpha_1}^{(\beta_1} \dots \delta_{\alpha_{2s})}^{\beta_{2s})} \times \left[p_2^{(\mu} p_2^{\nu)} + \frac{m^2}{2} \eta^{\mu\nu} \right], \end{array} \quad (41)$$

e.g. in terms of the perturbation of the “gothic inverse metric” $\sqrt{-g} g^{\mu\nu} = \eta^{\mu\nu} - \kappa \mathfrak{h}^{\mu\nu}$ [49, 50] and the coupling constant $\kappa = \sqrt{32\pi G_{(\text{Newton})}}$. In this formulation

of self-dual gravitational perturbation theory, all vertices involving two massive fields and multiple gravitons come exclusively from the mass term due to

$$\sqrt{-g} = 1 - \frac{\kappa}{2} \mathfrak{h} + \frac{\kappa^2}{8} (\mathfrak{h}^2 - 2\mathfrak{h}^{\mu\nu} \mathfrak{h}_{\mu\nu}) + O(\kappa^3), \quad (42)$$

where $\mathfrak{h} := \mathfrak{h}^{\mu\nu} \eta_{\mu\nu}$. In any case, in view of the vanishing anti-self-dual spin connection (40), all of the Feynman rules clearly factorize onto those for a massive scalar.

Therefore, for positive-helicity gravitational amplitudes involving two higher-spin matter particles, we do have the factorization property

$$\mathcal{M}(1_{\{a\}}, 2^{\{b\}}, 3^+, \dots, n^+) = \frac{\langle 1_a 2^b \rangle^{\odot 2s}}{m^{2s}} \mathcal{M}(1, 2, 3^+, \dots, n^+). \quad (43)$$

Equivalently, these amplitudes are constructible [11] from their factorization limits via on-shell recursion starting from the two 3-point amplitudes

$$\mathcal{M}(1^{\{a\}}, 2^{\{b\}}, 3^+) = -i\kappa \frac{\langle 1_a 2^b \rangle^{\odot 2s}}{m^{2s}} (p_1 \cdot \varepsilon_3^+)^2 \quad (44)$$

and

$$\mathcal{M}(1^+, 2^+, 3^-) = -\frac{i\kappa}{2} \frac{[12]^6}{[23]^2 [31]^2}, \quad (45a)$$

but with no reference to the second 3-graviton amplitude

$$\mathcal{M}(1^-, 2^-, 3^+) = -\frac{i\kappa}{2} \frac{\langle 12 \rangle^6}{\langle 23 \rangle^2 \langle 31 \rangle^2} \xrightarrow{\text{SDGR}} 0. \quad (45b)$$

However, perhaps the easiest way to express the amplitudes (43) is by using the Kawai-Lewellen-Tye-style double copy [51, 52], which is known [53–55] to hold for minimally coupled massive scalars. The gravitational all-plus amplitudes are thus given as a bilinear combination of their gauge-theoretic color-ordered counterparts (32):

$$\begin{aligned} \mathcal{M}(1^{\{a\}}, 2^{\{b\}}, 3^+, \dots, n^+) &= -i \left(\frac{\kappa}{2}\right)^{n-2} \frac{\langle 1_a 2^b \rangle^{\odot 2s}}{m^{2s}} \\ &\times \sum_{\sigma, \tau} A(1, 2, 3^+, \sigma) A(2, 1, 3^+, \tau) \prod_{i=1}^{n-3} \sum_{j \in X_{\sigma, \tau}^i} s_{\sigma(i)j}. \end{aligned} \quad (46)$$

Here the outer summation is over $\sigma, \tau \in S_{n-3}(\{4, \dots, n\})$, t_i is defined as the position of $\sigma(i)$ within permutation τ , i.e. $\tau(t_i) = \sigma(i)$, and $X_{\sigma, \tau}^i := \{3, \sigma(1), \dots, \sigma(i-1)\} \cap \{3, \tau(1), \dots, \tau(t_i-1)\}$. This expression is also consistent with the double copy for massive matter with spin [16, 56–59].

V. SUMMARY AND DISCUSSION

We have presented a chiral approach to massive higher-spin fields and formulated three theories, in which such

fields interact strongly, electromagnetically or gravitationally. We have focused on the self-dual sectors of these interactions, where scattering amplitudes coincide with those derived previously assuming parity conservation. On the one hand, all three theories may be consistently truncated down to their respective self-dual sectors. On the other hand, they may be extended by introducing additional vertex operators to their Lagrangians.

The problem of healthy interactions for higher-spin fields is important for various reasons. Higher-spin states might be indispensable for building consistent quantum gravity models, as indicated by string theory, the AdS/CFT correspondence and higher-spin gravities. Moreover, massive higher-spin particles can model many real physical systems within the effective field theory approach. A suitable implementation of the classical limit [11, 30, 31, 60–65] even allows to model gravitational dynamics of spinning black holes or other compact objects.

For marginal spins, the problem of consistent interactions has a solution for massive spin-1 fields, which always result from a spontaneously broken Yang-Mills theory, and consistent massive spin-2 theories are known as massive gravity [66–69]. For higher spins, this problem can be roughly subdivided into two: self-interactions of massive higher-spin fields and their gauge or gravitational interactions. The latter is more important for such applications as black-hole scattering. No solution to either problem has existed beyond spin-2, and the present paper aims to provide a new way forward.

Let us compare our chiral-field approach to other descriptions of massive higher-spin fields. In the Singh-Hagen formulation [4, 5], the auxiliary fields are fine-tuned to give enough differential consequences of the field equations to guarantee the correct number of physical degrees of freedom (p.d.o.f.), which is almost equivalent to a tedious analysis of Hamiltonian constraints. Zinoviev's approach [6] has a more transparent pattern of auxiliary fields intertwined by gauge symmetries à la Stückelberg, and the remaining challenge is of a purely technical nature [70–75]. If one is concerned only with field equations, more economic approaches to control the number of p.d.o.f. can be applied, see e.g. [76–79]. Furthermore, the light-front approach starts out with the correct p.d.o.f., see e.g. [80, 81] and especially [82] for the four-dimensional case, but the study beyond the cubic order can still be quite laborious. For twistorial constructions involving mass see e.g. [83] and the references therein.

Our present proposal for massive higher-spin particles is inspired by the recent attempts to bootstrap their on-shell amplitudes with as little off-shell input as possible [10, 16, 84, 85], which works surprisingly well for like-helicity configurations of the force-carrier bosons [11, 12]. The chiral description of massive higher-spin interactions is very close in spirit to the ideas originating from twistor theory [86–90] and from the covariantization of chiral higher-spin gravity [23, 26, 27, 91], where a chiral field $\Phi_{\alpha_1 \dots \alpha_{2s}}$ is used to describe massless spin- s states. It is also close to the light-cone gauge in not having redundant

degrees of freedom, the advantage being in maintaining manifest Lorentz symmetry. For spin-1/2 fields, the chiral description can be understood as integrating out half of the fermion components out of QCD [92], and likewise for spin-1 [93] after first integrating in an auxiliary field. However, no such interpretation is available for higher spin fields, while a parent action [94] may still exist.

Having presented the three chiral theories, we can already comment on their extensions. Similarly to the self-dual theories, our theories, while being consistent on their own, generate a subset of the amplitudes of their would-be non-chiral completions. In the case of SDYM (with matter) and SDGR the completions to full Yang-Mills theory (with matter) and gravity are known [35, 93, 95], and it would be interesting to find such completions for our proposals. Aiming at restoring parity can also be motivated by the applications to spinning black holes. Indeed, the gravitational AHH amplitude (44) was shown [30, 31] to contain the same spin-induced multipole moments as a spinning black hole [96–98], but only in combination with its antichiral version

$$\mathcal{M}_{\text{AHH}}(1^{\{a\}}, 2^{\{b\}}, 3^-) = -i\kappa \frac{[1^a 2^b]^{\odot 2s}}{m^{2s}} (p_1 \cdot \varepsilon_3^-)^2. \quad (47)$$

On the classical-gravity side, the tower of multipole moments can be extracted directly from the linearized Kerr solution [31, 99]. For a spin- s particle described by Eqs. (44) and (47), it appears truncated down to the $2s$ -pole with correct black-hole multipole coefficients — even before the classical limit is taken [11, 61–65]. (Interestingly, this “spin universality” property [100–102] is not obeyed by all three-point amplitudes [65], as recently

demonstrated [103] for the leading Regge states of the open and closed superstring.)

For simplicity, let us consider the gauge-theoretic case, where the classical analogue of the spinning black hole is known as the $\sqrt{\text{Kerr}}$ solution [63]. We can easily write the general form of the interaction terms that could modify the 3-point amplitude (29b) without spoiling (29a):

$$(D_{\alpha_1 \dot{\gamma}_1} \cdots D_{\alpha_k \dot{\gamma}_k} \Phi^{\alpha_1 \cdots \alpha_{2s}}) \epsilon_{\alpha_{k+1} \beta_{k+1}} \cdots \epsilon_{\alpha_{2s-1} \beta_{2s-1}} F_{\alpha_{2s} \beta_{2s}} \times (D_{\beta_1 \dot{\gamma}_1} \cdots D_{\beta_k \dot{\gamma}_k} \Phi^{\beta_1 \cdots \beta_{2s}}). \quad (48)$$

A single such term $\Phi^\alpha F_{\alpha\beta} \Phi^\beta$ is in fact known to be sufficient for restoring parity to the entire spin-1/2 theory, which then constitutes a chiral formulation of QCD with massive quarks due to Chalmers and Siegel [92]. As for higher spins, we will explore their gauge and gravitational interactions in more detail elsewhere.

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