Couplings between massless spin-2 fields, partially-massless spin-2 fields and vector gauge fields in de-Sitter spacetime

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Plan of the talk

1. Introduction

2. UIRs of $so(1,d+1)$: Dictionary for physicists

[Th. Basile, X. Bekaert, N.B. 2017]
1612.08166

3. Couplings between massless spin-2 fields, PM spin-2 and vector gauge fields

↳ Based on the PhD Thesis of Lucas Traina . UMONS .

4. A theory for multiple PM spin-2 fields

[N.B., C. Deffayet, S. Garcia-Saenz, L. Traina, 1906.03868]

1 Introduction

- De Sitter and anti-de Sitter spacetimes allow for partially massless (PM) gauge fields, with no counterpart in Minkowski spacetime.

  ↳ Possess a mass m_{PM} (⇝ eigenvalue of (A)dS covariant d'Alembertien

$$(\Box - m_{PM}^2)\,\varphi = 0$$

  intermediate between those of massive and massless fields

$$(\Box - m^2_{PM})\varphi = 0$$

↳ Non-zero mass m_{PM} proportional to λ, the (square root of) the cosmological constant $\boxed{\Lambda = -\sigma \frac{d(d-1)}{2} \lambda^2}$

where $AdS_{d+1} \rightsquigarrow \sigma = +1$

$dS_{d+1} \rightsquigarrow \sigma = -1$

and Einstein-Hilbert $\boxed{S^{EH} = \frac{1}{\kappa^2} \int_{M_D} \sqrt{-g}\,(R - 2\Lambda)\, d^D x}$

. Upper bound on graviton mass from BH mergers : $\boxed{\Delta m^2 \leqslant 10^{-110}}$ in Planck units

[Abbott et al. (2016)] where $\boxed{\Delta m^2 = m^2_{PM} - m^2_0}$, m^2_0 being the critical value for the massless graviton in dS.

↳ Theoretically $\Delta m^2 = 2\lambda^2$ (see below)

. Cosmological constant ~ 10^{-122} in Planck units (Wikipedia). There is still room for a PM spin-2.

AdS_{d+1} $\quad so(2,d)$	dS_{d+1} $\quad so(1,d+1)$
• $\mathcal{D}(e_0, \mathbb{Y}) \rightsquigarrow so(2) \oplus so(d)$	• $\mathcal{D}(\Delta_c, \mathbb{Y}) \rightsquigarrow so(1,1) \oplus so(d)$
• $C_2 = e_0(e_0 - d) + C_2[so(d)]$	• $C_2 = \Delta_c(\Delta_c - d) + C_2[so(d)]$

$$\boxed{\left(\Box - \lambda^2 m^2_{\mathbb{Y}}\right)\varphi_{\mathbb{Y}} = 0}$$

+ extra conditions
tracelessness - divergenceless

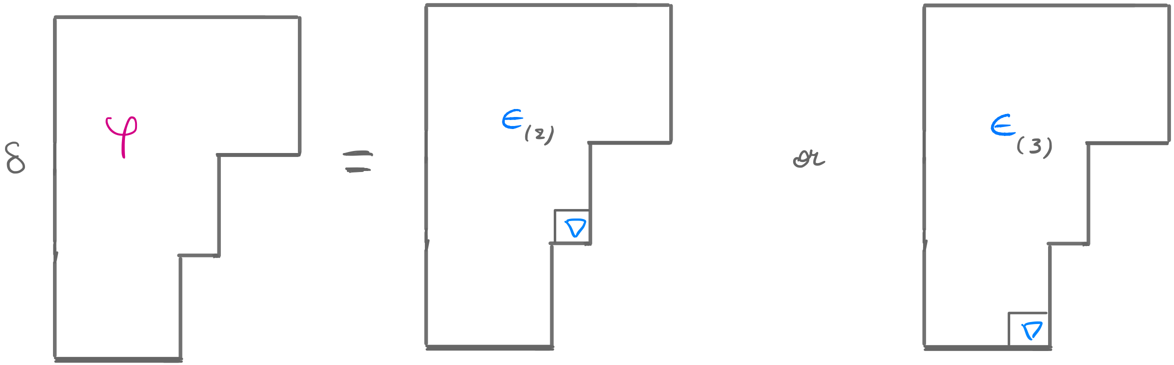
• $m^2_{\mathbb{Y}} = e_0(e_0 - d) - \sum_{k=1}^{r} s_k$	• $m^2_{\mathbb{Y}} = -\Delta_c(\Delta_c - d) + \sum_{k=1}^{r} s_k$
• (Partially) massless for $e_0 = e_t^I := s_I - p_I + d - t$, $\quad 1 \leq t \leq s_I - s_{I+1}$	• Various cases of "masslessness": Exceptional & Discrete UIR series

Exceptional: $\Delta_c = s_B - p + d - t$

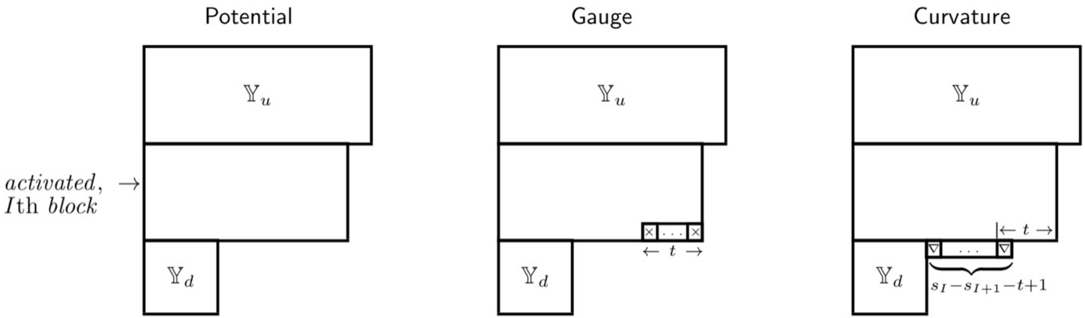
Discrete ($d+1$ even): • $\Delta_c = s_r - r + d - t$

$$\boxed{\Delta_c = s + 2 - t \text{ in } dS_4}$$

or



For a partially massless field in (A)dS [N.B., C. Iazeolla, P. Sundell 2008]



- Unitary in AdS_{d+1}: 1st block activated

② UIRs of $so(1,d+1)$: Dictionary for physicists

- **Principal series** : $\Delta_c = \frac{d}{2} + i\rho$, Y & $\underset{\omega}{\rho} \overset{\mathbb{R}}{}$ arbitrary

[scalar : $\nabla^2 \varphi_0 = (-\lambda^2) \Delta_c(\Delta_c - d)\varphi_0$ where $-\Delta_c(\Delta_c - d) = \rho^2 + \frac{d^2}{4}$ $\Rightarrow$ $\nabla^2 \geqslant 0$ in dS_{d+1}]

- **Complementary series** : $p < \Delta_c < d-p$, $p \in \{0, 1, \dots, [\frac{d-1}{2}]\}$

$s_i = 0$ for $p+1 \leqslant i \leqslant r$

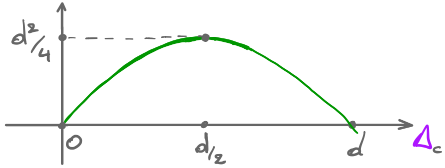
Rem : s_r may be $\neq 0$ for $d = 2r+1$, but then $s_r \in \mathbb{N}$

Rem : A scalar field can sit in these two UIRs

A scalar in the complementary series : $p=0$ and $\Delta_c \in \,]0, d[$

Example : scalar field

$R^2 m^2 = -\Delta_c(\Delta_c - d)$

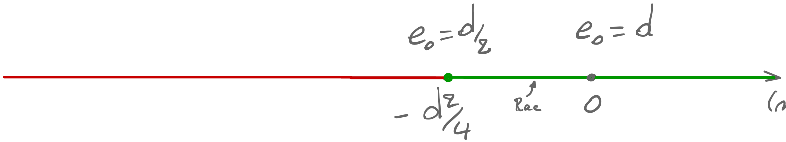


$\boxed{dS_{d+1}}$

$(mR)^2 = -\Delta_c(\Delta_c - d)$

compl. principal

$\boxed{AdS_{d+1}}$



$(mR)^2 = e_0(e_0 - d)$

$e_0 \geq s - p + d - 1$ for $s > 0$

$e_0 \geqslant \frac{d-2}{2}$ for $s = 0$

$e_0 \geqslant \frac{d-1}{2}$ for $s = 1/2$

- $m^2_{Rac} = -\frac{1}{4}(d^2 - 4)$

• Special case : $dS_4 \longleftrightarrow so(1,4) \longleftrightarrow r = [\frac{3}{2}] = 1$

$\hookrightarrow$ (Partially) Massless fields with $\varphi \sim \boxed{s = s_r}$ are in the discrete series

$$\Delta_c = s_r - r + d - t = \boxed{s + 2 - t = \Delta_c} \longleftrightarrow (\Box - \lambda^2 m^2)\varphi = 0$$

where $1 \leq t \leq s$ $\qquad$ $\boxed{m^2 = -\Delta_c(\Delta_c - d) + s}$

e.g. $\underline{s=1}$: $\Delta_c = 3 - t \to t = 1 \rightsquigarrow$ massless only , $m^2 = -2(2-3)+1 = 3$

$\underline{s=2}$: $\Delta_c = 4 - t \to t = 1$: massless (graviton) $\quad m^2 = -3(3-3)+2 = 2$

$\to t = 2$: PM spin-2 $\quad m^2 = -2(2-3)+2 = 4$

$\Longrightarrow$ Hence $\Delta m^2 = 4 - 2 = 2$ in unit of λ^2.

$\to$ Search for couplings among "massless" spin 2 , PM spin 2 and massless spin-1 vectors .

3. Couplings between massless spin-2 fields, PM spin-2 and vector gauge fields

3.1) NON-ABELIAN DEFORMATIONS

Interactions among the set $\{ h^i_{\mu\nu} , k^\Delta_{\mu\nu} , A^a_\mu \}$, g_{ij} , $g_{\Delta\Omega}$, g_{ab}

↳ Deformation of the gauge algebra asking for the existence of vertices with no more than 2 ∂'s

What is left is Diffeo., Yang-Mills, geometric-coupling algebra

$$a_2^{EH} = \xi^{*\mu}_i \, \xi^{j\nu} \, \nabla_{[\mu} \xi_{\nu]}^k \, g^i{}_{jk} \, , \qquad a_2^{Geom} = 2 a_2^{(7)} + \frac{\sigma}{L^2} (D-3) \, a_2^{(5)} \, , \qquad a_2^{YM} = \frac{1}{2} C^*_a \, C^b C^c$$

$$a_2^{(5)} = \xi^{*\mu}_i \, \chi^\Delta \, \nabla_\mu \chi^\Omega \, f_{(5)}{}^i{}_{\Delta\Omega} \, , \qquad a_2^{(7)} = \chi^*_\Delta \, \nabla^\mu \chi^\Omega \, \xi^i_\mu \, f_{(7)}{}^\Delta{}_{\Omega i}$$

$$a_2 = \kappa \, a_2^{EH} + \alpha_{PM} \, a_2^{Geom} + g_{YM} \, a_2^{YM}$$

↳ leads to a_0^{EH} , $a_0^{Geom} = h^i{}_{\mu\nu} \, T^{\mu\nu}_i (k, k)$ where $T^{\mu\nu}_i (k,k) \sim$ stress-energy tensor of PM field.

$\sim$ " $\nabla k \nabla k + \frac{\sigma}{L^2} k k$ "

$$a_0^{YM} = -\frac{1}{2} F^{\mu\nu}_c \, A^a_\mu \, A^b_\nu \, f^c{}_{ab} \, , \qquad g_{ijk} := g^l{}_{jk} \, g_{il} = g_{(ijk)}$$

$$f_{abc} := f^d{}_{bc} \, g_{ad} = f_{[abc]} \qquad f_{(5)}{}^i{}_{\Delta\Omega} = f_{(7)}{}^i{}_{\Delta\Omega} = a^i{}_{\Delta\Omega} = a^i{}_{(\Delta\Omega)}$$

$$a_1^{Geom} = 2 k^{*\Delta\mu\nu} \left(\nabla_\mu k^\Omega_{\nu\sigma} \, \xi^{i\sigma} + 2 k^\Omega_{\mu\sigma} \nabla_\nu \xi^{i\sigma} - \nabla_\mu h^i_{\nu\sigma} \nabla^\sigma \chi^\Omega + \frac{1}{2} \nabla_\sigma h^i_{\mu\nu} \nabla^\sigma \chi^\Omega - \frac{\sigma}{L^2} h^i_{\mu\nu} \chi^\Omega \right) a_{i\Delta\Omega}$$
$$+ \frac{\sigma}{L^2} (D-3) \, h^{*i\mu\nu} \, k^\Delta_{\mu\nu} \, \chi^\Omega \, a_{i\Delta\Omega} + h^{*i\mu\nu} \, F^\Delta_{\sigma\mu\nu} \nabla^\sigma \chi^\Omega \, a_{i\Delta\Omega} + 2 k^{*\Delta\mu\nu} \, F^\Omega_{\sigma\mu\nu} \, \xi^{i\sigma} \, a_{i\Delta\Omega}$$

3.2) ABELIAN DEFORMATIONS

1) Another type of coupling between $h^{i}_{\mu\nu}$ and $k^{\Delta}_{\mu\nu}$, that is abelian but $a_1^{\text{Non-Geo}} \neq$ trivial

$$\overset{(1)}{\delta} k_{\mu\nu\Delta} = \left(F^{\Omega}_{\sigma\mu\nu}\, \xi^{i\sigma} - \frac{\sigma L^2}{2(D-2)} \nabla_\mu F^{\Omega}_{\rho\sigma\nu}\, \nabla^{[\rho}\xi^{\sigma]i} - \frac{\sigma L^2}{2(D-2)} F^{\Omega}_{\rho\sigma\mu} \nabla_\nu \nabla^{[\rho}\xi^{\sigma]i} \right) b_{i\Delta\Omega}$$

$\delta h_{\mu\nu}$ undeformed. $\overset{(1)}{\delta} h_{\mu\nu} = 0$.

$$a_0^{\text{Non-Geom}} = \frac{1}{2} h^{i}_{\mu\nu} J^{\mu\nu}_{i}(F,F) \qquad \text{where} \quad J^{\mu\nu}_{i} = (F^{\Omega} F^{\Sigma})^{\mu\nu} b_{i\Omega\Sigma}$$

Conserved on-shell $\nabla_\mu J^{\mu\nu}_{i} \approx 0 \quad \Rightarrow$ Forces $b_{i\Delta\Omega} = b_{i(\Delta\Omega)}$

In case of a single PM $k_{\mu\nu}$ field, Y. Zinoviev had obtained the current $J_{\mu\nu}(F,F)$

2) In case only $k^{\Delta}_{\mu\nu}$'s fields involved $\Rightarrow \nexists$ theory! $\rightsquigarrow$ last part of talk.

$$\delta k^{\Delta}_{\mu\nu} = F^{\Omega}_{\sigma\mu\nu} \nabla^{\sigma}\chi^{\Sigma} c_{\Sigma}{}^{\Delta}{}_{\Omega} \;, \qquad a_0^{PM} = k^{\Delta}_{\mu\nu} J^{\mu\nu}_{\Delta} \quad \text{where} \quad b_{i\Delta\Omega} \to c_{\Sigma\Delta\Omega}$$

Requires $D=4$ and $c_{\Sigma\Delta\Omega} = c_{\Sigma(\Delta\Omega)} =: c_{\Sigma,\Delta\Omega}$

3) Between A^a_μ's and $h^i_{\mu\nu}$'s

$$a_0^{G1} = \frac{1}{2} h^i_{\mu\nu} T^{\mu\nu}_{1\,i} \quad , \quad T^{\mu\nu}_{i} = \left(F^{a}_{\mu\sigma} F^{b\sigma}_{\nu} - \frac{1}{4} \bar{g}_{\mu\nu} F^a_{c\sigma} F^{bc\sigma}\right) d_{i,ab}$$

minimal coupling to gravity.

Enforces $d_{i,ab} = d_{i,ba}$ symmetric.

$$\delta A^a_\mu = - F^b_{\mu\nu} \xi^i d_{i,}{}^a{}_b \sim \mathcal{L}_\xi A^a_\mu$$

4) Between A^a_μ's and $k^\Delta_{\mu\nu}$'s

$$a_0^{PM-1} = \frac{1}{2} k^\Delta_{\mu\nu} T^{\mu\nu}_{1\,\Delta} \quad \text{where} \quad T^\Delta_{1\,\mu\nu} = \left(F^a F^a - \frac{1}{4} \bar{g} F^a \cdot F^b\right) e^\Delta{}_{ab} \,.$$

↪ Requires $e_{\Delta ab} = e_{\Delta ba}$ symmetric

3.3) QUADRATIC CONSTRAINTS

Jacobi identity test on $\quad a_2 = \kappa\, a_2^{EH} + \alpha_{PM}\, a_2^{Geom} + g_{YM}\, a_2^{YM}$

And compatibility between gauge algebra and its realisation on the set of fields at hand

$$a_1 = \kappa\, a_1^{EH} + \frac{\kappa}{2}\, a_1^{Geom} + g_{YM}\, a_1^{YM} + \beta\, \bar{a}_1^{Non-Geom} + \omega\, \bar{a}_1^{PM} + \alpha_1\, \bar{a}_1^{G1} + \alpha_{PM-1}\, \bar{a}_1^{PM-1}$$

↳ requires $D=4$

↳ many quadratic constraints, they imply

$$\alpha_{PM} = \frac{\kappa}{2} \quad \text{and} \quad \alpha_1 = \kappa = \frac{\sqrt{2}}{2}\, \alpha_{PM-1} \, ,$$

Mixing of photons and (h^i, k^a) sector :

$$f_{d(a|c}\, d_i{}^d{}_{|b)} = 0 \, , \quad f_{d(a|c}\, e_\Delta{}^d{}_{|b)} = 0 \, , \quad d_{iac}\, d_j{}^c{}_b = g_{ij}{}^k\, d_{kab} \, ,$$

$$e_{\Delta ac}\, e_\Omega{}^c{}_b = a^i{}_{\Delta\Omega}\, d_{iab} \, , \quad d_{iac}\, e_\Delta{}^c{}_b = a_{i\Delta\Omega}\, e^\Omega{}_{ab} \, .$$

Pure $h_{\mu\nu} - k_{\mu\nu}$ sector: $\quad a_{i[\Delta|\Sigma}\, a_j{}^\Sigma{}_{\Omega]} = 0 = a_{[i|\Delta\Sigma}\, a_{j]}{}^\Sigma{}_\Omega$

$$g^k{}_{ij}\, a_{k\Delta\Omega} = a_{i\Delta\Sigma}\, a_j{}^\Sigma{}_\Omega \quad \text{and} \quad a_{i\Delta[\Sigma}\, a^i{}_{\Lambda]\Omega} = 0 \, , \quad a_{i\Sigma[\Delta}\, C_{\Omega],\Gamma}{}^\Sigma \, , \quad 0 = C_{[\Delta|,\Lambda\Sigma}\, C_{|\Omega],\Gamma}{}^\Sigma$$

3.4) ANALYSIS OF THE CONSTRAINTS

We assume each sector separately has same sign of kinetic terms:

$$g_{ij} = \delta_{ij} \quad , \qquad g_{\Delta\Omega} = \pm\, \delta_{\Delta\Omega} \quad , \qquad g_{ab} = \pm\, \delta_{ab}$$

↳ Rederive the known result that • $f^a{}_{bc} \sim$ compact semi-simple Lie algebra with δ_{ab} invar. tensor,

• $g_{ijk} = 0$ unless $i = j = k$

$$g^k{}_{ij}\, a_{k\Delta\Omega} = a_{i\Delta\Sigma}\, a_j{}^{\Sigma}{}_{\Omega} \Longrightarrow (a_i\, a_j)_{\Delta\Omega} = g_{iii}\, \delta_{ij}\, (a_i)_{\Delta\Omega}$$

$\Rightarrow (a_i)^{\Delta}{}_{\Omega}$ Projectors (up to rescaling)

$\Rightarrow a_{i\Delta\Omega} = g_{iii}\, \delta_{i\Delta}\, \delta_{i\Omega}$ (no sum over i) $\quad \begin{pmatrix} 0 & & & & & & 0 \\ & \ddots & & & & & \\ & & 0 & & & & \\ & & & 1 & & & \\ & & & & 0 & & \\ & & & & & \ddots & \\ 0 & & & & & & 0 \end{pmatrix}$ i^{th} entry

$\Rightarrow$ Massless spin-2 $h^i_{\mu\nu}$'s and PM $k^\Delta_{\mu\nu}$'s come in same number.

We can group them in pairs: $a_{1,11}$, $a_{2,22}$, etc $(h^1_{\mu\nu} \leftrightarrow k^1_{\mu\nu}), \dots, (h^N_{\mu\nu} \leftrightarrow k^N_{\mu\nu})$ interactions

1) Then, restricting to one of the pairs $(h_{\mu\nu}, k_{\mu\nu})$ and discarding the non-geometric coupling, $\beta = 0$ $\left(\rightsquigarrow a_0^{\text{Non. Geom}} = h_{\mu\nu} J^{\mu\nu}(F,F)\right)$

↳ recovers conformal gravity as non-linear theory of massless spin-2 $h_{\mu\nu}$ coupled to $k_{\mu\nu}$ PM. Only in 4D as $C_{\Lambda,\Delta\Omega}$ is only $D=4$.

requires opposite sign of kinetic terms.

- In case $\beta \neq 0$, no general solution of constraints.

- For spin-1 mixing

2) Coupling of spin-1 fields to one PM requires

(i) $D=4$

(ii) presence of a massless spin-2 to couple to Yang-Mills

↳ reconstructs YM coupled to conformal gravity

④ . $\boxed{\text{A theory for multiple PM spin-2 fields}}$

$\hookrightarrow \delta\, \varphi_{\mu\nu} = \nabla_\mu \nabla_\nu \epsilon - \sigma \lambda^2 \bar{g}_{\mu\nu}\, \epsilon \quad \rightsquigarrow \quad \delta\, \underset{\varphi}{\widetilde{\square\square}} = \widetilde{\boxed{\triangledown}\boxed{\triangledown}}\; \underset{\epsilon}{\bullet} \quad , \quad K \sim \begin{smallmatrix}\square\square\\ \square\end{smallmatrix}$

__Observations__ : $m^2 \lambda^2$ is small, so is λ if $g_{\mu\nu} = \varphi_{\mu\nu}$ PM $(m^2 = 4)$

- Several endeavours to find a consistent theory of non-linear PM spin-2 fields [Y. Zinoviev 2006, C. de Rham - S. Renaux-Petel 2012, S.F. Hassan, A. Schmidt-May, M. von Strauss 2012, S. Deser, E. Joung and A. Waldron 2012, Deser - Sandora - Waldron 2013, E. Joung, K. Mkrtchyan and G. Poghosyan 2019]

$\Rightarrow \exists$ 2-derivative (cubic) vertex for a single PM field in 4D [Y. Zinoviev 2006]

however it does not admit any consistent higher-order completion.

• A systematic analysis about a possible non-abelian deformation of PM spin-2 theory was suggested by the comment

> [26] In fact, since $s = 2$, $d = 4$ PM is gauge invariant, propagates on light cone [14], is conformally [15] and duality invariant [16], and couples consistently to charged matter, it might make more sense to search for non-abelian Yang–Mills-like interactions.

in [Deser - Joung - Waldron 2012] & [S. Deser - M. Sandora - A. Waldron 1301.5621]

• For a set of PM spin-2 [S. Garcia-Saenz, K. Hinterbichler, A. Joyce, E. Mitsou & R.A. Rosen 2015] show that there is NO non-abelian deformation $\boxed{\overset{(1)}{\delta}_\epsilon \varphi_{\mu\nu} = \vec{R}_{\mu\nu}(\epsilon)}$

with assumptions on # derivatives (max. 2) and taking $\vec{R}_{\mu\nu}$ linear in [h].

Felt the necessity to revisit this problem with more powerful methods

↳ BRST – BV from [G. Barnich & M. Henneaux 1993] : cohomological reformulation of [Berends – Burgers – van Dam 1985]

since the no-go result of [S. Garcia-Saenz, K. Hinterbichler, A. Joyce, E. Mitsou & R.A. Rosen 2015]

does not rule out non-abelian gauge algebras starting at higher orders in φ,

nor does it rule out transformations with more (than 2) derivatives.

→ We find that the abelian PM symetry admits no nonabelian deformation without any assumption on order of $\vec{R}_{\mu\nu}$ in $[\varphi]$ nor in the number of derivatives.

• Revisiting these analyses in the BV BRST-cohomological formulation

Start from $S_0[\varphi^a_{\mu\nu}] = -\frac{1}{4}\int d^n x \sqrt{-\bar{g}}\, k_{ab}\left[F^{a\,\mu\nu|\rho}F^b{}_{\mu\nu|\rho} - 2\,F^{a\,\mu}F^b{}_\mu\right]$

$F^a_{\mu\nu|\rho} := 2\,\nabla_{[\mu}\varphi^a_{\nu]\rho}$ curvature for PM

$\overset{(0)}{\delta}_\epsilon S_0 = 0$ under $\overset{(0)}{\delta}_\epsilon \varphi^a_{\mu\nu} = \nabla_\mu\nabla_\nu\epsilon^a - \sigma\lambda^2\,\bar{g}_{\mu\nu}\,\epsilon^a$

1) We prove that the most general deformation of the gauge algebra:

$$[\delta_{\epsilon_1}, \delta_{\epsilon_2}]\,\varphi^a_{\mu\nu} = \overset{(0)}{\delta}_\chi\,\varphi^a_{\mu\nu} \qquad (\text{off-shell})$$

where $\chi = (m^a{}_{bc}\,\epsilon_1^b\,\epsilon_2^c + n^a{}_{bc}\,\nabla^\mu\epsilon_1^b\,\nabla_\mu\epsilon_2^c)$ → no field dependence

2) Consistency requires $m^a{}_{bc} = 0 = n^a{}_{bc}$ ⇒ Abelian

3) We prove that there are NO higher-order corrections!

4) Deformation of gauge symmetry (but abelian g), if 2 ∂'s :

Consistency gives only (out of 6 candidates)

$$\boxed{\overset{(1)}{\delta}_{\epsilon}\, \varphi^{a}_{\mu\nu} = \alpha\, f^{a}{}_{b,c}\, F^{b}{}_{e(\mu|\nu)}\, \nabla^{e} \epsilon^{c}}$$

, only in $D = 4$.

5) Corresponding cubic vertex with 2 ∂'s : $S_1 = \int d^4x \sqrt{-\bar{g}}\, \varphi^{a}_{\mu\nu}\, J_{a}^{\mu\nu}$

where $J_{a}^{\mu\nu} = f_{bc,a} \left[F^{b\,\mu}{}_{e\sigma} F^{c\,\nu e\sigma} - \frac{1}{4} \bar{g}^{\mu\nu} F^{b\,e\sigma\lambda} F^{c}{}_{e\sigma\lambda} + \text{improvements} \right]$

$\Rightarrow$ # independent deformation : $\frac{1}{2} N^2 (N+1) \rightsquigarrow f_{ab,c} \sim \boxed{a \mid b} \otimes \boxed{c}$

$\longrightarrow$ Uniqueness result (existence not new)

- <u>Conservation</u> : Obviously $\nabla_\mu \nabla_\nu J^{\mu\nu}_a - \frac{\sigma}{L^2} \bar{g}_{\mu\nu} J^{\mu\nu}_a \approx 0$

but also, since $D=4$: $\boxed{\nabla_\mu J^{\mu\nu}_a \approx 0}$

$\Longrightarrow \mathcal{J}^\mu{}_{ab} := \sqrt{-\bar{g}}\, J^{\mu\nu}_a \nabla_\nu \bar{\epsilon}_b$ Noether current $\partial_\mu \mathcal{J}^\mu{}_{ab} \approx 0$ in 4D

rigid symmetry $\delta \varphi^a_{\mu\nu} = f^a{}_{b,c} K^b{}_{e(\mu|\nu)} \nabla^e \bar{\epsilon}^c$

Killing

6) Higher-order consistency :

Provided $f_{ae,b} f^e{}_{c,d} = 0$ (1) & $f_{ab,e} f^e{}_{c,d} = 0$ (2)

$S := S_0 + S_1$ fully consistent to all orders (!) .

But (1) & (2) non-trivial solution only if $k_{ab} \not> 0$

i.e. "wrong" relative signs.

$\Rightarrow$ First consistent interacting theory for PM spin-2 .

- Analogous to (but not all obtainable from) conformal gravity
  and its multi-conformal graviton extensions [N.B., M. Henneaux 2001]
  & [N.B., P. Henneaux, P. van Nieuwenhuizen 2002]

- Possibility that coupling to gravity or more general
  Einstein background might cure unitarity issue.